Wigner's Friend and Bell's Field Beables

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Abstract

A field-theoretic version of Wigner's friend (1961) illustrates how the quantum measurement problem arises for field theories. Similarly, considering spacelike separate measurements of entangled fields by observers akin to Wigner's friend shows the sense in which relativistic constraints make the measurement problem particularly difficult to resolve in the context of a relativistic field theory. We will consider proposals by Wigner (1961), Bloch (1967), Helwig and Kraus (1970), and Bell (1984) for resolving the quantum measurement problem for field theory. We will conclude by considering the possibility of giving up rich dynamical explanation in the context of a many-maps formulation of relativistic quantum field theory.

1. Wigner's Friend

When Eugene Wigner wrote his now famous paper (1961) on the dynamical role of observers in quantum mechanics, quantum field theory was flourishing. Wigner believed, however, that a dynamically complete formulation of quantum mechanics required a clear account of how quantum-mechanical states evolve to generate determinate physical records over the course of a measurement. In order to provide such an account, he believed that the dynamics of measurement interactions must be essentially different from that of all other physical interactions.

To explain what was required for a satisfactory dynamical account of quantum measurement, Wigner told a story of a friend F who measures an observable $\mathcal{F}$ of a system S that fails to be in an eigenstate $\mathcal{F}$. Then Wigner considered what it would be like to be the friend and what might happen if a second observer made a subsequent measurement of the composite system $F+S$. Wigner told his story in the context of the standard von Neumann-Dirac formulation of quantum mechanics, but it might easily be translated into the language of field theory.

Choose an inertial frame, and suppose, for simplicity, that the local state of a field $\mathcal{F}$ serves to characterize both the state of system S in spatial region R_S and the state of Wigner's friend F in a contiguous but disjoint spatial region R_F. Let $\psi_0^S(t_0)$ represent a state where the field $\mathcal{F}$ is zero in R_S and let $\psi_+^S(t_0)$ represent a state where the field is nonzero in R_S at time t_0 in the chosen frame. Suppose that the field $\mathcal{F}$ in region R_F is $\chi_r^F(t_0)$, a state that represents that the friend is ready to observe the field value of $\mathcal{F}$ in region R_S. Finally, suppose a Hamiltonian of interaction between the local fields such that when the friend observes the field $\mathcal{F}$ in region R_S, (i) if the state of $\mathcal{F}$ in R_S is $\psi_0^S(t_0)$, then the state of $\mathcal{F}$ in R_F and R_S at time t_1 after the interaction will be $\chi_0^F(t_1)\otimes\psi_0^S(t_1)$, where $\chi_0^F(t_1)$ represents a record in terms of the value of $\mathcal{F}$ in R_F that there was no observed field in R_S; and (ii) if the state of $\mathcal{F}$ in R_S is ψ_+^S, then the state of $\mathcal{F}$ in R_F and R_S will be $\chi_+^F(t_1)\otimes\psi_+^S(t_1)$ after the interaction, where $\chi_+^F(t_1)$ represents a record in terms of the field value of $\mathcal{F}$ in R_F that $\mathcal{F}$ was nonzero in R_S.

Now suppose that the field state in region R_S is a linear superposition of the zero field and the nonzero field $\alpha\psi_0^S(t_0)+\beta\psi_+^S(t_0)$ at time t_0 before the measurement interaction with the field in region R_F. It follows from the unitary dynamics and from the interaction Hamiltonian described above that the state of the field in regions R_F and R_S after the interaction will be $\alpha\chi_0^F(t_1)\otimes\phi_0^S(t_1)+\beta\chi_+^F(t_1)\otimes\phi_+^S(t_1)$ (Wigner 1961, 176). On the other hand, given the standard interpretation of quantum-mechanical states, the state of the field in region R_F after the interaction must be separable from the state of the field in region R_S and must be either $\chi_0^F(t_1)$ or $\chi_+^F(t_1)$ in order for there to be a determinate record of the friend's measurement in terms of his field state.[1] Further, in order for the field theory to be empirical adequate,

[1]On the standard eigenvalue-eigenstate link, there is a determinate value for field observable $\mathcal{O}$ in a local region if and only if the local state of the field is an eigenstate of $\mathcal{O}$. What it takes to have a determinate measurement record in terms of local field states is a more subtle question, but, for present purposes, we will suppose that all it requires is for there to be a determinate local value for whatever field observable the friend in fact

the probability of $\chi_0^F(t_1)$ must be $|\alpha|^2$ and the probability of $\chi_+^F(t_1)$ must be $|\beta|^2$.

So what is the state of the field $\mathcal{F}$ in regions R_F and R_S after the interaction: is it the pure state (1) $\alpha\chi_0^F(t_1)\otimes\phi_0^S(t_1)+\beta\chi_+^F(t_1)\otimes\phi_+^S(t_1)$ predicted by the unitary dynamics or is it (2) $\chi_0^F(t_1)\otimes\psi_0^S(t_1)$ or (3) $\chi_+^F(t_1)\otimes\psi_+^S(t_1)$ with probabilities $|\alpha|^2$ and $|\beta|^2$ respectively? If the dynamics of field theory is unitary, and it will be insofar as it satisfies the standard dynamical constraints of quantum mechanics, the state of the field after the measurement interaction must be the entangled state (1). But insofar as the quantum-mechanical state is supposed to provide a complete physical description, the local field state of region R_F must be described by either (2) or (3) in order for the friend to have a determinate physical record.

Note that, simply in terms of its possible information content, expression (1) cannot by itself succeed in selecting one of the two possible measurement outcomes. Consider the case where $\alpha=\beta=\frac{1}{\sqrt{2}}$. If there is a determinate physical measurement record in this state, then what is it? Here there is nothing in the representation of the physical state that might, even in principle, select a particular physical measurement record as the one that in fact obtains. Note also, that interactions with the environment will only serve to generate further field entanglements, which will do nothing whatsoever to select a particular one of the two possible measurement records as the one that obtains for Wigner's friend. The moral is that insofar as there are determinate physical measurement records, if the quantum-mechanical state is taken to be complete, then field theory gets the dynamics of measurement wrong; and insofar as there are determinate physical measurement records and the unitary dynamics is right, the quantum-mechanical state cannot be complete.

This is the quantum measurement problem. Wigner's proposed resolution of the problem was to stipulate a nonlinear dynamics for measurement interactions.[2] More specifically, Wigner stipulated that a determinate physical record is randomly generated in accord with the standard quantum

correlates to the field observable being measured. See Barrett (2002) for further discussion of measurement interactions in field theory.

[2]Wigner described the problem and his proposed solution in the context of classical, nonrelativistic quantum mechanics, but there is nothing in his description of arguments that is not directly applicable to any formulation of quantum mechanics that postulated a linear dynamics.

probabilities whenever such a record is required to explain the determinate experience of an observer who is conscious of the physical state of the system being measured. And since, by a principle of charity, Wigner believed that his friend must be conscious of a determinate measurement result, he concluded that the state must be either (2) or (3) after the measurement interaction.

One consequence of Wigner's proposal is that the dynamics that describes measurement interactions must be incompatible with the unitary dynamics typically taken for granted in field theory. As Wigner himself put it, "the quantum mechanical equation of motion cannot be linear if the preceding argument is accepted" (1961, 177).

While postulating a violation of the standard dynamics is clearly an unattractive option, the cost of maintaining a local unitary dynamics here would be to suppose that state of the Wigner's friend after the interaction is (1), which, if one assumes that the quantum-mechanical state provides a complete physical description, is not a state where the friend can have a determinate physical record. And this cost was unacceptable to Wigner: "to deny the existence of the consciousness of a friend to this extent is surely an unnatural attitude, approaching solipsism, and few people, in their hearts, will go along with it" (Wigner 1961, 177-178).

But giving the friend determinate records by stipulating a violation of the unitary dynamics involves a potential empirical cost. Wigner recognized that the pure state (1) predicted by the unitary dynamics had different empirical properties than either state, (2) or (3), that might randomly evolve on his stipulated collapse dynamics (1961, 180-181). In particular, there will be an observable $\mathcal{A}$ of the field in regions R_F and R_S that has state (1) as an eigenstate with eigenvalue $+1$ and every state orthogonal to (1) as an eigenstate with eigenvalue -1. So, if the state of the field is (1), then a measurement of $\mathcal{A}$ will always yield the result $+1$; but if the state of the field is (2) or (3), then a measurement of $\mathcal{A}$ will sometimes yield the result -1.

Rather than worry that the predicted violation of linearity might prove empirically false, Wigner characterized such predictions as a virtue of his formulation of quantum mechanics since they allowed one, at least in principle, to empirically determine which systems are conscious by testing which systems in fact cause collapses of the quantum-mechanical state. On the other hand, since Wigner's dynamical proposal has empirical consequences, one might be reluctant simply to stipulate that the standard unitary dynamics is violated during measurement interactions if there might be another way out, especially when it is entirely unclear how the fact that a system is conscious might affect a radical change in its dynamical properties.

Another cost of Wigner's stipulated measurement dynamics is that it is incompatible with the dynamical constraints of relativity. This incompatibility can be seen by considering a story where distant friends make almost simultaneous measurements of an entangled field.

2. Relativistic Considerations

Special relativity denies that there is a physical matter of fact regarding the temporal order of space-like separated events. If events **1** and **2** are space-like separated, then there will be an inertial frame $\mathcal{I}$ where **1** occurs first and another inertial frame $\mathcal{J}$ where **2** occurs first; and since the two inertial frames disagree, there can be no physical matter of fact regarding the temporal ordering of the measurements. That relativity denies that there is a physical matter of fact concerning the temporal order of spacelike separated events provides a straightforward way of showing the incompatibility of Wigner's proposed dynamics and the dynamical constraints of relativity.

Consider two boxes F_1 and F_2 equipped with alarm clocks. The clocks are synchronized, then the alarm on F_1 is set to noon on 1 January 2050, and the alarm on clock F_2 is set to noon plus one minute on the same date. The local field $\mathcal{F}$ in each box is put into an entangled superposition

$$\frac{1}{\sqrt{2}}(|0\rangle_{F_1} \otimes |+\rangle_{F_2} + |+\rangle_{F_1} \otimes |0\rangle_{F_2}), \tag{1}$$

where *0* and + indicate empirically different field configurations.[3] Then the two boxes are carefully moved apart so that the distance from one to the other is greater than one light minute. Friend 1 is instructed to observe the field in F_1 when her alarm rings and Friend 2 is instructed to observe the field in box F_2 when her alarm rings.

[3] On the unitary dynamics such entangled local field states will be ubiquitous. Indeed, it is taken to be a consequence of the Reeh-Schlieder theorem (1961) that local states of a relativistic field must be entangled for every pair of spacetime regions in even the vacuum state. See Schlieder (1965), Redhead (1995a) and (1995b), and Clifton and Halvorson (2000) for discussions of the physical significance of this theorem.

Now consider Wigner's dynamical proposal, and suppose that there is a collapse of the quantum-mechanical state when Friend 1 or Friend 2 observes the value of $\mathcal{F}$ in her respective box. What is the state of $\mathcal{F}$ in the boxes just before Friend 2 observes the field in her box?

Since the measurement events of the two friends are space-like separated, there is an inertial frame where Friend 1 makes her measurement first. In this frame, according to Wigner's proposal, Friend 1's measurement will collapse the state, so the state of the field just before Friend 2 makes her measurement will be either (I) $|0\rangle_{F_1} \otimes |+\rangle_{F_2}$ or (II) $|+\rangle_{F_1} \otimes |0\rangle_{F_2}$ with probability 1/2 for each possible outcome.

But since the measurement events are space-like separated, there is another inertial frame where Friend 2 is the first to measure the field in her box. In this frame, the state just before Friend 2 makes her measurement is (III) $\frac{1}{\sqrt{2}}(|0\rangle_{F_1} \otimes |+\rangle_{F_2} + |+\rangle_{F_1} \otimes |0\rangle_{F_2})$ since Friend 1 has not yet observed the entangled field.

But insofar as states (I), (II), and (III) are mutually incompatible, there can be no physical matter of fact concerning which obtains when Friend 2 observes the field in her box. State (I) describes the field configuration in Friend 2's box as +, state (II) describes the field configuration in Friend 2's box as 0, and state (III) describes a situation where, on the standard interpretation of states, there is no determinate matter of fact about which of the two field configurations obtains. And just as with Wigner's friend, there are experiments that would, at least in principle, empirically distinguish between state (I) or (II) and state (III).

Since Wigner's dynamical proposal requires mutually incompatible field states to obtain for the same spacetime region, it is manifestly incompatible with the constraints of relativity if one assumes that there is a single well-defined state of the field $\mathcal{F}$ in each spacetime region.[4]

[4]This is precisely what is denied by hyperplane-dependent collapse theories. The idea is to render a collapse of the quantum-mechanical state compatible with the constraints of relativity by allowing the physical state of a spacetime region to depend on the inertial frame one chooses. See Albert and Aharonov (1980) and (1981), Fleming (1988), Myrvold (2002) and (2003), and Woodcock (2007) for discussions of this approach. Such formulations of quantum mechanics then allow for the collapse of the quantum-mechanical state at the expense of postulating hyperplane-dependent quantum-mechanical states. It is, however, unclear that much is gained if, instead of explaining why the quantum-mechanical state evolves in a way that is compatible with the dynamical constraints of relativity, one simply denies that there is any frame-independent state that might satisfy such constraints.

3. Direct Records for Field Theory

Since Wigner's proposal involves stipulating a nonlinear collapse dynamics that depends on conscious intervention and is incompatible with relativity, one might hope that there is some other way to provide Wigner's friend with a determinate measurement record. If we cannot account for the friend's physical record, then we cannot account for ours; and insofar as we have good empirical evidence for anything, we presumably have good empirical evidence that we have determinate physical measurement records in pointer positions, marks in notebooks, electromagnetic fields on hard drives, etc.

Perhaps the most direct way to solve the quantum measurement problem for field theory is simply to stipulate local field states that would ensure that there are determinate measurement records precisely where and when one needs them. Rather than introduce a new dynamical law for measurement interactions, here one directly specifies a specific determinate-record state for the field that characterizes Wigner's friend after his measurement interaction. While this strategy is clearly ad hoc and immediately forfeits any dynamical explanation for the production of determinate measurement records, it does guarantee determinate measurement records in terms of the local field state of the friend.[5]

A version of this proposal was considered by I. Bloch (1967). Bloch was concerned with the difficulties one faces in reconciling quantum mechanics and relativity. He understood the problem as one of finding a collapse formulation of quantum mechanics that was both compatible with the constraints of special relativity and would explain the determinate measurement records generated by particle detection experiments. Using particle counters as quantum measuring devices, Bloch explained how one might get a weak sort of compatibility with relativity by supposing collapses to occur along the backward light cones of measurement interactions.[6]

This move renders the dynamical constraints of relativity potentially empty.

[5]This strategy, and spacetime maps of determinate records more generally, are discussed in Barrett (2005b).

[6]Bloch's discussion here is based on the earlier work of Aharonov, Bergmann, and Lebowitz and directly inspired the later work of Hellwig and Kraus (1970). A closely related approach was also suggested by Schlieder (1968). See Hellwig and Kraus (1970, 569) for a description of how Schlieder's spacetime maps are updated. These proposals also foreshadowed relativistic collapse proposals like Aharonov and Albert's (1983) and

Bloch concluded his discussion of relativistic quantum mechanics by describing how one might, if one pleased, define a "teleological wave function" by stipulating the value of the wave function so that it corresponded to whether or not each particle counter was triggered at each particular location in spacetime where a measurement was in fact made (1967, 156). That is, one might construct an empirically adequate *spacetime map* of the quantum-mechanical state by stipulating that the local state in each region of Minkowski spacetime is the corresponding eigenstate of the recording variable wherever and whenever there is in fact a measurement record. One might then complete the spacetime map by filling in local states for all other regions subject to the constraints imposed by the states in those regions where there is a determinate record.

There are several ways one might fill in the gaps. If one used the standard unitary dynamics to determine the local field states outside the determinate-record regions, then the resultant spacetime map might look as if collapses of the wave function had generated the determinate measurement records. Indeed, such a complete spacetime map might be constructed from Bloch's backward light-cone collapse prescription if one knew the result of each actual measurement. But if one knows the result of each actual measurement, there is a sense in which it does not really matter how one completes the spacetime map since one already has the right determinate local measurement records by stipulation. Such a *teleological spacetime map*, constructed from observed measurement records, would clearly be both empirically adequate (by stipulation) and perfectly compatible with special relativity (insofar as the theory is understood as just a map of local field states in Minkowski spacetime).[7]

While Bloch considered the construction of teleological spacetime maps for particle states, the general prescription would clearly also work to provide determinate local field states for all spacetime regions. Wigner's friend, characterized in terms of local field states, would then have a determinate measurement record, not by a special measurement dynamics, but by a direct

subsequent hyperplane-dependent collapse theories.

[7]One might naturally suppose that a formulation of quantum mechanics is compatible with relativity if it can be given without reference to any preferred inertial frame. More specifically, one might suppose it to be sufficient that one be able to a unique local quantum-mechanical state to all regions in Minkowski spacetime. But if this is all one requires, then it is easy (arguably too easy!) to get a formulation of quantum mechanics that is compatible with relativity.

stipulation of the determinate-record field state that obtains for the friend after the measurement interaction. It is the direct stipulation of local field states that makes the map of measurement records teleological. On this approach there is no dynamical explanation of the determinate measurement records. Indeed, such a formulation of field theory is so ad hoc as to fail to have any general rule, dynamical or not, for assigning local field states.

Concerning the construction of such teleological spacetime maps, Bloch concluded that "such a procedure appears to have little to recommend it" (1967, 156). While one cannot help but agree with Bloch's conclusion, given the difficulty of finding a satisfactory dynamical account of the quantum measurement process that is also compatible with the dynamical constraints of relativity, it is perhaps unsurprising to find such teleological constructions throughout the field theory literature.

The preferred practice in field theory, as in quantum mechanics more generally, is to avoid talking about measurement if at all possible.[8] If one does need a determinate local field state corresponding a possible measurement record, the typical practice is to stipulate whatever local state one needs in whatever spacetime region one needs it as a sort of boundary condition, without any dynamical explanation for how the particular local state came to be. The justification for such a practice is practical expedience rather than dynamical understanding.

Hellwig and Kraus (1970) codified this practice by explicitly endorsing precisely the sort of ad hoc teleological construction that Bloch found objectionable. Their proposal amounts to stipulating the corresponding local field state in those spacetime regions where determinate measurement records are in fact found, then filling in the local field states in other regions by applying a collapse along the backward light cone of each measurement event region in spacetime and applying the unitary dynamics everywhere else (1970, 567).

On this proposal, if Wigner's friend finds a determinate field configuration in region R_S, then one stipulates that this particular field configuration in

[8]It is typically taken to be sufficient for the purposes of empirical prediction to derive quantum amplitudes over a set of possible physical events with no discussion of how any particular event is in fact realized. The only passage in Ryder (1996), for example, that has anything to do with the dynamics of measurement is one where he points out that we only know that electrons exist because they interact with other matter via the electromagnetic field (1-2). But this point, of course, has nothing to do with quantum-mechanical considerations.

fact obtains in spacetime region R_S and that the corresponding field record obtains for spacetime region R_F. Similarly, in the relativistic story, if Friend 1 observes a 0 configuration and Friend 2 observes a + configuration when they observe the entangled field in their respective boxes, then one stipulates the corresponding local field states and records. Here one does not have the embarrassment of having to say which observer caused the collapse. Since there is no dynamical account of measurement records, there is nothing that might violate the dynamical constraints of relativity.

But again, while such a teleological construction clearly provides an empirically adequate model for any collection of actual determinate measurement records (by stipulation) and while there is a sense in which the resultant spacetime map of local field states is compatible with the dynamical constraints of special relativity (because there is no dynamical account of the evolution of the local field that determines measurement records), the strategy is clearly ad hoc. Rather than being predicted by the theory, determinate measurement records are put in by hand where and when one needs them. Such a construction guarantees empirical adequacy by stipulation rather than by theoretical success. While the resultant map of local field states may be empirically adequate in the sense of logically entailing the determinate records that were in fact found, the explanation for this predictive success is the most impoverished imaginable. It is only because the map of local field states was reverse-engineered from the actual measurement results that the theory is empirically adequate.

While the teleological strategy is clearly ad hoc, it also effective in its way. In particular, it provides a way to talk about determinate measurement records in the language of quantum field theory whenever such talk is unavoidable. The salient question is whether we can do better than such ad hoc teleological constructions.

One strategy for a principled account of determinate measurement records is to use a hidden-variable dynamics to construct a spacetime map for the determinate records. Here the hidden-variable dynamics is perhaps better understood as part of the characterization of possible physical worlds rather than as providing dynamical explanations for the determinate physical records. The resultant field theory then simply characterizes the set of possible determinate-record maps representing possible physical histories and provides a probability measure over the set representing the epistemic prior probability one should assign to each possible physical world in fact being ours. While such a theory would not provide a rich dynamical account of the production of determinate measurement records, it is clearly less ad hoc than the teleological practice of stipulating determinate field records of the sort one needs in whatever spacetime regions on needs them.

4. Beables for Quantum Field Theory

Bohmian mechanics is easily the most popular hidden-variable formulation of quantum mechanics. On David Bohm's (1952) theory, as characterized by John Bell (1981) and (1982), a complete physical description consists of the standard quantum-mechanical state ψ together with a specification of the always-determinate position Q of each particle. It is the determinate particle configurations relative to the wave function that are supposed to explain one's determinate measurement records in this theory.

According to the theory, the standard quantum-mechanical state always evolves in the standard deterministic unitary way. In the simplest nonrelativistic case, this evolution is described by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi \tag{2}$$

where $\hat{H}$ is the Hamiltonian of the system.

The determinate particle configuration Q also evolves in a deterministic way. For an N particle system, the particle configuration can be thought of as being pushed around in $3N$-dimensional configuration space by the flow of the norm-squared of the wave function (the probability current) just as a massless particle would be pushed by a compressible fluid. More specifically, the motion of the particles is given by

$$\frac{dQ_k}{dt} = \frac{1}{m_k}\frac{\mathrm{Im}(\psi^*\nabla_k\psi)}{\psi^*\psi} \tag{3}$$

evaluated at the current configuration Q, where m_k is the mass of particle k.

Since both the evolution of the wave function and the evolution of the particle configuration are fully deterministic in Bohmian mechanics, in order to get the standard quantum probabilities, one must assume a special statistical boundary condition. The distribution postulate requires that there be a time t_0 where the epistemic probability density for the configuration Q is given by $\rho(Q,t_0) = |\psi(Q,t_0)|^2$. If the distribution postulate is satisfied, one can show that Bohm's theory makes the standard quantum statistical predictions as epistemic probabilities for possible particle configurations.

Bell was a strong and persistent proponent of Bohmian mechanics, but he also realized that the theory was not entirely satisfactory. As Bell noted, "[w]hen the cogency of Bohm's reasoning is admitted, a final protest if often this: it is all nonrelativistic"(1984, 171). This can be seen from the fact that the auxiliary dynamics predicts that the velocity of a particle at a time is typically a function of the simultaneous positions of distant particles. In this sense, Bohmian mechanics is *dynamically* incompatible with relativity.

As a step toward reconciling Bohm's theory and relativity, Bell (1984) showed how to generalize the auxiliary dynamics (3) so that, rather than providing always determinate particle positions, the theory provides determinate local field quantities for all spacetime regions. Bell chose fermion number density as the determinate field quantity since "[t]he distribution of of fermion number in the world certainly includes the positions of instruments, instrument pointers, ink on paper,...and much much more" (1984, 175). The salient point here is that Bell believed that by making fermion density determinate, one makes all actual measurement records determinate. This is at least plausible. If the fermion number density is everywhere determinate, then regardless of whether Wigner's friend records the outcome of his measurement in ink marks in a spiral notebook, or data on a hard drive, or even the positions of calcium ions in his brain, there will be a determinate matter of fact concerning what result he got coded for in the determinate configuration of the fermion density field.

Bell characterization of the field dynamics requires a preferred inertial frame. Let three-space at a time be represented by a finite discrete lattice with points numbered $l = 1, 2, \ldots, L$. Define fermion number operators corresponding to each lattice point with eigenvalues $1, 2, \ldots, 4N$, where N is the number of Dirac fields. The determinate fermion number configuration of the world at a time $n(t)$ then is an assignment of a particular eigenvalue to each lattice point. This is the local beable that replaces determinate particle position for Bell's field-theoretic version of Bohmian mechanics. A complete specification of physical state at time t for Bell then is $(\psi(t), n(t))$, where $\psi(t)$ is the ordinary quantum field state at time t in the preferred frame, and the value of $n(t)$ provides determinate measurement records.

Just as in Bohmian mechanics, the quantum-mechanical state $\psi(t)$ evolves according to the time-dependent Schrödinger equation

$$\partial_t \psi(t) = -i\hat{H}\psi(t), \tag{4}$$

where $\hat{H}$ is the ordinary Hamiltonian. The stochastic evolution of the fermion configuration is characterized by the transition probabilities $T_{nm}dt$ for configuration m jumping to configuration n in interval dt. Let the quantum probability density D_m be defined as

$$D_m(t) = \sum_q |\langle mq|\psi(t)\rangle|^2 \tag{5}$$

and the current J_{nm} defined as

$$J_{nm} = \sum_{q,p} 2 \text{ Re } \langle\psi(t)|nq\rangle\langle nq| - iH|mp\rangle\langle mp|\psi(t)\rangle. \tag{6}$$

Then if $J_{nm} \geq 0$,

$$T_{nm} = J_{nm}/D_m; \tag{7}$$

otherwise, $T_{nm} = 0$. If the distribution postulate is satisfied, that is, if the epistemic probability distribution over possible fermion configurations at some initial time $P_n(t_0)$ is equal to the quantum probability density $D_n(t_0)$, then Bell shows that the transitions probabilities T_{nm} entail the standard quantum statistical predictions for the value of the local field quantity $n(t)$ at all time (Bell 1984, 175-9).[9]

Bell had two worries about his local beable field theory. The first was that choosing to make fermion number density determinate looks ad hoc since there are many other choices one might make. The puzzle of which field observable to make determinate is the preferred basis problem for quantum field theory. Bell worried that the choice of determinate field quantity might not be experimentally significant when measurement records are defined so grossly as by the positions of instrument pointers or of ink on paper. But this is not necessarily a bad thing for Bell's theory. While it might be experimentally difficult, or perhaps impossible, to determine which field observables must be determinate when such measurement records are determinate, Bell's choice of fermion density presumably does make determinate

[9] See Vink (1993) for a particularly clear generalization of Bell's dynamics.

what we typically take as measurement records. Bell briefly considered making more than one field observable determinate, but it is typically not possible to make different local field observables determinate while preserving the standard functional relations between the possessed values of the observables. This is a consequence the Kochen-Specker theorem (1967) in the context of field theory.[10]

Bell's second worry was that his local beable field theory is still incompatible with relativity. While the auxiliary dynamics is incompatible with relativity in that it requires a preferred inertial frame for its specification, the fact that Bell's theory makes the same statistical predictions as standard quantum mechanics has two immediate consequences. If the distribution postulate is satisfied, then *(i)* by the standard quantum no-signaling arguments, one cannot send superluminal signals in the theory and *(ii)* there is no empirical way to detect which preferred inertial frame was used to calculate the evolutions of the determinate field values. So while there is a sense in which it is *dynamically* incompatible, Bell's theory is nevertheless *observationally* compatible with relativity.[11]

Bell notes the observational compatibility between his field theory and relativity, but concludes that requiring a preferred frame for the dynamics "seems and eccentric way to make a world" (1984, 180).

5. A Many-Maps Formulation of Field Theory

The most direct way to eliminate an appeal to a preferred inertial frame in dynamical explanations of determinate measurement records is to give up dynamical explanation of determinate measurement records in field theory. Rather than provide a dynamical account of the possessed values of Bell's field beable n, the revised theory uses Bell's dynamics to characterize a set of

[10]See Clifton (2000) for a discussion of some of the constraints on stipulating more than one local field observable as determinate. One might make all local field observables determinate, but one would lose the functional relationships between local field observables. See Barrett (2005a) and Tomulka (2007) for discussions of the conceptual cost of doing this.

[11]This sort of compatibility between Bohmian mechanics and relativity has been often noted. See, for example, Albert (1992) and (1999), Barrett (1999), Bohm and Hiley (1993), Maudlin (1994) and (1996), and Dickson (1998).

physically possible spacetime maps with a determinate value for n specified for each local region and a probability measure over such maps. Each map represents a physically possible history of the world, and the probability measure represents the prior epistemic probability of each physically possible world being the actual physical world. As one learns the physical structure of the actual world, one conditions on the new evidence and updates the prior probabilities to get the standard quantum probabilities.

While the construction of physically possible maps of local field values makes use of Bell's auxiliary dynamics, the suggestion is that this be understood only as a part of the principled specification of possible physical worlds. That is, as opposed to the teleological strategy for constructing spacetime maps by stipulating determinate measurement records wherever one needs them, Bell's dynamics provides a principled way to identify determinate local field beables with each local spacetime region of a possible physical world.

The prescription for constructing a map of local field values that represents one physically possible world is as follows. Let $\mathcal{F}$ be a discrete-valued local field observable whose being determinate would guarantee determinate measurement records. Choose a preferred inertial frame for the purpose of mapping the local values of $\mathcal{F}$ and a countable spatial lattice of simultaneous points at each time in the preferred inertial frame. Consider a global quantum-mechanical state ψ defined at each point on the lattice and at all times in the inertial frame. Physically possible global quantum-mechanical states are characterized by a relativistic formulation of the unitary quantum dynamics for $\mathcal{F}$. It is in determining physically possible global states ψ, then, that relativistic constraints matter to the empirical content of the theory. Physically possible quantum-mechanical states, in turn, determine physically possible maps of determinate local field values and the epistemic probability measure over possible maps.

Choose an initial time t_0 in the preferred inertial frame and randomly assign values for $\mathcal{F}$ to each point in the spatial lattice so that the probability of particular determinate field value at a point r in the lattice is $|\psi(r, t_0)|^2$. Evolve these determinate values in the preferred inertial frame using Bell's stochastic dynamics. The result provides an assignment of a determinate value for $\mathcal{F}$ to the lattice points. Each such possible spacetime map $M_{\mathcal{F}}$ describes a possible physical world where the local value of $\mathcal{F}$ is everywhere determinate.

The probability that a particular map of local field values $M_{\mathcal{F}}$ will be generated by this prescription is determined by the global quantum-mechanical state ψ. Consequently, given a prior probability measure over possible global quantum-mechanical states, presumably a task for thermodynamics, the theory provides a prior probability measure μ over physical possible spacetime maps of the local field observable $\mathcal{F}$. The complete field theory then might be thought of as the set of possible spacetime field maps S together with the measure μ that represents the epistemic prior probability of each possible map providing an accurate description of the local field values of the actual physical world. One then updates the prior epistemic probabilities by conditioning on actual observed field configurations to get posterior predictions regarding unobserved regions of the actual world. As in Bohmian mechanics and Bell's local beable field theory, the epistemic probabilities one gets by conditioning on new empirical evidence here agree with the standard quantum probabilities for a no-collapse formulation of quantum mechanics.

Since $\mathcal{F}$ is determinate in each local region of spacetime, and since it is, by hypothesis, a field observable that makes all measurement records determinant, Wigner's friend, and everyone else, has fully determinate measurement records.

6. Conclusion

A many-maps formulation of field theory, like Bohmian mechanics and Bell's field theory, but unlike teleological constructions, provides determinate measurement records in a principle way. Further, the physically possible maps of local field values are determined by rules that, unlike Wigner's proposed dynamics, do not require one to say anything about when or where measurements are made. Finally, a many-maps formulation of field theory cannot be dynamically incompatible with relativity insofar as it does not provide a dynamical account of how determinate measurement records are produced. The dynamical constraints of relativity still have a role to play, but it is in characterizing possible global quantum-mechanical states, not in giving a causal account of the generation of measurement records.[12]

[12] This proposal thus violates one of Bell's deeply-held physical intuitions. He felt the that proper role of the quantum-mechanical state in a hidden-variable theory was to provide a causal explanation of the evolution of the beables: "For us [ψ] is an independent beable of the theory. Otherwise its appearance in the transition probabilities would be

There is no question that giving up on a dynamical explanation of physical measurement records involves a significant conceptual cost. Rather than providing a dynamical account of EPR correlations, for example, they are explained in a many-maps theory by the fact that a typical physical world, in the prior probability measure μ, will exhibit determinate local measurement records with just such nonlocal statistical correlations, and the actual world is expected to be typical in just this sense. On the other hand, to wish for a rich dynamical account of EPR correlations between determinate spacelike-separated field values, on at least one understanding of what such an account would involve, would be to wish for a dynamical account of measurement records that cannot possibly be compatible with the dynamical constraints of relativity.[13] If so, then perhaps we do not sacrifice as much as one might have thought in giving up on a dynamical explanation of determinate measurement records in quantum field theory.

One might still, of one wishes, think of the quantum-mechanical state ψ dynamically and insist that the unitary dynamics be put in a form compatible with the dynamical constraints of relativity. Indeed, the only way that relativity might play a meaningful role in a many-maps theory is in constraining possible global quantum-mechanical states.

quite unintelligible" (1984, 177).

[13]Bell understood his theorem as telling us that if we want the sort of rich dynamical account of the generation of determinate measurement records that is provided by a hidden-variable theory, then the dynamical account must be nonlocal. This was one of his reasons for thinking that one could do no better than a hidden-variable theory modeled on Bohmian mechanics.

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