ON THE FAITHFUL INTERPRETATION OF PURE WAVE MECHANICS

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In the long version of his Ph.D. thesis, Hugh Everett III developed pure wave mechanics as a way of solving the quantum measurement problem faced by the standard von Neumann-Dirac collapse formulation of quantum mechanics.[1] Pure wave mechanics, however, encounters problems of its own. I will briefly review Everett's description in his thesis of the standard measurement problem, how pure wave mechanics solves it, and the problems pure wave mechanics itself faces; then I will explain how one might nevertheless understand pure wave mechanics as a successful physical theory given the notion of faithfulness that Everett presents at the end of his long thesis. The result is a structural interpretation of pure wave mechanics as an empirically faithful theory.

The standard collapse formulation of quantum mechanics has two dynamical laws:

> *Process* 1: The discontinuous change brought about by the observation of a quantity with eigenstates ϕ_1, ϕ_2,..., in which the state ψ will be changed to the state ϕ_j with probability $|(\psi, \phi_j)|^2$.
> *Process* 2: The continuous, deterministic change of state of the (isolated) system with time according to a wave equation $\frac{\partial \psi}{\partial t} = U\psi$, where U is a linear operator.

The rule for when each applies is, on first pass simple: a physical system always evolves according to the deterministic Process 2 unless a measurement is made; in which case, it evolves in according to the random collapse Process 1 (1973, 3–4).

The quantum measurement problem, however, arises as a result of the conflict between these two dynamical laws. If we suppose that measuring devices are physical systems like any other, then the standard collapse theory is inconsistent because the incompatible laws might be applied to the same evolution; on the other hand, if measuring devices are somehow special, the standard theory is incomplete since it does not tell us what interactions should count as measurements.

Everett describes the problem with the standard theory as follows:

> The question of the consistency of the scheme arises if one contemplates regarding the observer and his object-system as a single (composite) physical system. Indeed, the situation becomes quite paradoxical if we allow for the existence of more than one

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[1] Since Everett's advisor, John Wheeler, was uncomfortable with Everett formulating his thesis as a direct attack on Bohr and the standard formulation of quantum mechanics, Everett ultimately defended a much shorter version of the thesis (1957a), which was essentially the same as the journal paper (1957b). While completed before the short version, the long version of Everett's theses was first published in DeWitt and Graham (eds.) (1973). References herein refer to that publication.

> observer. Let us consider the case of one observer A, who is performing measurements upon a system S, the totality $(A+S)$ in turn forming the object-system for another observer, B. . . . If we are to deny the possibility of B's use of a quantum mechanical description (wave function obeying wave equation) for $A+S$, then we must be supplied with some alternative description for systems which contain observers (or measuring apparatus). Furthermore, we would have to have a criterion for telling precisely what type of systems would have the preferred positions of "measuring apparatus" or "observer" and be subject to the alternate description. Such a criterion is probably not capable of rigorous formulation. (1973, 4)

Everett proceeds to tell his own version of a story that has come to be know as the Wigner's Friend story, then concludes from the story that "It is now clear that the interpretation of quantum mechanics with which we began [the standard von Neumann-Dirac collapse theory] is untenable if we are to consider a universe containing more than one observer. We must therefore seek a suitable modification of this scheme, or an entirely different system of interpretation" (1973, 6).[2] After briefly considering a handful of options for resolving the measurement problem, Everett describes his own proposal. His stated goal is simply to drop the collapse postulate, Process 1, from the standard formulation of quantum mechanics, then deduce the empirical predictions of the standard collapse theory as the subjective experiences of observers who are themselves treated as physical systems described by the theory. He calls his theory without without Process 1 *pure wave mechanics.*

Dropping Process 1 from the standard theory clearly solves the measurement problem in that it removes the possibility of a conflict between the two dynamical laws. But in dropping the collapse dynamics, one gives up the standard explanation for why we get determinate measurement records and the standard explanation for why these records are randomly distributed with the usual quantum statistics. The determinate-record and probability problems, respectively, are the problems of providing replacement explanations for determinate measurement records and quantum statistics in pure wave mechanics without appeal to Process 1.[3]

Consider an object system S and an observer A such that if S is initially in an eigenstate ϕ_i^S of the measured quantity, then the state of the composite system $\phi_i^S\psi_0^A$ will evolve to $\phi_i^S\psi_i^A$ over the course of a measurement interaction; in other words, suppose that the system S is undisturbed and the observer A's state is changed to ψ_i^A, which might represent determinately recording the measurement result i in a notebook. If the initial state of S is not an eigenstate of the observable being measured, but, rather $\sum_i a_i\phi_i^S$ then, by the linearity of Process 2, the

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[2] Everett here tells the Wigner's Friend story some four years before Wigner (1961) himself tells the story in his famous paper. That this story is being passed around illustrates part of the history of worrying over the foundations of quantum mechanics at Princeton during the 1950's. It is likely that the story was originally Wigner's and that Everett picked it up as a student; perhaps in the seminar on mathematical physics that Everett that took from Wigner. See Wigner (1961), Albert (1992), and Barrett (1999) for discussions of the story and how it illustrates the measurement problem in the standard theory.

[3] The determinate-record problem is often taken to involve a further problem of choosing a particular physical quantity with which to provide determinate values. This is the preferred basis problem. See Barrett (2008) for a discussion of these three problems in pure wave mechanics.

evolution form the initial state to the final state of the composite system in pure wave mechanics will be given by

$$\sum_i a_i \phi_i^S \psi_0^A \rightarrow \sum_i a_i \phi_i^S \psi_i^A = \Psi^{SA},$$

which, as Everett himself points out, is not a state that describes an observer with any particular measurement result:

> Thus in general after a measurement has been performed there will be no definite system state, even though there is a correlation. It seems as though nothing can ever be settled by such a measurement. Furthermore this result is independent of the *size* of the apparatus, and remains true for apparatus of quite macroscopic dimensions. ... This behavior seems to be quite at variance with our observations, since macroscopic objects always appear to us to have definite positions. Can we reconcile this prediction of the purely wave mechanical theory with experience or must we abandon it as untenable? (1973, 61–62)

Further, regarding quantum statistics, it is not at all clear how one is to get the standard statistical predictions of Process 1 for the measurement records (*i*) when one apparently has no determinate measurement records to which the statistics might apply and (*ii*) when, as Everett explains, "nothing resembling Process 1 can take place" (1973, 61).

Everett's goal then was to explain both determinate measurement records and the statistical predictions of quantum mechanics in pure wave mechanics. More specifically, he said that his strategy for providing this explanation would be to "deduce the probabilistic assertions of Process 1 as *subjective* appearances . . . thus placing the theory in correspondence with experience. We are then led to the novel situation in which the formal theory is objectively continuous and causal, while subjectively discontinuous and probabilistic"(1973, 9). That said, it has never been entirely clear how Everett intended to resolve either the determinate-record or the probability problems. It is not that Everett had nothing to say about these problems; indeed, as we have just seen, he shows that he clearly understood both in the very statement of his goal. The difficulty in interpreting Everett arises from the fact that Everett had several suggestive things to say in response to each problem, none of these suggestive things do quite what Everett seems to be describing himself as doing, at least in his strongest statements of his project, and it is unclear that his various considerations can be put together into a single account of how one is to understand the theory as predicting determinant records distributed according to the standard quantum statistics.[4]

Everett's discussion of the goals of theoretical physics near the end of the long thesis, however, suggests a conservative strategy for how one might understand his deduction of determinate measurement records and quantum statistics. This strategy provides a concrete sense in which pure wave mechanics can explain both determinate measurement records and quantum statistics. While the sort of explanation provided is relatively weak, (*i*) it can be made perfectly clear, (*ii*) there is textual evidence that something like this is what Everett had in mind as the proper standard for the empirical acceptability of physical theories more generally, and

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[4] See Barrett (2008) for a brief description of the considerations involved.

(*iii*) it is closely related to a type of explanation that has a long history of being taken seriously by both physicists and philosophers of science.

In the second appendix to the long thesis Everett explains that an essential goal of theoretical physics is to produce *faithful* physical theories. A faithful theory is one that can be put into a close structural correspondence with the elements of the perceived world:

> Every theory can be divided into two separate parts, the formal part, and the interpretive part. The formal part consists of a purely logico-mathematical structure, i.e., a collection of symbols together with rules for their manipulations, while the interpretive part consists of a set of "associations," which are rules which put some of the elements of the formal part into correspondence with the perceived world. The essential point of a theory, then, is that it is a *mathematical model*, together with an *isomorphism* between the model and the world of experience (i.e., the sense perceptions of the individual, or the "real world"— depending upon one's choice of epistemology).

And, in an associated footnote, he explains that

> By isomorphism we mean a mapping of some elements of the model into elements of the perceived world which has the property that the model is faithful, that is, if in the model a symbol A implies a symbol B, and A corresponds to the happening of an event in the perceived world, then the event corresponding to B must also obtain. The word homomorphism would be technically more correct, since there may not be a one-one correspondence between the model and the external world. (1973, 133)

To begin, note that here, in his most careful description of the aims of theoretical physics, Everett adopts a broadly empiricist position; and, in this spirit, he is careful to argue that it is a mistake to require a successful physical theory to be descriptive of the ontology of the world. Indeed, he argues for the stronger line that it is a mistake to understand theories as descriptive of metaphysics at all.

> [W]hen a theory is highly successful and becomes firmly established, the model tends to become identified with "reality" itself, and the model nature of the theory becomes obscured. The rise of classical physics offers an excellent example of this process. The constructs of classical physics are just as much fictions of our own minds as those of any other theory we simply have a great deal more confidence in them. It must be deemed a mistake, therefore, to attribute any more "reality" here than elsewhere" (1973, 134).

He then uses this stronger line to characterize his version of empiricism more precisely.

> Once we have granted that any physical theory is essentially only a model for the world of experience, we must renounce all hope of finding anything like '*the* correct theory.' There is nothing which prevents any number of quite distinct models from being in correspondence with experience (i.e., all 'correct'), and furthermore no

> way of ever verifying that any model is completely correct, simply because the totality of all experience is never accessible to us" (1973, 134).

Given the strong formulation of empiricism Everett develops here and the associated metaphysical ambivalence that we find here and throughout the long thesis, it is difficult to imagine that he might ever have held that any particular set of commitments concerning the metaphysical structure of the world was required for a proper understanding of pure wave mechanics.[5] The suggestion is that his metaphysical ambivalence might explain both why Everett did not make the careful distinctions that would have selected one set of metaphysical commitments over another for the interpretation of pure wave mechanics and, consequently, why readers can find room in his description of pure wave mechanics for talk of such diverse ontologies as those suggested by splitting worlds, many minds, many histories, and such.

Rather than describing the metaphysical structure of the world, a successful physical theory for Everett is supposed to be somehow isomorphic to the world of experience. But in the quotations above Everett conflates what one might take as the theory itself and a formal model of the theory in his description of what a physical theory is. Adopting the standard distinction between theory and formal model, a theory on Everett's view is *faithful* when one can find some elements of the model of the theory that are in fact isomorphic to elements of the perceived world. It is arguably a short step from this reconstruction of faithfulness to something like the constructive empiricist's description of what it is for a theory to be empirically adequate.[6] Taking Everett's goal to be to show that pure wave mechanics is empirical faithful might then be thought of as a structural empiricist interpretation of Everett.

While empirical faithfulness is an essential virtue of a successful physical theory, Everett argues that it is also desirable to have a theory that is *comprehensive* and *simple*. While he allows for yet other theoretical virtues, he takes these two virtues, in particular, to be important to inquiry. His argument is that if one's current theory is comprehensive and simple, then it is more likely to provide a suitable context for engineering future theories that might exhibit a yet higher standard of empirical faithfulness. He concludes that "it may be impossible to give a total ordering of [rival physical] theories according to "goodness," since different ones may rate highest according to the different criteria" (1973, 136). Nevertheless, it is in the context of a sort of cost-benefit analysis of faithful theories for the purpose of *engineering* future faithful theories that Everett considers the comparative virtues of pure wave mechanics.

One might grant that pure wave mechanics is comprehensive in that it treats all physical interactions in precisely the same way and simple in that it involves only one dynamical law and this law is deterministic and unitary, but it remains to show that pure wave mechanics has the essential virtue of faithfulness. While Everett

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[5] Everett's pervasive ambivalence regarding metaphysical issues has been most compellingly put to me both in conversation and in forthcoming work by Brett Bevers.

[6] The constructive empiricist would say that an empirically adequate physical theory is one that has a model with a substructure that is isomorphic to the phenomena as one chooses to represent them. This statement is meant to capture some degree of flexibility in what one takes as the relevant empirical substructure of the model and to capture van Fraassen's most recent formulation (e.g. 2008, 253) where he also allows for a corresponding pragmatic degree of freedom in how one chooses to represent empirical phenomena.

provides sufficient material for several different approaches for reconstructing the details of an argument for the faithfulness of pure wave mechanics, I will briefly sketch just one such argument here with the aim of showing that the model of pure wave mechanics does indeed have enough structure that *one can find* a substructure in the model that is isomorphic to the quantum-mechanical expectations supported by experience.

One might think of the model of pure wave mechanics as a structure of complex-valued weighted correlations between the observable properties of different physical systems at each time. On the unitary dynamics, when one physical system interacts with another, the properties of the two systems typically become correlated, and the composite system ends up in an nonseparable state. It is enough to characterize an interaction to say which properties become correlated and how and to what degree they become correlated.

Everett developed several ways of talking about the correlations between observable properties that fully characterize the correlation model of pure wave mechanics. The most important of these was his notion of a *relative state*. The basic idea is this: while the global state of the world may be a complicated entangled state involving most every physical system, one can always understand systems as having states relative to each other simply by dint of the precise way in which their observable properties are correlated. More specifically, one can think of relative states as what one gets when one chooses a physical system S and a state of that system ϕ_S then ignores all components of the entangled global state of the world that characterize S as being in any state other than ϕ_S. If the single remaining component also characterizes the system R as being in state χ_R, then we say that the state of R is χ_R relative to the state of S being ϕ_S.

Consider again an ideal observer A beginning in a state corresponding to being ready to make a measurement of a system S that is initially in a superposition of states corresponding to different values of the observable being measured. Given the linear dynamics and the perfect correlations produced by an ideal observer, the postmeasurement state of the observer A and her object system will be the entangled state Ψ^{SA} above. Since the observer's notebook record of the measurement outcome is perfectly correlated to the observable being measured (that is, since every term in the representation of the global state in the determinate-record-$\otimes$-determinate-value basis has the form $a_k\phi_k^S\psi_k^A$), Everett would say that A's notebook is in a state where she determinately recorded the result k relative to the system S being in the state ϕ_k^S. But since the global state of the world will typically be a complex entangled state, however, it is likely that no such simple measurements ever actually occur; nevertheless *relative to one having performed a simple measurement*, one may end up in a postmeasurement relative state like Ψ^{SA}. If so, then relative to there being a record that the measurement was performed and relative to A recording the particular result k, S is in the corresponding relative state ϕ_k^S.

Consequently, if the concern is empirical faithfulness as characterized earlier, the determinate-record problem is solved simply by noting that the values of determinate measurement records for those measurements we take ourselves to have performed can be found in the correlation structure of pure wave mechanics as relative states, relative to there being a record of the measurements being performed and relative to the properties we take the measured systems to have. Given a clear picture of such nested relative states in the correlation model, addressing the probability problem requires only slightly more subtlety.

There is a parameter determined by the correlation model that covaries with our standard quantum statistical expectations. It is not the norm-squared of the coefficients on the global state, but it is closely related. Consider the state of the observer and her object system relative to the observer having a record that the measurement was performed but not relative to any particular record of the result. One might renormalize this relative state, then think of it as a state describing the superposition of possible measurement records that would result from a simple measurement interaction on the linear dynamics. Suppose that the coefficient associated with the term characterizing the notebook as recording the result k is a_k. Our quantum expectations for result k covary with the parameter $|a_k|^2$; in other words, the parameter $|a_k|^2$ can be taken as representing the degree to which the result k is expected given the usual quantum statistics. The faithfulness of pure wave mechanics with respect to the usual quantum statistics simply consists in one being able to find such a parameter in the correlation model. And, taken together, that pure wave mechanics is faithful to our determinate measurement records and the statistical distribution of these records simply amounts to the fact that *one can find* an isomorphic substructure to our statistical experience with determinate records in the model of pure wave mechanics.[7]

While there is a sense in which showing that pure wave mechanics is empirically faithful involves deducing determinate records and the standard probabilities from pure wave mechanics, there is also a sense in which this is not a deduction of determinate records or of the standard quantum probabilities at all. We have not deduced that one should expect measurement records to be determinate in a world described by pure wave mechanics nor have we deduced quantum probabilities as probabilities in such a world nor have we shown that there is something like a canonical way to rationally assign expectations given the theory; rather, we have shown that we can find a parameter in the correlation model that covaries with the standard quantum expectations for determinate records that we have from empirical experience *and just that.* Pure wave mechanics nowhere tells us that relative states have the metaphysical virtues of determinate physical records nor that the quantity $|a_k|^2$ represents an objective probability or a constraint on rational choice given the nature of the physical world. Not only would such conclusions require just the sort of metaphysical commitments that Everett seeks to avoid, but he also repeatedly insists that pure wave mechanics makes no assertions concerning the probabilities of

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[7] Note that the preferred basis problem does not even arise on this standard of empirical acceptability since faithfulness requires only that one be able to find our determinate records in the model. If one required rather that there be a substructure in the model of pure wave mechanics that one might *on theoretical grounds alone* identify as the empirically relevant substructure and if one required that this substructure be isomorphic to our experience, as a constructive empiricist might, then one might argue that pure wave mechanics fails to account for our concrete experience because there is much more in the correlation than our concrete experience since the correlation model contains something more like all physically possible experience. Even so, one might reply that there is a sense in which the entire structure of pure wave mechanics is isomorphic to the *general* statistical structure of our experience. Indeed, van Fraassen himself makes a move like this in his discussion of quantum mechanics at the end of (2008). But just as with Everett's notion of faithfulness, one should wonder whether this is all one should want from a successful physical theory.

any sort.[8] Rather than claim that we have somehow deduced determinate measurement records distributed with the standard quantum probabilities from pure wave mechanics alone, the procedure of showing that pure wave mechanics is empirically faithful is better characterized as one where we start with our empirically informed expectations, then find something in the correlation model that covaries with our empirically grounded expectations. The point is just that the determinate record and probability problems are solved here in just the sense that one can find our actual empirical records and our empirically supported quantum expectations in the correlation model of pure wave mechanics. That there is much more than just our actual determinate records in the correlation model is something that Everett can, and does, embrace.

The remaining question is whether a theory being empirically faithful in this sense is enough for it to be considered empirically acceptable. While one might worry that faithfulness is a relatively weak standard for the acceptability of physical theory, it is clearly not empty since not every physical theory has a model with a parameter associated with representations of possible measurement records that covaries with the quantum expectations we find in experience. Further, pure wave mechanics has significant virtues beyond empirical faithfulness—it is, after all, difficult to disagree with Everett's claim that pure wave mechanics is both comprehensive and simple. That said, one clearly might want more than even this from a successful physical theory, but what more and why are questions for another occasion.

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[8] The centrality of this point is reflected in the fact that Everett originally titled his long thesis *Quantum Mechanics without Probability.* He also repeatedly insists on the stronger point that pure wave mechanics itself makes no statistical assertions (e.g. 1973, 8).

Bibliography

Albert, David, Z (1992) *Quantum Mechanics and Experience.* Cambridge, MA: Harvard University Press.

Barrett, Jeffrey A. (2008) “Everett’s Relative-State Formulation of Quantum Mechanics”, The Stanford Encyclopedia of Philosophy (Winter 2008 Edition), Edward N. Zalta (ed.), URL = http://plato.stanford.edu/archives/win2008/entries/qm-everett/.

Barrett, Jeffrey A. (1999) *The Quantum Mechanics of Minds and Worlds*, Oxford: Oxford University Press.

DeWitt, Bryce S., and R. Neill Graham (eds.) (1973) The Many-Worlds Interpretation of Quantum Mechanics, Princeton: Princeton University Press.

Everett, Hugh (1957a) *On the Foundations of Quantum Mechanics*, Ph.D. thesis, Princeton University, Department of Physics.

Everett, Hugh, (1957b) “Relative State Formulation of Quantum Mechanics”, Reviews of Modern Physics, 29: 454462. Reprinted in Wheeler and Zurek 1983, pp. 315323.

Everett, Hugh (1973) “The Theory of the Universal Wave Function”, in DeWitt and Graham 1973, 3140.

van Fraassen, Bas (2008) *Scientific Representation: Paradoxes of Perspective*, Oxford university Press.

Wigner, Eugene (1961) “Remarks on the Mind-Body Problem”, in *The Scientist Speculates*, I. J. Good, ed. pp. 284-302, Heinemann, London (1961); Basic Books, New York (1962).