On the Thermodynamical Character of Black Holes in Classical General Relativity I: Surface Gravity and Temperature

Erik Curiel

November 16, 2011

Contents

PROVISO

This is not even a rough draft of a paper. It is a brief sketch of a work in progress. Please forgive the inelegant, the redundant, and, worst of all, the sketchy.

1 Introduction

The ultimate goal of the project of which this paper forms only a part is to figure out what exactly the status of the analogy is between black-hole mechanics restricted to General Relativity on the one hand (*i.e.*, with no input from quantum field theory on curved spacetime or from any other type of semi-classical calculation) and classical thermodynamics on the other ("classical" in the sense that no statistical or quantum considerations come into play). Based on the striking formal similarities of the respective mathematical formulæ of the so-called Zeroth, First, Second and Third Laws of classical thermodynamics and of the mechanics of black holes in stationary, asymptotically flat spacetimes, the best particular analogies seem to be: (1) that between the surface gravity of a black hole as measured on its event horizon and the temperature of a classical system; and (2) that between surface area of the horizon and entropy.[1] When it is also noted that each version of the First Law states a conservation principle for essentially the same quantity as the other, *viz.* mass-energy, it becomes tempting to surmise that some deep or fundamental connection between black holes and thermodynamics is being uncovered. But is it of real physical significance in some sense?

The conventional answer to this question is 'no'. Because classical black holes are perfect absorbers, they would seem to have a temperature of absolute zero, even when they have non-zero surface gravity. It is only with the introduction of quantum considerations, the standard account runs, in particular the derivation of Hawking radiation, that one finds grounds for taking the analogy seriously. And yet the startling and suggestive fact remains that one can derive laws for black holes formally identical to those of classical thermodynamical systems from the fundamental principles of General Relativity itself with no aid from quantum field theory in curved spacetime. Does the use of quantum field theory in curved spacetime offer the only hope for taking the analogy seriously? I think the answer is 'no'. To attempt to justify that answer, I shall begin by arguing that the standard argument to the contrary begs the question. Looking at the various ways that the idea of "temperature" enters classical thermodynamics then will suggest arguments that, I claim, show the analogy between classical black-hole mechanics and classical thermodynamics should be taken seriously indeed, at least so far as temperature goes, without the need to rely on or invoke quantum mechanics. If this is correct, then there may be a deep connection between classical general relativity and classical thermodynamics on their own, independent of quantum mechanics. This paper is not aiming for any such grand conclusions, however, but only attempts to show that there are grounds for taking the analogy seriously even without the introduction of quantum mechanics.[2] Towards that end, I will consider in this paper only the analogy between surface gravity and temperature; I will not consider the problems with taking the area of a black hole to be a measure of its true physical entropy. That is a job for a future paper.

I want to stress that I do not consider quantum effects to be irrelevant when considering possible relations between gravitational physics and thermodynamics. I want only to motivate the idea that

---

[1] Both the surface gravity and the surface area in question are defined with respect to the orbits of the Killing field in virtue of which the spacetime is qualified as 'stationary'. See Wald (1984, ch. 12) for details.

[2] Hayward (1998) has also argued that one does not need to invoke quantum mechanics in order to take the analogy seriously, by analysis of the interaction of thermal matter with black holes as defined by way of his theory of dynamical trapping horizons. Since Hayward uses a different definition of a black hole than I shall consider in this paper, his arguments are not directly relevant to what I say here. I plan to investigate his work in a later paper, in large part because I agree with him that his characterization of black holes is more physically meaningful, and certainly appropriate as a model for a far broader class of systems, than that of the traditional view.

the analogy between the laws of classical thermodynamics and those of black hole mechanics in classical General Relativity is robust and deep in its own right. Before diving in to any details, I also want to stress that nothing in this paper is meant to be a rigorous or thoroughly worked out argument. They are tentative arguments and conjectures put forward to see whether they offer enough promise to warrant further work.

I should perhaps say, by way of background, that I am curious about this question in the first place in part because of my curiousity about the larger question of the relation between thermodynamical characteristics of a physical system and the possibility of always being able to or indeed always being required to find an underlying statistical interpretation of those thermodynamical characteristics. That the laws of black hole mechanics follow from the fundamental theory itself (in this case, General Relativity), and are not as with classical thermodynamics an independent adjunct connected to the underlying fundamental (Newtonian) theory through the use of statistical devices, could suggest that thermodynamics is itself more of the nature of a fundamental theory than has been thought since the advent of statistical mechanics—or at least that thermodynamical characteristics and quantities of physical systems may be fundamental to them in some way analogous to that of other fundamental characteristics and dynamical quantities, such as the possession of a stress-energy tensor, for example, and its satisfaction of some form of covariant conservation principle.[3] I do not intend to investigate these larger issues, however. I intend to investigate only the status of the analogy between the laws of classical thermodynamics on the one hand and those of black hole mechanics in classical General Relativity on the other. I mention these larger issues only to give some of my motivation for this work.

There are other motivations behind this project as well. Although philosophers of physics have recently begun to work on issues arising from proposals for theories of quantum gravity, many of which take as their starting points the seemingly thermodynamical character of gravitational phenomena as exemplified by the laws of black hole mechanics, almost no philosophical work has been done investigating the nature of this seemingly thermodynamical character as revealed by work in general relativity itself and in quantum field theory formulated on curved, relativistic spacetimes. Because general relativity and quantum field theory are well entrenched physical theories, I believe it behooves philosophers to study it, if not before, at least in conjunction with work done on quantum gravity.

2 The Laws of Black Hole Mechanics and the Laws of Thermodynamics

Within the context of General Relativity, one can derive laws describing the behavior of black holes in stationary, asymptotically flat spacetimes bearing a remarkable resemblance to the classical laws

---

[3] Although I tend to find myself more in agreement in general with Unruh and Wald than with Bekenstein on most disputed issues in black hole mechanics, I nevertheless was somewhat reassured by my discovery that a physicist of Bekenstein's stature has been at least toying with the same sort of ideas—see, *e.g.*, Bekenstein (2000).

of equilibrium thermodynamics:[4]

Zeroth Law

[**Thermodynamics**] The temperature T is constant throughout a body in thermal equilibrium.

[**Black Holes**] The surface gravity κ is constant over the event horizon of a stationary black hole.

First Law

[**Thermodynamics**]

$$\mathrm{d}E = T\mathrm{d}S + p\mathrm{d}V + \Omega\mathrm{d}J$$

where E is the total energy of the system, S the entropy, p the pressure, V the volume, Ω the rotational velocity and J the angular momentum.

[**Black Holes**]

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega_{\mathrm{H}}\delta J$$

where M is the total black hole mass, A the surface area of its horizon, Ω_{H} the 'rotational velocity' of its horizon,[5], J its total angular momentum and 'δ' denotes the result of a first-order, linear perturbation of the spacetime.[6]

Second Law

[**Thermodynamics**] $\delta S \geq 0$ in any process.

[**Black Holes**] $\delta A \geq 0$ in any process.

Third Law

[**Thermodynamics**] $T = 0$ is not achievable by any process.

[**Black Holes**] $\kappa = 0$ is not achievable by any process.

The formal analogy should be obvious.

3 The Standard Argument Begs the Question

There are well-known difficulties with taking the surface gravity of a classical black hole to represent a physical temperature. One important method for defining the thermodynamic temperature of an object derives from the theory of heat radiation from black bodies. If a normal object immersed in a bath of thermal radiation settles down to thermal equilibrium, it will itself emit thermal radiation

---

[4] The following enumeration and most of the technical discussion in this note are derived from Wald (1984, ch. 12) and Wald (1999b).

[5] See Wald (1984, pp. 319–320).

[6] For an exact definition and thorough discussion of the sort of perturbations envisaged, see Gao and Wald (2001).

with a power spectrum characteristic of its equilibrium temperature as measured using a gas thermometer. This power spectrum can then be used to define a temperature scale. It is this definition of thermodynamic temperature that is always (at times implicitly) invoked when the claim is made that if one considers classical General Relativity alone then black holes, being perfect absorbers and perfect non-emitters, have an effective temperature of absolute zero.[7] On account of this fact, it seems that the surface gravity κ, which is never zero for a non-extremal Kerr black hole, cannot represent a physical temperature of the black hole in classical general relativity. As a result, conventional wisdom holds that if the formal similarities mentioned above were all there were to the matter then they would most likely represent a merely accidental resemblance or perhaps would indicate at best a superficial relationship between thermodynamics and black holes, but in any event would not represent the laws of classical thermodynamics as extended into the realm of black holes.[8]

In 1974 using semi-classical approximation techniques Hawking discovered that black holes appear to radiate with a thermal spectrum as though they were perfect black-body emitters in thermal equilibrium with temperature $\frac{\hbar}{2\pi}\kappa$ when quantum particle-creation effects near the black hole horizon are taken into account (Hawking 1975). It is this result that is generally taken to justify the view that the resemblances between the laws of black hole mechanics and the laws of classical thermodynamics point to a fundamental and deep connection among General Relativity, quantum field theory and thermodynamics, and in particular that κ *does* in fact represent the physical temperature of a black hole.[9]

I do not think this definition of temperature is the appropriate one to use in the context of a purely classical description of black holes, for the radiative thermal equilibrium of systems immersed in a thermal bath is essentially a *quantum* phenomenon, by which I mean one that can be correctly modeled only by using the hypothesis that thermal energy is exchanged in discrete quanta. To use that characterization of temperature, therefore, to argue that we must use quantum mechanics in order to take surface gravity seriously as a physical temperature is to beg the question. If that is correct, it follows that the standard argument does not bear on the strength of the analogy as indicating a real physical connection between classical general relativity and thermodynamics. After all, if one is trying to determine the status of the analogy between *classical* gravitational theory and *classical* thermodynamics independently of any quantum considerations, then the definition of temperature from the theory of radiative heat is not the most appropriate one to use, for the radiative thermal equilibrium of systems immersed in a thermal bath is essentially a *quantum* phenomenon, by which I mean one that can be correctly modeled only by using the hypothesis that thermal energy is exchanged in discrete quanta.

---

[7] See for example the remarks in Bardeen, Carter, and Hawking (1973), Carter (1973) and Wald (1999a).

[8] The remarks of Wald (1984, p. 337), for example, are exemplary in this regard.

[9] See again, for example, the remarks of Wald (1984, p. 337).

4 Temperature in Classical Thermodynamics

I think there are grounds for taking the analogy very seriously even when one restricts oneself to the classical theories, by looking at other ways that the idea of "temperature" enters classical thermodynamics. To make the case more poignant, imagine that we are physicists who know only classical general relativity and classical thermodynamics, but have no knowledge of quantum field theory or the statistical foundation of classical thermodynamical phenomena. How could we determine whether or not to take black holes as thermodynamical objects in a substantive, physical sense, given that we know the deep formal analogy between the two sets of laws? I argue that we ought to look to the way that temperature is introduced in classical thermodynamics and the various roles it plays there. If the surface gravity of black holes can be introduced in the analogous ways and plays the analogous physical roles, I contend that the global analogy is already on strong ground. In other words, the surface gravity must play the same role in the new theory *vis-à-vis* other theoretical quantities as temperature does in the original theory *vis-à-vis* the analogous theoretical entities there. If, moreover, it can be shown that surface gravity couples to ordinary classical thermodynamical systems in the same formal way as ordinary temperature does, then there are no grounds for denying that it is a true physical temperature. Indeed, it was exactly on grounds such as these that physicists in the 19th century concluded that the power spectrum of blackbody radiation itself encoded a *physical* temperature, not merely that there was an analogy between thermodynamics and the theory of blackbody radiation.

There are three fundamental, related ways that temperature is introduced in classical thermodynamics, which themselves mediate the various physical roles temperature can play in the theory (how it serves as the mediator of particular forms of coupling between different types of physical system, *e.g.*). The first derives from perhaps the most basic of the thermodynamic characteristics of temperature and is perhaps most definitive of the cluster of ideas surrounding the concepts of "temperature" and "heat": it is that when two bodies are brought into contact, heat will spontaneously flow only from the one of higher temperature to the one of lower temperature. The second arises from the fact that increase in temperature is positively correlated with increases in the capacity of a system to do work. This fact allows one to define an empirical scale of temperature through, *e.g.*, the use of a gas thermometer: the temperature reading of the thermometer is made directly proportional to the volume of the thermometric gas used. The utility of such a scale is underwritten by the empirical verification that such empirical scales defined using a multitude of different gases under a multitude of different conditions are consistent among one another.[10] The third arises from an investigation of the efficiency of reversible, cyclic engines, *viz.*, Carnot engines, which yields a definition of the so-called absolute temperature scale associated with the name of Kelvin.[11] It is the possibility of physically identifying the formally derived absolute scale with the empirically derived scale based on capacity to do work (increase in volumes, *e.g.*) that warrants the assertion that they

---

[10] Planck (1926, §1, p. 1) remarks that quantitative exactness is introduced into thermodynamics through this observation, for changes of volume admit of exact measurements, whereas sensations of heat and cold do not.

[11] See, *e.g.*, Fermi (1956, §§8–10).

both measure the same physical quantity.

Now, the following fundamental theorem of classical thermodynamics provides the basis for the definition of the absolute temperature scale:

Theorem 4.1 *Any two reversible, cyclic engines operating between temperatures T_2 and T_1 have the same efficiency. The efficiency of any non-reversible engine operating between T_2 and T_1 is always less than this.*

This theorem is a direct consequence of either of two classical postulates, which can be argued on physical grounds both to be equivalent to each other and to directly imply the Second Law of thermodynamics (*cf*. Fermi 1956):

Postulate 4.2 (Lord Kelvin) *A transformation whose only final result is to transform into work heat extracted from a source that is at the same temperature throughout is impossible.*

Postulate 4.3 (Clausius) *A transformation whose only final result is to transfer heat from a body at a given temperature to a body of a higher temperature is impossible.*

I claim that these last two postulates, and the fact that they provide grounds for proof of the efficiency theorem, encode essentially all that is of physical significance in the three rough ways I sketched that temperature enters into classical thermodynamics. The Clausius Postulate captures the idea that when two bodies are brought into thermal contact, heat flows from the body of higher temperature to the other. The Kelvin Postulate captures the idea that the capacity of a body to do work on its environment tends to increase as its temperature increases. If one could show that appropriately formulated analogues to these two propositions about classical black holes hold in general relativity, with surface gravity playing the role of temperature, one would have gone a long way towards showing that surface gravity *is* a true thermodynamical temperature. If one could moreover show that those analogues imply the properly formulated analogue of the efficiency theorem, and so use them to define an absolute temperature scale for black holes that was essentially equivalent to surface gravity, the result would be even more secure.

5 Taking Surface Gravity Seriously as a Physical Temperature

5.1 Irreducible Mass, Rotational Energy and "Heat" of Black Holes

- Irreducible mass M_{irr} of a black hole of mass M and angular momentum J, *à la* Christodoulou (1970):

$$M_{\text{irr}}^2 = \frac{1}{2}[M^2 + (M^4 - J^2)^{\frac{1}{2}}]$$

- follows from Second Law that initial mass M_0 of black hole cannot be reduced below M_{irr} by any physical process

- thus: "rotational energy" of a black hole = $M - M_{\text{irr}}$
- define "quantity of heat" transferred in any process to be change in total black hole energy minus change in its rotational energy

5.2 What Is "Thermal Contact" for a Black Hole?

I claim, based on characterization of "quantity of heat": exchange of "non-rotational gravitational energy", *e.g.*, emission and absorption of gravitational radiation.

5.3 "Heat Flow" between Orbiting Kerr Black Holes—the Clausius Postulate

Conjecture 5.1 *A transformation whose only final result is that a "quantity of heat" (as defined in §5.1) is transferred from a black hole at a given surface gravity to one of a higher surface gravity is impossible.*

5.4 "Angular Momentum" as Measure of Capacity for a Kerr Black Hole to Do Work—the Kelvin Postulate

Theorem 5.1 *A transformation whose only final result is to decrease the angular momentum and increase the irreducible mass of a black hole while leaving its surface gravity unchanged is impossible.*

5.5 Carnot Cycles for Kerr Black Holes and an Absolute Temperature Scale

The Penrose Process for Kerr black holes:

> Because the stationary Killing field of a Kerr black hole turns spacelike before it hits the event horizon (the "ergosphere"), it is possible to shoot composite systems into the black hole in such a way that the system initially has positive energy and angular momentum, but splits in the ergosphere in such a way that a bit with negative energy and angular momentum enters the black hole while the rest (with, eventually, positive energy greater than that of the original system) escapes: "energy and angular momentum extracted from the black hole".

Use this to define a "Carnot Cycle" for Kerr black holes, based on:

- isothermal transformation: Penrose process during which κ stays constant
- adiabatic transformation: Penrose process during which irreducible mass stays constant

Then, to capture the idea that one can define an absolute temperature scale for black holes from Carnot cycles as one does in classical thermodynamics:

Conjecture 5.1 *Taking the ratio of exchanged quantities of "heat" (as characterized in §5.1) to measure efficiency of the total process, it follows by the standard argument that the ratio of exchanged "heat" in a reversible process (meaning one in which the irreducible mass does not change) equals the ratio of the values of κ at which "heat" was exchanged.*

5.6 Coupling of Kerr Black Holes to Ordinary Thermal Systems

Conjecture 5.1 *Put a small classical thermodynamical system with temperature T into stable orbit about a Kerr black hole with surface gravity κ. The two systems will be in "thermal contact" through emission and absorption of gravitational radiation. They will reach a state of joint equilibrium only when $T = \kappa$.*

Indirect evidence for this conjecture comes from the arguments of Hayward (1998).

References

Bardeen, J., B. Carter, and S. Hawking (1973). The four laws of black hole mechanics. *Communications in Mathematical Physics 31*, 161–170.

Bekenstein, J. (2000). The limits of information. *arXiv:gr-qc/0009019v2* (`http://xxx.lanl.gov/abs/gr-qc/0009019`).

Carter, B. (1973). Black hole equilibrium states. In B. DeWitt and C. DeWitt (Eds.), *Black Holes*, pp. 56–214. New York: Gordon and Breach.

Christodoulou, D. (1970). Reversible and irreversible transformations in general relativity. *Physical Review Letters 25*, 1596–1597.

Fermi, E. (1937[1956]). *Thermodynamics*. Dover Publications, Inc. The Dover 1956 edition is an unabridged, unaltered republication of the 1937 Prentice-Hall edition.

Gao, S. and R. Wald (2001). The "physical process version" of the first law and the generalized second law for charged and rotating black holes. *arXiv:gr-qc/0106071* (`http://xxx.lanl.gov/abs/gr-qc/0106071`).

Hawking, S. (1975). Particle creation by black holes. *Communications in Mathematical Physics 43*, 199–220.

Hayward, S. (1998). Unified first law of black-hole dynamics and relativistic thermodynamics. *Classical and Quantum Gravity 15*, 3147–3162. Also available online as *arXiv:gr-qc/9710089v2* (`http://xxx.lanl.gov/abs/gr-qc/9710089`).

Israel, W. (1979). Dark stars: The evolution of an idea. In S. Hawking and W. Israel (Eds.), *General Relativity: An Einstein Centenary Survey*, pp. 199–276. Cambridge university Press.

Penrose, R. (1969). Gravitational collapse: The role of general relativity. *Revista del Nuovo Cimento 1*, 272–276.

Planck, M. (1926). *Thermodynamics*. Dover Publications, Inc. The Dover reprint of the third English edition of 1926, translated by A. Ogg from the 7th German edition of 1922.

Unruh, W. and R. Wald (1982). Acceleration radiation and and the generalized second law of thermodynamics. *Physical Review D25*, 942–958.

Wald, R. (1984). *General Relativity*. Chicago: University of Chicago Press.

Wald, R. (1999a). Gravitation, thermodynamics and quantum theory. *arXiv:gr-qc/9901033v1* (`http://xxx.lanl.gov/abs/gr-qc/9901033`).

Wald, R. (1999b). The thermodynamics of black holes. *arXiv:gr-qc/9912119v1* (`http://xxx.lanl.gov/abs/gr-qc/9912119`).