PHYSICS AND NARRATIVE

I'm not going to attempt anything along the lines of a summing up of what Hilary has meant, and what he continues to mean, to my own particular corner of philosophy here. But it needs at least to be mentioned - on an occasion like this - that Hilary has thought about the foundations of physics harder, and longer, and more deeply, and with more openness, and with more wonder, and with more determination, and with more courage, than anyone now living. Sidney Morgenbesser used to say that Hilary was the most quantum-mechanical philosopher in the world, because he and his philosophical position could not simultaneously be identified. And that (as with all of Sidney's jokes) is exactly right. Nobody, to this day, is as young, and as curious, and as willing to be surprised, and as ready to turn his back on everything he has ever believed, and as full of the exuberant expectation of the impossible, as Hilary. And this is how to learn about the world.

And I want to do something in that spirit here. There is a topic that Hilary and I have been talking and talking and talking to one another about for decades, and about which we have both lately been surprised, about which we have both lately come to understand that we never really knew anything at all. And that's what I want to talk about today.

I'm going to start of with some mildly technical remarks about the behaviors of quantum-mechanical wave-functions under Lorentz transformations. But bear with me just a bit - I think something of what's at stake in these considerations, I think that something of what's philosophically interesting in these considerations, will come through well enough later on,

whether you're able to follow the quantum mechanics or not.

Consider a system of four distinguishable quantum-mechanical spin-1/2 particles. Call it S. And suppose that the complete history of the motions of those particles in position-space - as viewed from the perspective of some particular Lorentz-frame K - is as follows: Particle 1 is permanently located in the vicinity of some particular spatial point, and particle 2 is permanently located in the vicinity of some other spatial point, and particles 3 and 4 both move with uniform velocity along parallel trajectories in space-time! The trajectory of particle 3 intersects the trajectory of particle 1 at space-time point P - as in figure 1 - and the trajectory of particle 4 intersects the trajectory of particle 2 at space-time point Q. And P and Q are simultaneous, from the perspective of K.

And suppose that the state of the <u>spin</u> degrees of freedom of S, at $t = -\infty$, is $[\phi>_{12} [\phi>_{34},$ where

$$[\varphi\rangle_{AB} = 1/\sqrt{2}[\uparrow\rangle_{A}[\downarrow\rangle_{B} - 1/\sqrt{2}[\downarrow\rangle_{A}[\uparrow\rangle_{B}.$$
(1)

I want to compare the effects of two different possible <u>Hamiltonians</u> on this system. In one, S evolves freely throughout the interval from $t = -\infty$ to $t = +\infty$. The other includes an impulsive contact interaction term that <u>exchanges spins</u> - a term (that is) which is zero except when two of the particles occupy the same point, and which (when it <u>isn't</u> zero) generates precisely the following unitary evolution:

$$[\uparrow >_{A}[\downarrow >_{B} \rightarrow [\downarrow >_{A}[\uparrow >_{B}$$

$$[\downarrow >_{A}[\uparrow >_{B} \rightarrow [\uparrow >_{A}[\downarrow >_{B}$$

$$[\uparrow >_{A}[\uparrow >_{B} \rightarrow [\uparrow >_{A}[\downarrow >_{B}$$

$$[\downarrow >_{A}[\downarrow >_{B} \rightarrow [\downarrow >_{A}[\downarrow >_{B}$$

$$(2)$$

A minute's reflection will show that the entire history of the quantum state of this system, from the perspective of K - the complete temporal sequence (that is) of the instantaneous quantum-mechanical <u>wave-functions</u> of this system, even down to the overall phase, from the perspective of K - will be <u>identical</u> on these two scenarios. On both scenarios (that is) the state of S, from the perspective of K, throughout the interval from $t = -\infty$ to $t = +\infty$, will be precisely $[\phi>_{12} [\phi>_{34}]$.

And what's interesting is that the situation is altogether <u>different</u> from the perspective of every <u>other</u> frame. On the first scenario - the scenario in which S evolves freely - the state of S is going to be precisely $[\phi>_{12} [\phi>_{34}, \text{ in every frame, at every time, throughout the interval from t' = <math>-\infty$ to $t' = +\infty$.² But on the <u>second</u> scenario, when viewed from the perspective of frames other than K, the interactions at P and Q occur at <u>different times</u>. In those other frames, then, at all times throughout the interval between P and Q, the state of S is going to be $[\phi>_{14} [\phi>_{23}]$.

And it follows immediately that the complete temporal sequence of the quantum states of S in frames other that K cannot be deduced, either by means of the application of a geometrical space-time point-transformation or in any other way, from the complete temporal sequence of the quantum states of S in K - because transformation in question would need (per impossible!) to map precisely the same history in K into one of two entirely distinct histories in K', depending on

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All of this is as easy as can be. And all of it has been taken note of, on a number different occasions, in the literature of the foundations of quantum mechanics. It was pointed to in a 1984 paper by Yakir Aharonov and myself - for example - and in a paper by Wayne Myrvold from 2002, and it must at least have occurred in passing to a great many people.³ But nobody seems to have been able to look it straight in the face, nobody seems to have entirely taken it in.⁴

Let's back up (then) and slow down, and see if we can figure out what it means.

Call a world <u>narratable</u> if the entirety of what there is to say about it can be presented as a single <u>story</u>, if the entirety of what there is to say about it can be presented as a <u>single temporal</u> <u>sequence</u> of <u>instantaneous global physical situations</u>.

The possible worlds of Newtonian Mechanics can each be presented, in its entirety, by means of a specification of the local physical conditions at every point in a four-dimensional manifold. And there is a way of slicing that manifold up into a one-parameter collection of infinite three-dimensional hyperplanes such that the dynamical significance of the parameter in question - the dynamical role of the parameter in question - is precisely that of a time. A Newtonian-Mechanical instantaneous global physical situation, then, is a specification of the local physical conditions at each one of the points on any particular one of those infinite three-dimensional hyperplanes. And since all of those instantaneous global Newtonian-Mechanical physical situations taken together amount - by construction - to a specification of the local physical conditions at every point in the manifold, the possible worlds of Newtonian Mechanics

are invariably <u>narratable</u>. Moreover, they are <u>uniquely</u> narratable, in the sense that the number of different ways of slicing the manifold up in such a way as to <u>satisfy</u> the conditions described above - in a Newtonian-Mechanical world - is invariably, precisely, <u>one</u>.

The possible worlds of Non-Relativistic Quantum Mechanics can each be presented, in its entirety, by means of a specification of the values of a real two-component field - a specification, that is, of the quantum-mechanical wave-function - at every point in a 3N+1 dimensional manifold (where N is the number of particles in the world in question). And there is a way of slicing that manifold up into a one-parameter collection of infinite 3N-dimensional hyperplanes such that the dynamical role of the parameter in question is precisely that of a time. A Non-Relativistic Quantum-Mechanical instantaneous global physical situation, then, is a specification of the local physical conditions at each one of the points on any particular one of those infinite 3N-dimensional hyperplanes. And since all of those instantaneous global Non-Relativistic Quantum-Mechanical physical situations taken together amount to a specification of the local physical conditions at every point in the manifold, the possible worlds of Non-Relativistic Quantum-Mechanics are invariably narratable. And the narratability here is again unique, in the sense that the number of different ways of slicing the manifold up in such a way as to satisfy the conditions described above is invariably, precisely, one.

The possible worlds of <u>Classical Relativistic Maxwellian Electrodynamics</u> - just like those of Newtonian Mechanics - can each be presented, in its entirety, by means of a specification of the local physical conditions at every point in a four-dimensional manifold. And there is, again, a way of <u>slicing that manifold up</u> into a one-parameter collection of infinite three-dimensional hyperplanes such that the dynamical significance of the parameter in question is

precisely that of a <u>time</u>. And so a Classical Relativistic Maxwellian instantaneous global physical situation is a specification of the local physical conditions at each one of the points on any particular one of those infinite three-dimensional hyperplanes. And since all of those instantaneous global Classical Maxwellian physical situations taken together amount to a specification of the local physical conditions at every point in the manifold, the possible worlds of Classical Relativistic Maxwellian Electrodynamics are narratable. But in <u>this</u> case the narratability is manifestly <u>not</u> unique - Classical Relativistic Maxwellian Electrodynamics is (rather) <u>multiply</u> narratable. In the case of Classical Relativistic Maxwellian Electrodynamics, each different <u>Lorentz-frame</u> is plainly going to correspond to a different way of slicing the manifold up in so as to satisfy the conditions described above.

But Relativistic <u>Quantum</u> Theories are an altogether different matter. In both the non-relativistic and the relativistic cases, an instantaneous Quantum-Mechanical <u>state of the world</u> - an instantaneous Quantum-Mechanical <u>global physical situation</u> - is a specification if the expectation-values of all of the local and non-local quantum-mechanical observables that refer exclusively to the time in question. And the lesson of the example we went through above is that the entirety of what there is to say about a Relativistic Quantum-Mechanical world can <u>not</u> be presented as a one-parameter family of situations like that. The lesson of the example we went through above (more particularly) is that any one-parameter family of situations like that is necessarily going to leave the expectation-values of non-local Quantum-Mechanical observables that refer to several <u>different</u> times - the expectation-values of non-local Quantum-Mechanical observables (that is) which are instantaneous from the perspective of <u>other</u> Lorentz-frames - unspecified. In order to present the entirety of what there is to say about a Relativistic Quantum-

Mechanical world, we need to specify, <u>separately</u>, the Quantum-Mechanical state of the world associated with <u>every seperate space-like hypersurface</u>. If the theory is to be relativistic in the sense of Einstein, in the sense of Minkowski, nothing less is going to do.

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The relationship between the quantum-mechanical states of the world associated with any set of space-like hypersurfaces and the quantum-mechanical states of the world associated with any other set of space-like hypersurfaces is therefore, invariably, a matter of dynamical evolution - even (for example) if one of those sets happens to be the complete family of equal-time hyperplanes for K and the other one of those sets happens to be the complete family of equal-time hyperplanes for K'.⁶

Moreover, the elementary <u>unit</u> of dynamical evolution here is plainly not an infinitesimal <u>translation in time</u> (which is generated by the global Hamiltonian of the world H, as in figure 2a) but an arbitrary infinitesimal <u>deformation</u>, an arbitrary infinitesimal <u>undulation</u>, of the <u>space-like</u> <u>hypersurface</u> (which is generated by the <u>local Hamiltonian density</u> of the world δ H, as in figure 2b).

The dynamical laws of the evolutions of relativistic quantum-mechanical systems therefore have a much richer mathematical structure than the laws of the evolutions of <u>non-relativistic quantum-mechanical systems</u> do. Suppose (for example) that we should like to calculate the wave-function of some particular isolated quantum-mechanical system on hypersurface b, given the wave-function of that system on some <u>other</u> hypersurface a - where a may be either in the past of b or in its future. In the <u>non-relativistic case</u>, which is depicted in figure 3a, there is always exactly <u>one</u> continuous one-parameter family of hypersurfaces - the

continuous one-parameter family of <u>absolute simultanieties</u> between a and b - along which a calculation like that is going to have to <u>proceed</u>, along which the system in question can be pictured as <u>evolving</u>. In the relativistic case, on the other hand, there are invariably an <u>infinity</u> of continuous one-parameter families of space-like hypersurfaces along which such a calculation can proceed, and along which the system in question can be pictured as evolving. Two such families are displayed in figures 3b and 3c.

If the laws of the evolution of the system in question can be written down in a mathematically consistent way, then the calculation that proceeds along the route pictured in figure 3b and the calculation that proceeds along the route pictured in figure 3c, so long as they both start out with precisely the same wave-function at a, will both necessarily produce precisely the same wave-function at b. Moreover, if the laws can be written down in an entirely Lorentz-invariant language, then the wave-functions associated with any two space-like hypersurfaces will invariably generate identical expectation-values for all local observables at points at which those two hypersurfaces happen to intersect.

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Here are two stories - which pull in very different directions - about where all this might leave us:

STORY 1:

This story is about the <u>collapse</u> of the wave-function. And it will work best - for the moment - to tell it in the language of the old-fashioned and idealized and unscientific and altogether outmoded postulate of collapse on which collapses are brought about by means of the

intervention of localized, external, un-quantum-mechanical <u>measuring-devices</u>. On this picture, collapses involve a discontinuous and probabilistic projection of the wave-function of the <u>measured</u> system, the <u>quantum-mechanical</u> system, onto an eiginfunction of some particular one of its <u>local observables</u> (the observable, that is, which the external device in question is designed to measure) at some particular <u>space-time point</u> (the so-called 'measurement-event' - the point at which the measured system <u>interacts</u> with the measuring-device). The probability of a projection onto this or that particular eigenfunction of the measured observable is determined, in the familiar way, by the Born rule.

On the <u>non-relativistic</u> version of the collapse postulate (which is depicted in figure 4a) the collapse occurs as the 'now' sweeps forwards across the measurement-event - the collapse (that is) affects the wave-function of the system in question in the <u>future</u> of that event, but not in its <u>past</u>. And twenty or so years ago, I wrote a paper with Yakir Aharonov which proposed a manifestly Lorentz-invariant <u>relativistic</u> version of that postulate (which is depicted in figure 4b) on which the collapse occurs as an <u>undulating space-like hypersurface</u>, <u>any</u> undulating space-like hypersurface, <u>deforms</u> forwards across the measurement-event - on which (that is) the collapse affects the wave-function of the system in question on those space-like hypersurfaces that intersect the <u>future light-cone</u> of measurement-event, but not on those space-like hypersurfaces that intersect its <u>past light-cone</u>.

Suppose (then) that we are given the wave-function of some isolated relativistic quantum-mechanical system S along some space-like hypersurface a, and suppose that we are given the addresses of all of the space-time points in the future of a at which measurements of local observables of S are to be carried out, and suppose that we are told what particular local

observable of S each particular one of those measurements is to be a measurement of. The relativistic postulate of collapse just described - together with the deterministic laws of the ordinary dynamical evolutions of the wave-functions of isolated relativistic quantum-mechanical systems under infinitesimal deformations of the space-like hypersurface - will assign a definite probability to any particular assignment of outcomes to those measurements, and it will assign a definite probability to any particular assignment of a quantum-mechanical wave-function to any particular space-like hypersurface b which is entirely in the future of a, and (moreover) it will do both of those things uniquely - completely independent (that is) of which one of the above-mentioned routes the calculation of those probabilities take.

The <u>trouble</u> here - or so I imagined until now - is that the possible worlds of this sort of a theory aren't going to be <u>narratable</u>. Suppose (for example) that the momentum of a free particle is measured along the hypersurface marked t=0 in figure 5, and that later on a collapse leaves the particle localized at P. Then the projection-postulate that Aharonov and I proposed is going to stipulate (among other things) that the wave-function of the particle along hypersurface h is an eigenstate of momentum, and that the wave-function of the particle along hypersurface j is (very nearly) an eigenstate of <u>position</u>. And so the quantum-mechanical wave-functions associated with hypersurfaces h and j, in this example, are going to disagree with one another even about the expectation-values of <u>local</u> quantum-mechanical observables at points like Q, where they intersect.⁷ And that (of course) puts narratability quite decisively out of the question.

But a case might be made that the example we went through at the outset of this paper sheds a very different light on all this. We can now see - it might be argued - that the narratability of Relativistic Quantum Theories is dead before the measurement problem ever even

comes up, before the non-locality that Bell discovered ever even enters the picture. Adding a postulate of the collapse of the wave-function to a Relativistic Quantum Theory, on this view, solves the measurement problem, and costs <u>nothing</u>. The Lorentz-invariance of the theory is preserved perfectly intact, and as for the failure of <u>narratability</u>, <u>that</u> price turns out to have been <u>paid</u>, unbeknownst to us, long before the question of measurement ever arose.

If all this is right, then many-worlds and many-minds and many-histories theories have no advantage whatever - in so far as questions of Lorentz-invariance are concerned - over collapse theories. The Lorentz-invariance of many-worlds and many-minds and many-histories theories comes, after all, at the price of a failure of narratability - just as that of collapse theories does.

Moreover, there is now reason to hope that these considerations may turn out <u>not</u> to depend all that sensitively on the unrealistic idealizations of the measurement-process I described a few paragraphs back. A talented young German physicist named Roderich Tamulka has recently published a fully relativistic version of the GRW collapse-theory for non-interacting particles - a theory (as it turns out) that fits around the schematic general principles that Yakir and I laid out 20 years ago like skin. It still remains - and it may turn out to be a highly non-trivial business - to generalize Tamulka's theory to the case of interacting particles and to fields. We shall have to wait and see. But what Tamulka has already accomplished represents an immense and encouraging step in the right direction.

STORY 2:

This is a story about the linear, unitary, deterministic evolution of the wave-functions of quantum-mechanical systems, altogether un-adorned by any mechanism of collapse. Consider a

Relativistic Quantum-Mechanical world W in which the free Hamiltonian of a certain pair of electrons is identically zero, and in which the wave-function of that pair, along every space-like hypersurface whatever, is precisely the wave-function $[\phi>_{12}$ of equation (1). And let $t'=\alpha$ be a flat space-like hypersurface all of whose points are simultaneous with respect to some particular Lorentzian frame of reference K'. And imagine an experiment designed to measure and record the total spin of that pair of electrons along $t' = \alpha$. The experiment involves two localized pieces of apparatus, which have previously been brought together, and prepared in a state in which certain of their internal variables are quantum-mechanically entangled with one another, and then separated in space. One of those pieces of apparatus then interacts with particle 1 at point L (in figure 6) and the other interacts with particle 2 at point Q. And the positions of the relevant pointers on those two pieces of apparatus, at the conclusions of those interactions, are measured, and the values of those positions are transmitted to F, and those values are mathematically combined with one another in such a way as to determine the outcome of the measurement of the total spin of the pair of electrons along $t' = \alpha$, and (finally) that outcome is recorded, in ink (say), in English, on a piece of paper, at G.8 No such experiment is actually carried out in W - mind you - but it is a fact about W that if such an experiment were to have been carried out, it would with certainty have been recorded at G that the total spin of that pair along $t' = \alpha$ was zero.

Now, the most obvious and most straightforward way of <u>accounting</u> for that fact, the most obvious and most straightforward way of <u>explaining</u> that fact, is to point out (1) that the state of the electron-pair, along the hypersurface $t' = \alpha$, is $[\phi>_{12}$, and (2) that $[\phi>_{12}$ is an eigenstate of the total spin of that pair, with eigenvalue zero, and (3) that a <u>measurement</u> of the total spin of that pair along $t' = \alpha$ - if it had been carried out - would therefore, with certainty, have found that the

total spin of that pair <u>is</u> zero. Note that this explanation depends only on the <u>state</u> of the pair of electrons at $t' = \alpha$, and not <u>at all</u> on the <u>dynamical laws</u> by which that state <u>evolves</u>.

But another explanation - or rather a continuous infinity of other explanations - have plainly got to be available as well. If (for example) we trace out the development of the world exclusively along the continuous one-parameter family of hypersurfaces of simultaniety in \underline{K} , the experiment in question is going to look not so much like an instantaneous measurement as an extended sequence of <u>dynamical interactions</u>. At t=0, state of the electron-pair is $[\phi>_{12}]$, and the pair of apparatuses are in the specially prepared quantum-mechanically entangled state - call it [Θ > - alluded to above. Then - at L- electron 1 interacts with one of the localized pieces of apparatus, and this interaction leaves the electron-pair quantum-mechanically entangled with the pair of apparatuses. Then - at Q - electron 2 interacts with the other localized piece of apparatus in precisely such a way as to undo that latter entanglement - leaving the electron-pair once again in the state $[\phi>_{12}]$ and the pair of apparatuses once again in the state $[\Theta>]$. Thereafter, the various transmitters and receivers and compilers and recorders go to work, and the end-product of all this activity - the end-product (that is) which is entailed with certainty by the state of the world along t=0 and the deterministic quantum-mechanical equations of motion, no matter which continuous one-parameter family of space-like hypersurfaces the intervening calculation traces through - is a sheet of paper at G, bearing the inscription "total spin equals Zero".

This particular experiment's having this particular outcome, then, can be given a complete and satisfactory and deterministic explanation which traces out the development of the world exclusively along the continuous one-parameter family of hypersurfaces of simultaniety in K, and which makes no mention whatsoever of the state of the pair of electrons - or of anything

else - at t'= α .

And we are plainly going to be able to produce very much the same sort of an explanation - very much the same <u>continuous infinity</u> of explanations - of the outcome any hypothetical experiment whatsoever.

And this suggests a way of picturing relativistic quantum-mechanical worlds - for a price - as narratable. All that needs to be given up is the Einsteinian insistence that the unfolding of the world in every separate Lorentz-frame and along every continuous one-parameter family of space-like hypersurfaces all be put on an equal metaphysical footing. Suppose - on the contrary - that it is stipulated that an assignment of a quantum state of the world to every one of the hypersurfaces of simultaniety of (say) K - and to no other space-like hypersurfces - amounts in and of itself to a complete and exhaustive and unaugmentable account of the world's history. Then there would be no facts at all about the 'state of the world' along, say, t'=\alpha. And all talk of such 'facts' in the physical literature would need to be re-interpreted as shorthand for counterfactual talk about how this or that hypothetical experiment - if it were to be performed - would come out. The world would be narratable - and (moreover) uniquely so.

On this way of thinking, the impulse away from an <u>Einsteinian</u> understanding of special relativity - the impulse (that is) towards a <u>Lorentzian</u> understanding of special relativity - would arise not (in the first instance) from the <u>non-locality</u> of the <u>collapse</u>, but <u>earlier</u> and <u>farther down</u>, from <u>the geometry of the Hilbert space</u> and the demand for <u>narratability</u>. And the way would seem to be open to trying out new fundamental theories of the world which violate Lorentz-invariance - a little bit - even in their empirical predictions.

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- 1. This sort of permanent localization can be accomplished, say, by placing the particles in boxes, or by making their masses large.
- 2. I am going to be supposing, throughout, that the velocities of these other frames with respect to K are small compared to the speed of light, so that the effects of Lorentz-transformations on the spins can be neglected. The effects of transforming to other frames that are going to interest us here can all be made as large as one likes, even at small relative velocities, by separating the two particles from one another by a great spatial distance.
- 3. The example presented here, however, is a good deal cleaner and more perspicuous than either the one discussed by Aharonov and I in 1984 or the one discussed by Myrvold in 2002. The example cited in the paper by Aharonov and myself involves <u>measurement</u>-type interactions, and the one Myrvold presents involves an external field that violates Poincare-invariance. Neither of those sorts of distractions come up, however, in the example presented here.
- 4. What Yakir and I had to say about it in the 1984 paper was that in so far as frame K is concerned, the interaction "disrupts (as it were) the *transformation* properties of the state and

disrupts its *covariance*, without in any way disrupting the history of the state itself". But precisely how it is that the <u>transformation</u> properties of something can be disrupted without <u>in any way</u> disrupting <u>the history of the thing itself</u> I confess I can no longer imagine. It seems panicked - looking back on it now - and incoherent, and mad.

Professor Myrvold (on the other hand) thinks it shows that the Lorentz-transformation of quantum-mechanical wave-functions is not so much a <u>geometrical</u> or even a <u>kinematical</u> matter as it is a matter of <u>dynamics</u>, a matter of the <u>Hamiltonian</u> of the system whose wave-function is being transformed. According to Professor Myrvold, the business of performing a Lornetz-transformation on the complete temporal history of the wave-function of an isolated system is in general going to require that we know, and are able to solve, the system's <u>dynamical equations of motion</u>. But if we go <u>that</u> route, nothing whatever is going to remain of the intuition that carrying out such a transformation is merely a matter of looking at <u>precisely the same set of physical events</u> from two different <u>perspectives</u>, from two different <u>points of view</u>. Dynamics - after all is not the business of changing one's perspective on already <u>existing</u> events, but of generating entirely <u>new</u> ones!

5. It means a host of things, by the way, to speak of the parameter in question here as "playing the dynamical role of a time". It means (for example) that the trajectory of every particle in the world intersects every one of the three-dimensional hyperplanes in question here exactly once, and it means that the total energy on any one of these hypersurfaces is the same as the total energy on any other one of them, and it means (principally and fundamentally and in sum) that the equation

$$F = m(\partial^2 x/\partial \rho^2),$$

where ρ is the parameter in question, is true.

- 6. In this respect, then, Professor Myrvold (see note 4) is perfectly right. Where Myrvold goes wrong is in imagining that a relationship like that is consistent with the claim that an assignment of a quantum state of the system in question to every one of the equal-time hyperplanes of K can amount to a complete <u>history</u> of that system where he goes wrong (that is) is in imagining that a relationship like that can leave the world narratable.
- 7. Note, however, that the expectation-values of all local observables at Q given the state along t=0 will still be completely independent of the route by which one chooses to calculate from t=0 to Q. On certain routes (for example) Q is going to come up as an element of h, and on certain others it will come up as an element of j. If Q comes up as an element of h, then the expectation-values of all local observables at Q, given the state along t=0, will be determined in the familiar way by the state at h. But if Q comes up as an element of j, then the expectation-values of all local observables at Q, given the state along t=0, will be determined by a probability-distribution over various different possible states at j corresponding to the different possible outcomes of the measurement at P. The Lorentz-invariance of the dynamical equations of motion and the collapse-postulate, however, will guarantee that those two sets of expectation-values will invariably be identical.

- 8. Detailed instructions for the construction and preparation of measuring-apparatuses like these using only local interactions can be found in an old paper of Yakir Aharonov's and mine...
- 9. The account <u>does</u> depend on the dynamics of the two pieces of <u>measuring-apparatus</u> of course and on the dynamics of the mechanism whereby the positions of the relevant pointers on those two pieces of apparatus are <u>transmitted to F</u>, and on the dynamics of the mechanism whereby those position-values are mathematically combined with one another in such a way as to determine the <u>outcome</u> of the total-spin measurement, and (finally) on the dynamics of the mechanism whereby that outcome is <u>recorded</u> at \underline{G} but it doesn't depend <u>at all</u> on the dynamics of the pair of electrons <u>themselves</u>.