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What time reversal invariance is and why it matters

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Abstract *David Albert's* Time and Chance *(2000) provides a fresh and interesting perspective on the problem of the direction of time. Unfortunately, the book opens with a highly non-standard exposition of time reversal invariance that distorts the subsequent discussion. The present article not only has the remedial goal of setting the record straight about the meaning of time reversal invariance, but it also aims to show how the niceties of this symmetry concept matter to the problem of the direction of time and to related foundation issues in physics.*

1. Introduction

David Albert's *Time and Chance* (2000) represents yet another tilting with that perennial philosophical windmill, the problem of the direction of time. Although the writing style occasionally gets in the way of the presentation, the book rewards the reader with fresh insights into a cluster of issues about the implications of statistical thermodynamics for time's asymmetries and also provides a fascinating, if somewhat speculative and sketchy, hypothesis about how the measurement problem in quantum mechanics and the problem of the direction of time intersect. Unfortunately the book gets off on the wrong foot with a bizarre treatment of time reversal invariance. This treatment leads to the claim that, contrary to the conventional wisdom, classical electrodynamics is "*not* invariant under time reversal" (p. 14). Furthermore

> neither (it turns out) is quantum mechanics, and neither is relativistic quantum field theory, and neither is general relativity, and neither is supergravity, and neither is supersymmetric string theory, and neither (for that matter) are any of the candidates for fundamental theory that anybody has taken seriously since Newton. And everything everybody has said to the contrary … is wrong. (p. 14)

It is precisely because Albert's book is so valuable and will be widely discussed that the record needs to be set straight about these matters.

Sections 2 and 3 deal with Albert's treatment of time reversal invariance and my proposals for an alternative account. Section 4 shows why the choice between these alternative accounts matters for a number of issues that fall under the umbrella of the problem of the direction of time. Section 5 contains concluding remarks.

2. What is wrong with Albert's treatment of time reversal invariance

Albert's basic account of time reversal invariance goes like this: (putative) laws L are time reversal invariant just in case for any sequence $S_i, \ldots, S_f$ of instantaneous states (where the temporal order of states runs from left to right) in accord with L, the reverse sequence $S_f, \ldots, S_i$ is also in accord with L.[1] Albert imposes two conditions on the specification of an instantaneous state:

(i) It must be *genuinely instantaneous*, which is to say that state descriptions at different times must be logically, conceptually, and metaphysically independent.

(ii) It must be *complete*, which is to say that all of the physical facts about the world can be read off the full set of state descriptions.

In the case of a theory which posits a pure particle ontology it is clear—to Albert—that the instantaneous state $S(t^\star)$ at some time $t^\star$ is specified by giving the positions $\mathbf{x}(t^\star)$ of all the particles at $t^\star$. This is contrary to the standard practice in physics which takes the state to be $(\mathbf{x}(t^\star), \mathbf{v}(t^\star))$ where $\mathbf{v}(t^\star)$ is a specification of the instantaneous velocities of the particles.[2] The reason for the standard practice is that physicists have in mind a different completeness condition; namely

(ii′) It must be *complete*, which is to say that all of the physical facts about the world *at the time in question* can be read off the state description *at that time*.

Apart from any sophisticated reasons deriving from modern physics, there is a simple motivation for preferring (ii′) to (ii): we would like to be able to say to Zeno of Elea that there is a difference between an arrow moving at $t^\star$ ($\mathbf{v}(t^\star) \neq 0$) and a stationary arrow ($\mathbf{v}(t^\star) = 0$). But physicists also want *dynamical completeness* of state descriptions; that is, they do not want to preclude *ab initio* the possibility of Laplacian determinism by excluding information from the instantaneous state description that is relevant to determining, via the laws of motion, the future and past histories of the system. In the case where the equations of motion are second (or higher) order with respect to time, the exclusion of velocity has precisely this preclusive effect.

Albert, of course, recognizes all of this, but he still stands fast against including instantaneous velocity in the state description since he thinks the inclusion violates the independence condition (i). But, as he admits, independence does hold for the standard state description in the sense that for any two distinct times t_1 and t_2, $(\mathbf{x}(t_1), \mathbf{v}(t_1))$ and $(\mathbf{x}(t_2), \mathbf{v}(t_2))$ are logically, conceptually, and metaphysically independent. What is violated is the much stronger requirement that $(\mathbf{x}(t), \mathbf{v}(t))$ is independent of *all* of the other states in an interval of time $(t_1, t_2) \ni t, \ t_1 < t_2$. But this violation is hardly shocking—indeed, if instantaneous velocity means what it is usually taken to mean, the strong form independence in question must fail in order that the limit, that defines the velocity as the time derivative of position, exists.[3] Someone who believes in the importance of completeness in the sense of (ii′) will be ready to tolerate the very mild dependence of states needed to secure (ii′).

Albert admits that instead of using his preferred notion of instantaneous state one could work with what he calls the *dynamical condition*, which is just what physicists call the state. But then a more elaborate explanation of time reversal invariance is required; in Albert's nomenclature, if the sequence of dynamical conditions $D_i, \ldots, D_f$ is in accord with L, then so is $^R D_f, \ldots, ^R D_i$ where $^R D$ stands for the time reverse of D. More perspicuously, if the history $t \mapsto D(t)$ is in accord with L, then so is the time reverse

history $t \mapsto {}^{T}D(t)$, where ${}^{t}D(t)$, $= {}^{R}D(-t)$. In the case of particle mechanics $D(t) := (\mathbf{x}(t), \mathbf{v}(t))$ and "R" is implemented by reversing the velocities so that the requirement of time reversal invariance becomes: if the history $t \mapsto (\mathbf{x}(t), \mathbf{v}(t))$ is in accord with the laws, so is $t \mapsto (\mathbf{x}(-t), -\mathbf{v}(-t))$. In particle mechanics the difference between the Albert and the standard treatment doesn't come to much since, if $t \mapsto \mathbf{x}(t)$ is a differentiable trajectory, $t \mapsto \mathbf{v}(t)$ can be read off the Albert history $t \mapsto S(t)$ where $S(t) := \mathbf{x}(t)$.

So is the difference between Albert's treatment and standard treatment just a tempest in a tea pot? Albert thinks not, for he thinks that outside of particle mechanics a substantive difference emerges. The usual treatment using the dynamical conditions commits one to defining the operation $D \mapsto {}^{T}D$, or equivalently the operation $D \mapsto {}^{R}D$; but the textbook stories usually told about these operations are, Albert thinks, nonsense. Take, for example, classical electromagnetism. The physics texts implement time reversal by transforming the dynamical condition by reversing the velocities of the charges, reversing the magnetic fields, and leaving the electric fields the same. Here is Albert's comment:

> The thing is that this identification is *wrong*. Magnetic fields are *not* the sorts of things that any proper time-reversal transformation can possibly turn around. Magnetic fields are not—either logically or conceptually—the *rates of change* of anything. If S_i, ..., S_f is a sequence of instantaneous states of a classical electrodynamical world, and if the sequence of dynamical conditions corresponding to S_i, ..., S_f is D_i, ..., D_f, and if we write the sequence of dynamical conditions corresponding to S_f, ..., S_i as ${}^{R}D_f$, ..., ${}^{R}D_i$, then the transformation from D_k to ${}^{R}D_k$ can involve nothing whatsoever other than *reversing the velocities of the particles*. And if that is the case, and if D_f, ..., D_f is in accord with the classical electrodynamical laws of motion, then, in general ${}^{R}D_f$, ..., ${}^{R}D_i$ will *not* be.[4] (pp. 20–21)

How does it follow that since magnetic fields are not the rates of change of anything, they cannot be "turned around" by a time reversal operation? If X is produced by Y and Y *is* "turned around" by time reversal, then X must also be affected by time reversal. And so it is with the magnetic field: if the velocities of the charges travelling, say, along a wire are reversed, then the magnetic field produced by the electric current is also reversed. In the following section it will be seen in detail how the reversal of motion prescription for particles forces out the standard account of the time reversal behaviour of electromagnetic fields. Since it follows from this account that classical electromagnetism is time reversal invariant, we have the beginnings of a rebuttal to Albert's claim that "(notwithstanding what all the books say) there have been dynamical distinctions between past and future written into the fundamental laws of physics for a century and a half now" (p. 21). But it will turn out (and this is the really important thing) that until we have a proper grip on what time reversal invariance means, we cannot even give a precise sense to phrases like "dynamical distinctions between past and future".

Before turning to the details of electromagnetism, I want to take up Albert's treatment of time reversal invariance in quantum mechanics (QM). QM is an especially interesting case since Albert's notion of instantaneous state coincides with (what he calls) the dynamical condition: both are given by the state vector ψ (or more precisely, the ray of Hilbert space in which this vector lies). When you think about how Albert wants to understand time reversal invariance, it follows immediately that the law of motion in QM—the Schrödinger equation—is not time reversal invariant: in general, it

is not the case that if $t \mapsto \psi(t)$ satisfies this equation, then so does $t \mapsto \psi(-t)$, and by Albert's lights this means a failure of time reversal invariance.

Albert makes a stronger claim, which I reconstruct as follows: in any theory where (a) the instantaneous state and the dynamical condition coincide, (b) the laws are time translation invariant, (c) the laws are deterministic, and (d) the laws are time reversal invariant, then the laws do not allow the state to change with time (see p. 132, an especially n. 3). The basic idea is that since by (a) $S(t) = D(t)$ and since $S(t)$ cannot be "turned around" or non-trivially transformed by time reversal, $^T D(t) = {}^R D(-t) = D(-t)$ and, consequently, $^T D(0) = D(0)$. If time reversal invariance did hold, then determinism applies equally to past and future[5], with the upshot that for every history $t \mapsto D(t)$ in accord with the laws, $D(+t) = D(-t)$ for all t. Now add time translation invariance which implies that if $t \mapsto D(t)$ is in accord with the laws, then so is the new history $t \mapsto D'(t)$ where $D'(t) = D(t + \Delta)$ for any chosen constant Δ. As with the original history, so also with the new one: $D'(+t) = D'(-t)$. But $D'(+\Delta/2) = D'(-\Delta/2) = D(+\Delta/2)$. Then by determinism, $D'(t) = D(t + \Delta) = D(t)$ for all t, which means that the dynamical condition in any history in accord with the laws is the same at every time.

To escape the absurd conclusion a *modus tollens* must be performed. Albert's version of the *modus tollens* is to say that QM is not time reversal invariant. But a more plausible move is to reject the notion that, because of (a), the dynamical condition cannot be "turned around" or non-trivially transformed by time reversal. To see why this is so in QM let's work in the position representation where the quantum state is given by a complex valued square integrable function $\psi(\mathbf{x})$ of $\mathbf{x}$. Consider a solution $t \mapsto \psi(\mathbf{x}, t)$ of the Schrödinger equation that describes the motion of a wave packet. Then the state $\psi(\mathbf{x}, 0)$ at $t = 0$ not only determines the probability distribution for finding the particle in some region of space at $t = 0$ *but it also determines whether at $t = 0$ the wave packet is moving, say, in the $+x$ direction or in the $-x$ direction*. Since $\psi(\mathbf{x}, 0)$ encodes information about the momentum of the particle it must be "turned around" or non-trivially transformed by the time reversal operation so that the reversed state at $t = 0$ describes the wave packet propagating in the opposite direction. So instead of making armchair philosophical pronouncements about how the state cannot transform, one should instead be asking: How can the information about the direction of motion of the wave packet be encoded in $\psi(\mathbf{x}, 0)$? Well (when you think about it) the information has to reside in the phase relations of the components of the superposition that make up the wave packet. And from this it follows that the time reversal operation must change the phase relations.

Good! Now we are getting somewhere. And let's go a little further and find out how ψ must transform. Write $t \mapsto {}^T \psi(t) = {}^R \psi(-t) = \hat{R} \psi(-t)$. In general the operator $\hat{R}$ should be consistent with the commutation relations. And it should reproduce classical results; in particular, if $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$ are respectively the position and momentum operators, we should have $\hat{R} \hat{\mathbf{x}} \hat{R}^{-1} = \hat{\mathbf{x}}$ and $\hat{R} \hat{\mathbf{p}} \hat{R}^{-1} = -\hat{\mathbf{p}}$.[6] For a single spinless particle these constraints turn out to fix $\hat{R}$ to be of the form $\hat{U} \hat{K}$ where $\hat{U}$ is a unitary operator (that depends on the representation we are using) and $\hat{K}$ is complex conjugation (see Sachs, 1987, Ch. 3). It is the latter that changes the phase relations among the components of the superposition that makes up the wave packet and effects a reversal of the direction of motion of the wave packet. With a little more work it can be shown that the following conditions are equivalent: (i) if $\psi(t)$ solves the Schrödinger equation $\hat{H} \psi(t) = i\hbar (\partial \psi(t)/\partial t)$ (where $\hat{H}$ is the Hamiltonian operator), then so does $\hat{R} \psi(-t)$, (ii) $\hat{R} \hat{H} = \hat{H} \hat{R}$, and (iii) for any ψ_1 to ψ_2, the transition probability from ψ_1 to ψ_2 in time Δt is equal to the

transition probability from $\hat{R}\psi_2$ to $\hat{R}\psi_1$ in the same Δt. Condition (ii) serves as the theoretical criterion for time reversal invariance for a quantum system with a specified Hamiltonian. And, of course, it turns out that the quantum counterparts of time reversal invariant classical Hamiltonian systems pass this criterion. Condition (iii) is useful in experimentally testing time reversal invariance in cases where the Hamiltonian is unknown and (ii) cannot be applied but the transition probabilities can be measured.

Squaring Albert's view of time reversal in QM with the Correspondence Principle calls for contortions, even for the simplest case of a single spinless particle moving in a velocity independent potential. Albert agrees that the classical equations of motion for this case are time reversal invariant (it's just that he doesn't want to include the velocity or momentum of the particle in the instantaneous state description). If the Correspondence Principle holds then there must be a solution to the Schrödinger equation corresponding to any allowed classical motion of the particle—in particular, there must be solutions corresponding to the given motion of the classical particle and to its time reversed motion. The relation between these solutions to the Schrödinger equation is characterized by the conditions stated in the preceding paragraph; in particular, the operator $\hat{R}$ is chosen so that if $t \mapsto \psi(t)$ is a solution describes a wave packet propagating in the $+x$ direction, then $t \mapsto {}^T\psi(t) = \hat{R}\psi(-t)$ describing a wave packet propagating in the $-x$ direction. But Albert's view of time reversal in QM prevents him from calling these solutions the time reverses of one another even though they correspond to the classical motions that, he agrees, are the time reverses of one another. This, I submit, is the symptom of a perverse view.

In sum, not only has Albert's armchair philosophizing led him up a blind alley, it also obscures some elementary but extremely interesting physics. I do not mean to suggest by the above examples that fixing the properties of the time reversal operation is always such an easy or straightforward matter. It is not, and in some instances the quest may not end in any clear answer. But to intimate that the quest is meaningless or else of no interest for the issue of the direction of time is to do a serious injustice to both the physics and the philosophy.

I now turn to the task of making good on my claims about electromagnetism. In doing so I hope also to bring further clarity to the meaning of time reversal invariance by way of indicating how the time reversal transformation is determined in various settings.

3. Time reversal in particle mechanics and electromagnetism

Let's begin by considering in more detail the case of the mechanics of particles in Newtonian or Minkowski spacetime. Without begging any questions about time reversal we can describe some kinematically possible history of the system by specifying the particle worldlines in the spacetime, along with a (constant) mass associated with each worldline. (Here I am in agreement with Albert: that the history of the system can be described in this manner captures the core meaning of saying that the system is a pure particle system.) To keep life simple, assume that the particles are point masses, that they never collide, and that their worldlines are everywhere differentiable. Thus far, there is no preferred parametrization associated with these worldlines. The picture is, therefore, that given in Figure 1.

Now add to the picture a time orientation by choosing a continuous non-vanishing timelike vector field—the arrows in this field tell us the direction of the future.[7] Such an orientation picks out a preferred parametrization of the particle worldlines; namely, the
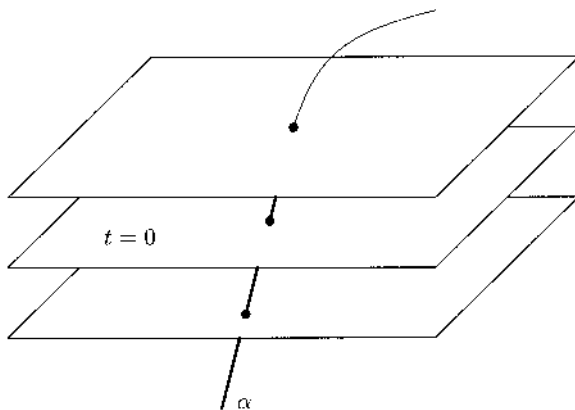
Figure 1. Particle history α in a non-temporally oriented world.

one in which the four-vector tangent field V^a, $a = 1, 2, 3, 4$ (with the parameter being absolute time in the Newtonian case and proper time in the Minkowski case) points to the future.[8] The picture now is given by Figure 2.

Literal time reversal is given by reversing the time orientation, and this reversal induces a reversal of the four velocities of the particles, as in Figure 3. One would like to say that the laws governing the particle motions are literally time reversal invariant just in case whenever they allow a history, as in Figure 2, they also allow the time reversed history, as in Figure 3. But the trouble is that in the actual world we can't dial the time orientation. Nor can we check time reversal invariance by travelling between the worlds of Figure 2 and 3—at least not without the help of a magical rocket ship that lets us traverse metaphysical space between different possible worlds.

The solution is to ask: In the world of Figure 2, what is the counterpart of the process pictured in Figure 3? The answer is ambiguous up to continuous symmetries of the spacetime—the inhomogeneous Galilean transformations in the case of Newtonian spacetime and the inhomogeneous Lorentz transformations in the case of Minkowski spacetime. This ambiguity seems at first disturbing since it leads to an ambiguity in the
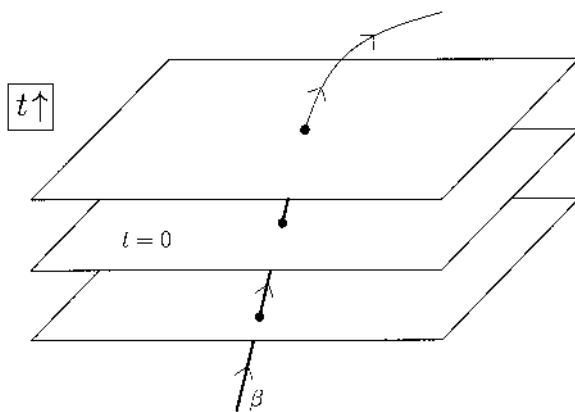

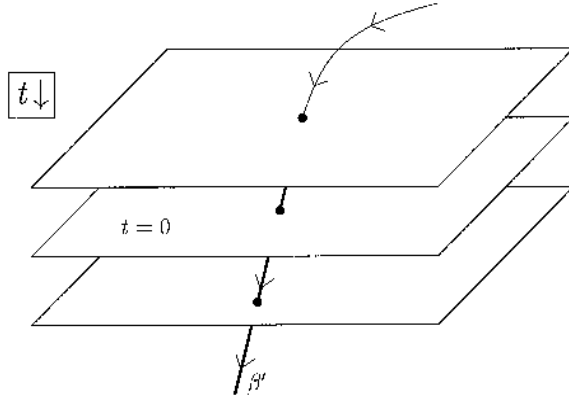**Figure 2.** Particle history β in a temporally oriented world.

Figure 3. Particle history β' in a world with the opposite temporal orientation.

definition of the time reversal transformation. But note that the laws based on these spacetimes must be invariant under these continuous symmetry transformations; for example, laws of motion for the particles can fail to be invariant under spatial translations only by picking out a preferred origin of space and, thus, presupposing more structure to spacetime than is actually present if spacetime is Newtonian or Minkowskian. In effect, the choice of different counterparts of the process of Figure 3 will result in different admixtures of time reversal invariance and the invariances corresponding to the continuous spacetime symmetries. But this is not a bad thing if what we are ultimately interested in is the *total* invariance of the laws. For sake of simplicity I will choose the counterpart that effects a clean separation between time reversal invariance on the one hand and space translation invariance, time translation invariance, etc. on the other. Figure 4 shows the original time oriented process of Figure 2 together with the chosen counterpart of Figure 3. From this we can read off the transformation rule for the four-velocity V^a:

$$^{T}V^{4}(\mathbf{x},\ t) = V^{4}(\mathbf{x},\ -t),\ ^{T}V^{\alpha}(\mathbf{x},\ t) = -\ V^{\alpha}(\mathbf{x},\ -t),\ \alpha = 1,\ 2,\ 3. \tag{1}$$
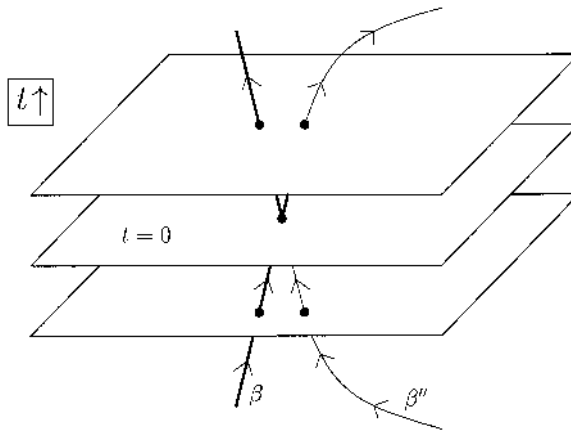

Figure 4. Particle history β and its temporal reverse β'' in the same temporally oriented world.

Thus, the standard account of time reversal for particles is vindicated. To check whether some putative laws of motion are or are not time reversal invariant the transformation rule for mass is needed. But since mass is assigned to the worldlines prior to a choice of time orientation, the rule must be that $^{T}m = m$. It turns out, of course, that the familiar laws of particle mechanics are time reversal invariant.

Turning to the Maxwell theory of electromagnetism in Minkowski spacetime, Maxwell's equations can be written in covariant form using the Maxwell tensor $F^{ab} := \nabla^{[a}A^{b]}$, where $A^a(\mathbf{x}, t)$ is the four-potential.[9] In this notation, Maxwell's equations are $\nabla_b F^{ab} = \mathcal{J}^a$ and $\nabla_{[a}F_{bc]} = 0$. In the Lorentz gauge, Maxwell's equations imply that the four-potential satisfies the wave equation $\nabla^b\nabla_b A^a = \mathcal{J}^a$, where $\mathcal{J}^a := \rho V^a$ is the four-current and ρ is the charge density. The general solution to the inhomogeneous wave equation can be written as

$$A^a(\mathbf{x}, t) = \int G(x, x')\mathcal{J}^a(x')\mathrm{d}^4x' \tag{2}$$

where $x := (\mathbf{x}, t)$ and $G(x, x')$ is the Green's function satisfying $\nabla^b\nabla_b G(x, x') = \delta^4(x, x')$.[10] I propose that (2) be taken as the definition of the four-potential arising from $\mathcal{J}^a$ or, equivalently, that (2) together with $F^{ab} := \nabla^{[a}A^{b]}$ be taken as the definition of the Maxwell tensor arising from $\mathcal{J}^a$. The time reversed counterpart $^{T}A^a$ to A^a arising from the time reversed current $^{T}\mathcal{J}^a$ is then given by

$$^{T}A^a(\mathbf{x}, t) = \int G(x, x')(^{T}\mathcal{J}^a(x'))\mathrm{d}^4x'. \tag{3}$$

Comparing (2) and (3) and using (1) and the transformation rule $^{T}\rho = \rho$ (which, I take it, is agreeable to Albert) it follows from the transformation rule for V^a that

$$^{T}A^4(\mathbf{x}, t) = A^4(\mathbf{x}, -t), \quad ^{T}A^\alpha(\mathbf{x}, t) = -A^\alpha(\mathbf{x}, -t), \quad \alpha = 1,2,3. \tag{4}$$

The gauge invariant definitions of the electric and magnetic fields are

$$\mathbf{E} := -\nabla A^4 + \frac{d}{dt}\mathbf{A}, \quad \mathbf{B} := \nabla x\mathbf{A}. \tag{5}$$

From (4) and (5) it follows that $^{T}\mathbf{B}(\mathbf{x}, t) = \nabla x(^{T}\mathbf{A}(\mathbf{x}, t)) = -\nabla x\mathbf{A}(\mathbf{x}, -t)$, with the upshot

$$^{T}\mathbf{B}(\mathbf{x}, t) = -\mathbf{B}(\mathbf{x}, -t). \tag{6}$$

And for the electric field it follows that

$$^{T}\mathbf{E}(\mathbf{x}, t) = -\nabla(^{T}A^4(\mathbf{x}, t)) + \frac{d}{dt}(^{T}\mathbf{A}(\mathbf{x}, t)) = -\nabla A^4(\mathbf{x}, -t) +$$
$$\lim_{\Delta t \to 0}[^{T}\mathbf{A}(\mathbf{x}, t + \Delta t) - {}^{T}\mathbf{A}(\mathbf{x}, t)] = -\nabla A^4(\mathbf{x}, -t) + \lim_{\Delta t \to 0}$$
$$[-\mathbf{A}(\mathbf{x}, -t - \Delta t) - \mathbf{A}(\mathbf{x}, -t)] = -\nabla A^4(\mathbf{x}, -t) + \frac{d}{dt}\mathbf{A}(\mathbf{x}, -t).$$

The upshot is that

$$^{T}\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}, -t). \tag{7}$$

One then verifies that the time reversed fields and currents satisfy Maxwell's equations if the original fields and currents do.

To deal with source-free fields, note that any homogeneous solution to Maxwell's equations can be written as the difference of two inhomogeneous solutions. Then assuming that the linear superposition of the four-potentials corresponding to the

superposition of two solutions to Maxwell's equations transforms as $^{T}(A_1^a + A_2^a) = {}^{T}A_1^a + {}^{T}A_2^a$, it follows that the source-free fields also obey (6) and (7).[11]

One might worry that the crucial assumption in my construction—namely, that equation (2) defines the four-potential—smuggles in too much of Maxwell's equations. To assuage this worry I will mention a result of David Malament.[12] For any unit timelike vector field ξ^a (which may be thought of as the four-velocity of a reference frame), the four-vector electric field E^a relative to ξ^a is defined from the Maxwell tensor F^{ab} by $F_n^a \xi^n$. If, as Albert assumes, $^{T}E^a = E^a$ and if $^{T}E^a$ is understood as $(^{T}F_n^a)(^{T}\xi^n)$, then it follows from the assumed transformation property of four-velocity that the magnetic field must obey the standard transformation law. This result makes it clear that the time reversal properties of the electromagnetic field can be derived using assumptions that Albert should accept and that do *not* presuppose that transformed fields satisfy Maxwell's equations. The disadvantage is that, unlike my approach, it assumes rather than derives the transformation of the electric field.

In sum, I hope that the reader is convinced that, at least in the cases of particle mechanics, electromagnetism, and ordinary QM it is not at all mysterious or controversial how the time reversal properties of physical quantities are determined.[13] Given that time reversal inverts the direction of motion of a classical particle, the time reversal behaviour of the electric current, the electromagnetic field, and the quantum wave function are forced out with the help of entirely natural assumptions. Of course, the further away one gets from these simple cases, the more difficult it can become to fix the action of the time reversal transformation; but the situation with respect to time reversal is no different from that of any other concept which has a secure core but becomes progressively vaguer as it is extended to non-core cases.

4. Why it matters how we understand the time reversal operation

The details of time reversal invariance matter little for Albert's main concerns with the foundations of statistical mechanics. But they do matter a great deal for other aspects of the problem of the direction of time, some of which are engaged in Albert's book and others of which are not. In this section I will outline what I take to be the most interesting of these issues.

Suppose that all the fundamental laws of physics are time reversal invariant. What follows from this supposition? Albert tells us that it means that the fundamental laws "fail to distinguish between past and future" and that this in turn is said to mean that (1) whatever can happen can also happen backwards in time, and (2) the algorithms of physics for inferring towards the past and future are identical. What claim (1) amounts to depends, of course, on the proper understanding of time reversal invariance, and I hope to have brought some clarity to that issue. In particular, it is worth reiterating the following.

Past and future as temporal mirror images

Time reversal invariance of the laws L does *not* imply that in a history $t \mapsto D(t)$ in accord with L, then the future and the past are mirror images of each other in the sense that $D(-t) = {}^{R}D(t)$ and $D(t) = {}^{R}D(-t)$. Given determinism and the assumption that $D(0) = {}^{R}D(0)$ this conclusion does follow. But the latter assumption holds only in very special cases. (a) In classical mechanics $D(0) = {}^{R}D(0)$ holds just in case the three velocities of all the particles vanish at $t = 0$.[14] Such cases have unpleasant consequences

for cats, as will be seen shortly.[15] (b) For the case of electromagnetism $D(0) = {}^R D(0)$ holds just in case $\mathbf{B}(0) = 0$ and $\mathbf{j}(0) = 0$. (c) For the quantum mechanical treatment of a spinless particle $D(0) = {}^R D(0)$ holds just in case $\psi^\star(\mathbf{x}, 0) = \exp(i\alpha)\psi(\mathbf{x}, 0)$, $\alpha = \text{const.}$; for in the position representation for a spinless particle, the reversal operator $\hat{R}$ is just complex conjugation, and ψ and $\hat{R}\psi$ correspond to the same state just in case $\hat{R}\psi$ and ψ belong to the same ray. It is then an easy exercise to show that the condition that $\psi^\star(\mathbf{x}, 0) = \exp(i\alpha)\psi(\mathbf{x}, 0)$ implies that the expectation value of momentum is zero at $t = 0$. (So if you want to move, your wave function can't be real and it can't be purely imaginary; rather, your wave function has to be complex, and more complex than the condition in question allows.)

In the more typical cases where $D(0) \neq {}^R D(0)$ a failure of past and future to be temporal mirror images of one another is perfectly compatible with time reversal invariance of the laws. This is just an instance of the more general fact that contingent conditions don't have to exhibit the symmetries of the laws that govern them, e.g. spatial translation invariance of the laws doesn't mean that the matter content of space has to be arranged homogeneously. A tension between the time reversal invariance of laws and the observed asymmetries in time in the actual universe arises only if it is insisted that these temporal asymmetries cannot be de facto.

Claim (2) also calls for clarification.

Algorithms for inferring the past and future

I will concentrate exclusively on one sense of this notion of algorithm of inference—determinism. I will assume throughout that time translation invariance holds.[16] Say that the laws L are *future* (respectively, *past*) *deterministic* just in case for any histories $t \mapsto D_1(t)$ and $t \mapsto D_2(t)$ that are in accord with L, if $D_1(t^\star) = D_2(t^\star)$ for any particular $t^\star$, then $D_1(t) = D_2(t)$ for any $t > t^\star$ (respectively, $t < t^\star$). Then if the definition of time reversed history $t \mapsto {}^T D(t) = {}^R D(-t)$ is such that ${}^R({}^R D) = D$, time reversal invariance for L implies that future and past determinism stand or fall together—either both hold or both fail. Since this involutional character of the "R" operation is a common feature of all the cases studied in the physics literature—and, arguably, is an essential feature of time reversal—it follows that time reversal invariance requires that the deterministic algorithms for inferring the past and future are identical at any given point of time. And by time translation invariance, the algorithms are the same at all points of time. It will be seen below that if objective state vector reduction is allowed in QM, not only is determinism broken but so is the symmetry of inference to past and future.

If time reversal invariance fails this symmetry of inference can be broken. *But it need not be broken.* Indeed, in the case of elementary QM it is hard to see how it could possibly be broken in the presence of time translation invariance. To quantize a classical system means first putting the classical mechanical treatment in Hamiltonian form and then finding a suitable Hilbert space that admits the quantum analogue $\hat{H}$ of the classical Hamiltonian H as a self-adjoint operator. QM does not provide any unequivocal prescription for carrying out this task. But if it can't be done, then there is no QM for the system in question. If it can be done, then $\hat{U}(t) := \exp(-i\hat{H}t), -\infty < t < +\infty$, is a strongly continuous one-parameter family of unitary operators. $\hat{U}(t_1)\psi(0)$ is the "algorithm" for inferring the future state at $t = t_1 > 0$ and $\hat{U}(-t_1)\psi(0) = \hat{U}^{-1}(t_1)\psi(0)$ is the "algorithm" for inferring the past state at $t = -t_1 < 0$. So there is symmetry with respect to inferring the past and future whether or not $\hat{H}$ is such that time reversal invariance holds (which, recall, requires that $\hat{R}\hat{H} = \hat{H}\hat{R}$). Of course, it might be that a

classical system where time reversal invariance fails does not admit a Hamiltonian formulation, or if it does admit such a formulation there may be no corresponding quantum $\hat{H}$ that is self-adjoint. But in these cases there is no quantum dynamics at all and, hence, no quantum "algorithm" for inferring either the past or the future.

In classical mechanics one can give mathematical examples where time reversal invariance fails in such a way that, say, future determinism holds but past determinism fails. These are necessarily examples where different past histories can all eventuate in the same present state. As far as I am aware, however, all such examples resist Hamiltonian formulation[17] and, thus, the standard route to quantization is blocked. And in any case, as we saw above, quantum dynamics, when it exists, offers symmetric "algorithms" for inferring the past and future. It thus seems that standard QM is incapable of implementing one of the differences between past and future that Albert takes to be fundamental. It remains to be seen whether this fact tells us something about the nature of time or about the (in)adequacy of QM.

In addition to Albert's claims (1) and (2) there are a number of other issues about time reversal invariance which are not broached in Albert's treatment but which beg for attention.

Physical implications of time reversal invariance

Given the conceptual importance of time reversal invariance, it is more than a little surprising that it does not have many immediate implications for physics. In the regime of classical physics the only example the author of *The Physics of Time Reversal* can cite of how time reversal invariance figures in the solution of a problem in dynamics is Painléve's Theorem (aka the Bad News for Cats Theorem), which says that if a cat is composed of a collection of particles between which only conservative forces act, if the cat is initially upside down, and if the initial velocities of all the cat's particles vanish (and thus $D(0) = {}^{R}D(0)$), then the cat in free fall cannot land on its feet (see Sachs, 1987, pp. 14–18). Though important to cats and cat owners, this result does not have major importance for physics. In terms of concrete physical implications for non-quantum physics, time reversal invariance seems more interesting in the breach than in the observance.

More examples are forthcoming in QM. The involutional character of the reversal operator $\hat{R}$, which requires that ψ and $\hat{R}\psi$ belong to the same ray, and the fact that $\hat{R} = \hat{U}\hat{K}$ where $\hat{U}$ is unitary and $\hat{K}$ is complex conjugation, imply that $\hat{R}\hat{R} = \pm \hat{I}$. In a case where $\hat{R}$ satisfies $\hat{R}\hat{R} = -\hat{I}$ it follows from time reversal invariance that each energy eigenvalue E has a two-fold degeneracy (*Kramer's degeneracy*). For if $\hat{H}\psi = E\psi$ then (by time reversal invariance) also $\hat{H}\hat{R}\psi = E\hat{R}\psi$; but ψ and $\hat{R}\psi$ are orthogonal (see Sachs, 1987, Section 4.1). Examples of experimentally testable implications of time reversal invariance are more numerous in quantum field theory but are also harder to describe. The interested reader is referred to Sachs (1987).

Distinguishing past and future

In describing time reversal invariance in the preceding section I supposed that one of the two possible time orientations was singled out as *the correct* orientation, meaning that vectors in the field that specifies the orientation point towards the future. In the case of time reversal invariant laws that choice is arbitrary as far as the laws are concerned. But for pure particle theories something else seems to be true; namely, the histories of the

system of particles can be fully described without using any time orientation at all—i.e. by using bare world lines without preferred orientation—much less without having to broach the issue of which of the two possible time orientations is the right one. If this is correct, it would seem to follow that in a pure particle world obeying time reversal invariant laws of motion, there is no need to postulate that there is a time orientation or, equivalently, that relatively timelike events are time ordered. From the point of view of physics it suffices that triples of relatively timelike events are ordered by *temporal betweeness* $bet(p, q, r)$. If a time orientation is added, $bet(p, q, r)$ can be defined as: either p is earlier than q which is earlier than r, or else r is earlier than q which is earlier than p. But in the envisioned situation, $bet(p, q, r)$ can be taken as a primitive relation which does not presuppose a time orientation. This seems to have been Reichenbach's view in *The Direction of Time* (1971), where he speaks of a time oriented history and its time reversed counterpart as being "equivalent descriptions" of the same physical situation.[18]

If electromagnetic fields were merely auxiliary devices for talking about the interactions among charged particles,[19] then we would have a similar situation. But within standard Maxwell theory the electromagnetic field has a life of its own, as witnessed by the existence of source-free solutions. One could rule out source-free solutions representing radiation coming in from spatial infinity by imposing the so-called Sommerfeld radiation condition. But then the resulting theory is not time reversal invariant. Time reversal invariance can be restored by also imposing the time reverse of the Sommerfeld radiation condition to rule out electromagnetic fields propagating off to infinity. But in this modified theory with both radiation conditions in place, fields are being tied exclusively to sources and absorbers, and one is well on the way to viewing the fields as auxiliary devices. Here I want to assume that we are working with the standard time reversal invariant Maxwell theory without radiation conditions.

A specification of an ordinary (i.e. non-distributional) solution to Maxwell's equations requires that the Maxwell tensor F^{ab} is differentiable. Thus, the energy-momentum tensor field associated with the electromagnetic field, namely $T^{ab} := F^a_c F^{xb} + 1/4\eta^{ab}F^{cd}F_{cd}$, where η_{ab} is the Minkowski metric, is a smooth field. Let U^a be a continuous non-vanishing timelike vector field. Then the four-momentum $W^a := -T^{ab}U_b$ as measured relative to U^a is a continuous timelike field. If U^a and W^a point in the same direction at some point where F^{ab} is non-zero, they will point in the same direction at every point where F^{ab} is non-zero. If they do point in the same direction, U^a defines the future pointing (relative to F^{ab}) time orientation. The point is not that Maxwell electromagnetism picks out a unique preferred time orientation—of course it cannot since it is time reversal invariant. If $(F^{ab}, \mathcal{J}^a)$ is a solution to Maxwell's then so is $(\mathcal{T}F^{ab}, \mathcal{T}\mathcal{J}^a)$, where $\mathcal{T}F^{ab} := -F^{ab}$ and $\mathcal{T}\mathcal{J}^a := -\mathcal{J}^a$. $(F^{ab}, \mathcal{J}^a)$ and $(\mathcal{T}F^{ab}, \mathcal{T}\mathcal{J}^a)$ are the electromagnetic counterparts of the particle worlds of Figures 2 and 3 which have opposite time orientations. Just as time reversal invariant particle mechanics cannot tell us whether we reside in the world of Figure 2 or the world of Figure 3, so Maxwell electromagnetism cannot tell us whether we reside in the world of $(F^{ab}, \mathcal{J}^a)$ or the world of $(\mathcal{T}F^{ab}, \mathcal{T}\mathcal{J}^a)$. But unlike pure particle mechanics, an electromagnetic world obeying Maxwell's equations is not happily described as a world that lacks a time orientation and is endowed with only a temporal betweenness relation. In the case of time reversal invariant particle mechanics the unoriented world lines of Figure 1 serve as the time-orientation neutral content that is common to the worlds of Figure 2 and 3. What serves as the time-orientation neutral content common to $(F^{ab}, \mathcal{J}^a)$ and $(\mathcal{T}F^{ab}, \mathcal{T}\mathcal{J}^a)$? If the disjunction of $(F^{ab}, \mathcal{J}^a)$ and $(\mathcal{T}F^{ab}, \mathcal{T}\mathcal{J}^a)$ or the double-valued ($\pm F^{ab}$, $\pm\mathcal{J}^a$) is allowed as an answer, then electromagnetism as well as particle mechanics falls under Reichenbach's doctrine of equivalent descriptions.

Such answers, however, might be rejected as question-begging. This matter is worth pursuing, but instead of pursuing it here I will instead ask what a violation of time reversal invariance can do for us.

Is our universe temporally orientable? Is it temporally oriented?

Above I have been assuming that physics is done in Newtonian spacetime or Minkowski spacetime. But we live not in such a world but in a world of general relativity. The very existence of a Lorentz signature metric (as presupposed by Einstein's general theory of relativity) already imposes some non-trivial conditions on the global topology of the spacetime manifold $\mathcal{M}$; but it does not guarantee that spacetime is temporally orientable. Locally, of course, spacetime must be temporally orientable since for any point $p \in \mathcal{M}$ a small enough neighbourhood $N(p)$ can be chosen that N is simply connected, and any simply connected manifold with a Lorentz signature metric admits a continuous non-vanishing timelike vector field. But how can we be sure that our spacetime doesn't have some ghastly multiple connectivity in the large that prevents us from making a globally consistent distinction between past and future? We can't be certain, just as we can't be certain about any non-trivial issue about the global structure of spacetime. Nor does crude enumerative induction from local experience seem to help since, as we have just seen, temporal orientability can hold everywhere locally without holding globally. But for the issue in question a more sophisticated kind of induction can give the basis for a conditional form of certainty: *if* the laws of physics are "universal" in the sense that they are the same in every region of spacetime, *if* by local investigations we manage to find (some of) the basic laws of physics, and *if* these laws are not time reversal invariant, then we can infer that in our universe there is a globally consistent distinction between past and future. Roughly the idea is this. Choose any closed path in the spacetime. Suppose for purposes of *reductio* that the transport of a timelike vector around some such path by some method that is continuous and keeps timelike vectors timelike results in a flip in time sense when the vector returns to the starting point. Along this path choose a chain of overlapping simply connected neighbourhoods. Use the failure of time reversal invariance of the laws to pick out the future direction of time in each of these neighbourhoods. By choosing these neighbourhoods sufficiently small we can be assured that the time directions assigned to points within the neighbourhood are self-consistent when compared by any method that is continuous and keeps timelike vectors timelike; for otherwise the laws do not make consistent predictions. Thus the future direction picked out in two adjacent neighbourhoods N and N' must agree in the overlap $N \cap N'$. But by the *reductio* assumption this agreement must fail when the chain of neighbourhoods closes. Since a contradiction has been reached the *reductio* assumption must be false and the spacetime is globally temporally orientable.

It is sometimes claimed that the failures of time reversal invariance which have been experimentally detected in elementary particle physics[20] are irrelevant to the sorts of temporal asymmetries that we humans encounter. Even if this were so—and I think the jury is still out on this issue—it does not follow there are no important implications for the problem of the direction of time. For if what has been experimentally detected is in fact a failure of time reversal invariance of the fundamental laws of physics—and if these laws do have a universal character—then we can be sure that our universe is temporally orientable and that it is in fact temporally oriented. The first surety follows from the argument of the previous paragraph. The second surety follows from the first plus the

fact that a temporal orientation is needed to sort the dynamically possible from the dynamically impossible histories when time reversal invariance fails.

However, the laws themselves may not imply which of the two possible time orientations is the correct one. For a lawlike equation of motion L that is not time reversal invariant will be matched by a lawlike ^{T}L that is also not time reversal invariant and that makes exactly the opposite division of histories into those that are allowed and those that are not. So unless L and ^{T}L are distinguished by time orientation neutral predictions, the failure of time reversal invariance tells us that there is a time orientation but not which is the correct one. As far as I am aware, there are no plausible candidates for the role of what might be called *time reversal super-non-invariant laws SL* having the property that not only are SL not time reversal invariant but also that ST and ^{T}SL are distinguished by time orientation neutral predictions. It would be interesting if a proof that there could be no such laws could be found.

The measurement problem in quantum mechanics and the failure of T- and CPT-invariance

Crudely speaking solutions to the measurement problem in quantum mechanics can be divided into two main camps: non-collapse vs. collapse.[21] Those from the first camp maintain Schrödinger evolution but explain how measurements have outcomes by breaking the eigenvalue–eigenvector link and assigning definite values in a state ψ to selected quantum observables even though ψ is not an eigenstate of the operators corresponding to these observables.[22] Those from the second camp maintain the eigenvalue–eigenvector link but modify or selectively violate Schrödinger evolution so as to allow that the collapse of the wave packet or state vector reduction is an objective process. In *Time and Chance* Albert takes seriously the implementation of the collapse solution proposed by Ghirardi *et al.* (1986), and he speculates about how the time asymmetry entailed by the GRW collapse mechanism may help to underwrite the asymmetry of inferences we want to make about past and future. Here I do not want to comment on either the viability of the GRW solution or the plausibility of Albert's speculations. Rather I want to call attention to a general implication of objective state vector reduction.

To allow for the possibility of pure-to-mixed state transitions, take the state space to be the set of density matrices $\mathscr{D}(\mathscr{H})$ on the appropriate Hilbert space $\mathscr{H}$. Construe dynamics over some time interval (say, from t_i to t_f) in terms of a "superscattering matrix", i.e. a map $\mathbf{S}: \mathscr{D}(\mathscr{H}) \rightarrow \mathscr{D}(\mathscr{H})$.[23] If $\tilde{R}$ is the time reversal operator (acting on density matrices), then time reversal invariance requires that for any $\rho_i \in \mathscr{D}(\mathscr{H})$, if $\rho_f = \mathbf{S}\rho_i$ then also $\tilde{R}\rho_i = \mathbf{S}\tilde{R}\rho_f$. The only property of $\tilde{R}$ needed for the following result is its involutional character, i.e. $\tilde{R}\tilde{R} = \hat{I}$.[24]

> *Wald's (1980) theorem.* If $\mathbf{S}$ allows for pure-to-mixed transitions in that for some initial pure state $\rho_i \in \mathscr{D}(\mathscr{H})$ (i.e. $\rho_i^2 = 1$) the final state $\rho_f = \mathbf{S}\rho_i$ is mixed (i.e. $\rho_f^2 \neq 1$) and if $\mathbf{S}$ is linear (i.e. for any ρ_1, $\rho_2 \in \mathscr{D}(\mathscr{H})$ and any $0 < \lambda < 1$, $\mathbf{S}(\lambda\rho_1 + (1 - \lambda)\rho_2) = \lambda\mathbf{S}\rho_1 + (1 - \lambda)\mathbf{S}\rho_2)$, then $\mathbf{S}$ violates time reversal invariance.

In contrast to cases of time reversal invariance by unitary dynamics (which necessarily does not involve pure-to-mixed transitions), here the symmetry of inferences to the past and future is broken. For the present set-up, time reversal invariance for $\mathbf{S}$ (i.e. $\tilde{R}\mathbf{S} = \mathbf{S}\tilde{R}$) implies that $\mathbf{S}^{-1}$ exists; but Wald's proof shows that the existence of $\mathbf{S}^{-1}$ leads to contradiction in the presence of pure-to-mixed transitions. Wald's proof also

shows that, assuming that the total charge–parity–time-reversal operator $\tilde{C}\tilde{P}\tilde{R}$ is involutional, pure-to-mixed transitions violate CPT-invariance! As Wald notes, even if T- (or CPT-) invariance fails, effective T- (or CPT-) invariance can hold in the sense that the transition probabilities for a process and its T (or CPT) inverse can be the same.[25] Nevertheless, enough measurements will reveal that a pure-to-mixed transition has taken place, and this information, together with Wald's theorem, enables the experimenter to infer that time reversal invariance has been violated.

One way to escape Wald's result is to reject the assumption of linearity of the dynamics. However, he shows that this escape comes at the price of introducing non-causal propagation of information. The other escape route is to postulate hidden variables and to construct a dynamics on the total state space that is time reversal invariant and that, nevertheless, reproduces the transition from a pure to a mixed quantum state in the regime where the hidden variables are indeed hidden. How viable this latter escape route is can only be settled by appropriate go or no-go proofs.

In closing, it should be noted that objective state vector reduction, whether it is supposed to happen as a result of measurement interaction or spontaneously by means of a GRW mechanism, is highly controversial—I would estimate that the majority of workers in the foundations of QM deny it. The situation changes dramatically in the cosmological context.

Cosmology and time reversal invariance

The standard treatment of the initial value problem for Einstein's gravitational field equations for empty space takes the instantaneous dynamical state to be specified by the three-metric of the initial value hypersurface together with its "time derivative" (in the guise of the second fundamental form that specifies how the three-manifold is embedded as a spacelike hypersurface of the spacetime; see Wald (1984) for details). The natural time reversal operation leaves the three-metric unchanged and adds a minus sign to its time derivative.[26] Under this operation the source-free Einstein field equations are time reversal invariant. When source fields are present the issue of time reversal invariance passes to the character of the coupled Einstein–matter field equations. In the case where the source field is the electromagnetic field, the standard way of coupling electromagnetism and gravity produces coupled Einstein–Maxwell equations that are time reversal invariant. Similarly, time reversal invariance holds for the coupled Einstein–matter equations resulting from all known classical source fields.

However, when quantum fields are evolved on general relativistic spacetimes the verdict for time reversal invariance is rather different. In particular, the phenomenon of Hawking radiation indicates that black holes are not entirely black but radiate with a thermal spectrum. Furthermore, this radiation leads to an evaporation of the black hole.[27] And if the evaporation is complete (see Figure 5), then a quantum field evolving on this background spacetime will exhibit a pure-to-mixed state transition.[28] By the result discussed in the preceding subsection, T- and CPT-invariance will be violated.[29] I will indicate how the crucial pure-to-mixed transition is derived for the simplest case of a scalar quantum field governed by the Klein–Gordon equation.

The spacetime pictured in Figure 5 is not globally hyperbolic.[30] This poses a problem for talking about dynamics of the quantum field since existence and uniqueness results for the initial value problem for the Klein–Gordon field do not extend to such spacetimes. In general, determinism fails in such spacetimes, leaving no natural way to construe dynamics as an automorphism of some fixed algebra of observables; indeed it
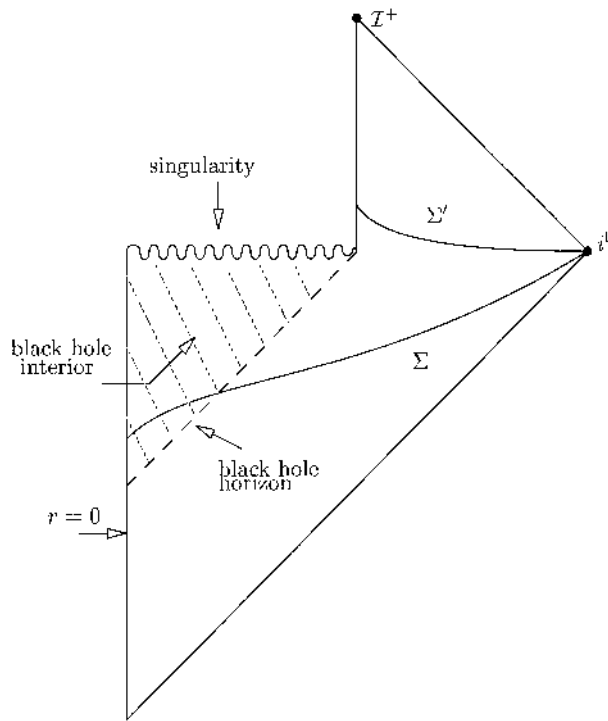
Figure 5. Conformal diagram of the evaporation of a spherically symmetric black hole.

is not even clear how to construct the algebra of observables of the quantum field in such an ill-behaved setting. Nevertheless, I claim that there is a clear and interesting sense in which, in some non-globally hyperbolic spacetimes, quantum evolution involves a pure-to-mixed state transition and, consequently, a violation of time reversal invariance.

The approach I will use is necessarily axiomatic rather than constructive. It uses the algebraic formulation of quantum field theory in which observables are elements of $C^\star$-algebras and states are positive linear functionals from algebras of observables to $\mathbb{C}$. In particular I will work here with algebras of four-smeared fields. For a globally hyperbolic spacetime $(\mathcal{M}, g_{ab})$ it is typically postulated that these fields satisfy a variety of axioms, including the following:

Axiom 1. There is a global $C^\star$-algebra $\mathscr{A}(\mathcal{M})$ associated with the entire spacetime $\mathcal{M}$. For any open region $\mathfrak{R} \subset \mathcal{M}$ of compact closure there is an associated $C^\star$-subalgebra $\mathscr{A}(\mathfrak{R}) \subset \mathscr{A}(\mathcal{M})$, and for any two such regions $\mathfrak{R}$ and $\mathfrak{R}'$ such that $\mathfrak{R} \subset \mathfrak{R}'$, $\mathscr{A}(\mathfrak{R}) \subset \mathscr{A}(\mathfrak{R}')$.

Axiom 2. If $\mathfrak{R}$, $\mathfrak{R}' \subset \mathcal{M}$ are relatively spacelike, then $[\mathscr{A}(\mathfrak{R}), \mathscr{A}(\mathfrak{R}')] = 0$.

Provably, these axioms do hold for the Klein–Gordon field on a globally hyperbolic spacetime (see Dimock, 1980). I assume that Axioms 1 and 2 continue to hold for the non-globally hyperbolic spacetime of Figure 5. And I also assume that in any given physical situation, the actual state on the $C^\star$-subalgebras $\mathscr{A}(\mathfrak{R})$, $\mathfrak{R} \subset \mathcal{M}$, is obtained by restricting the actual global state ω on $\mathscr{A}(\mathcal{M})$ to the respective subalgebras.

By a *time slice* I mean a spacelike hypersurface without edges. In the absence of dynamics in the usual sense, one can still talk about time evolution in a non-globally hyperbolic spacetime from one (necessarily non-Cauchy) time slice to another slice by taking the state at a given "time" Σ to be a normalized positive linear functional on the algebra $\mathscr{A}(\delta(\Sigma))$ of smeared fields associated with some thin sandwich $\delta(\Sigma)$ about Σ. It can then be established that for the non-globally hyperbolic spacetimes thought to describe the process of black hole evaporation, there is a large class of physically relevant global states on $\mathscr{A}(\mathscr{M})$ for each member of which it is the case that for some time slice Σ corresponding to the pre-evaporation regime (see Figure 5), the state on $\mathscr{A}(\delta(\Sigma))$ is pure, whereas for no time slice Σ' corresponding to the post-evaporation regime is the state on $\mathscr{A}(\delta(\Sigma'))$ pure. In this sense time evolution leads from a pure state to a mixed state. The proof is based on a simple lemma for $C^\star$-algebras:

> **Lemma.** If $\mathscr{A}$ is a $C^\star$-subalgebra of a $C^\star$-algebra $\mathscr{C}$ and if the restriction $\omega_{\mathscr{A}}$ of a state ω on $\mathscr{C}$ to $\mathscr{A}$ is pure, then $\omega(AC) = \omega(A)\omega(C)$ for all $A \in \mathscr{A}$ and any $C \in \mathscr{C}$ that commutes with every element of $\mathscr{A}$.

In the post evaporation regime of a spacetime representing black hole evaporation, a thin sandwich $\delta(\Sigma')$ of any time slice Σ' is relatively spacelike with respect to the interior region of the black hole. So by Axiom 2 the algebras associated with these regions commute. But physically relevant states on the global algebra are such that some elements of the two algebras are correlated because, for example, the ingoing and outgoing Hawking radiation associated with the black hole horizon establishes correlations between regions interior and exterior to the black hole. Thus, for such a state ω on the global algebra, there will be observables $A \in \mathscr{A}(\delta(\Sigma'))$ and $C \in \mathfrak{R}$ with $\mathfrak{R}$ belonging to the black hole interior region such that $\omega(AC) \neq \omega(A)\omega(C)$. It follows from the Lemma that the restriction of ω to $\mathscr{A}(\delta(\Sigma'))$ is mixed. Thus, if the restriction of ω to $\mathscr{A}(\delta(\Sigma))$, where Σ is a pre-evaporation time slice, is a pure state, a pure-to-mixed transition takes place.

Needless to say, there are many possible escape routes from this conclusion: one can deny that black hole evaporation is ever complete, or assert that even if evaporation is complete it does not leave a naked singularity, or assert that the correlations between regions interior to the black hole and regions exterior to the black hole are somehow washed out, etc. (see Belot *et al.*, 1999). So the connection of black hole evaporation to pure-to-mixed state transitions and, thence, to the failure of time reversal invariance remains controversial. Nevertheless, it provides a intriguing hint of how cosmology and the direction of time may be linked. Whether or not this linkage will be preserved in a complete quantum theory of gravity remains to be seen.

5. Conclusion

I have endeavoured to correct a number of misimpressions about time reversal invariance engendered by Albert's *Time and Chance*, and in doing so I have tried to remove some of the mystery about time reversal and at the same time have tried to show how both time reversal invariance and its failure are connected not only to the problem of the direction of time but to related foundation issues in physics. Little of what I have said is of direct relevance to some of the knottier aspects of the problem of the direction of time, e.g. why are we justified in inferring that it is much more likely that, in the future direction of time, the entropy of an isolated system will increase rather than decrease? This should not come as a surprise. "The problem of the direction of time" is a label

for a grab bag of problems, some of which are only loosely related. There has been an unfortunate tendency in the philosophical literature to try to stitch together these various problems into a unified picture—witness, for example, the many attempts to link together the many so-called "arrows of time" into a unified explanatory hierarchy. On close inspection the linkages and the hierarchical ordering turn out to rest on assumptions that are dubious or else are so vague that they do not even have the virtue of being false. Part of this unfortunate tendency stems from the desire to the roll together various mysteries and by doing so to find in the solution to one the solution of the others. Here too I am sceptical, especially of the claim that the solution of the mystery of measurement in QM will turn out to have much to do with solutions to any of the problems in the grab bag of "the problem of the direction of time". But that is the subject for another paper.

Acknowledgement

Notes

1. A similar view of time reversal invariance is found in Horwich (1987).
2. In the Hamiltonian formulation of mechanics the state is $(\mathbf{x}(t^\star), \mathbf{p}(t^\star))$ where $\mathbf{p}(t^\star)$ is a specification of the instantaneous momenta of the particles. The canonical momentum is not necessarily mass times velocity; see Section 3.
3. And if $\mathbf{x}(t)$ is a continuous function of t, then the Albert state $\mathbf{x}(t^\star)$ at $t^\star$ cannot be independent of *all* of the other Albert states in an interval of time $(t_1, t_2) \ni t^\star$.
4. I have slightly altered Albert's notation to conform to mine.
5. More on this below in Section 4.
6. In the Hamiltonian formulation of classical mechanics, which provides the basis for quantization, the canonical momentum is defined as $\mathbf{p} := \partial L/\partial \mathbf{v}$, where $L = T - V$ is the Lagrangian with T and V respectively the kinetic and potential energy. If V is velocity dependent, then $\mathbf{p}$ may not be simply $m\mathbf{v}$. I ignore this complication here. Thus, it is fair to assume that classically $\mathbf{p}$ transforms like $\mathbf{v}$ under time reversal. So by the Correspondence Principle, $\hat{R}\hat{\mathbf{p}}\,\hat{R}^{-1} = -\hat{\mathbf{p}}$.
7. One venerable position on the problem of the direction of time is that only time reversal non-invariant laws can provide a sufficient basis for assigning such an orientation to time itself. I will return to this issue in Section 4 in the subsection on *Distinguishing past and future*.
8. My convention is that the fourth component of V^a is the time component. General relativistic spacetimes may not be globally temporally orientable—they may not admit a continuous non-vanishing timelike vector field. In Section 4 I will raise the issue of how time reversal invariance, or the lack of it, is related to temporal orientability.
9. A^4 is called the scalar potential and is typically denoted by Φ, whereas the spatial components A^a, $\alpha = 1$, 2, 3, of A^a form what is called the vector potential.
10. The retarded and advanced Green's functions—leading to retarded and advanced solutions—are particular solutions. In some treatments of time reversal invariance there is an interchanging of retarded and advanced potentials. This idea does not have to be invoked in order to obtain the time reversal properties of the electromagnetic field.
11. I am grateful to John Norton for helping me to clarify this point. There are many consistency checks on the above results, e.g. using the Lorentz force law or the Lorentz transformation properties of the electric and magnetic fields. All of these checks are passed.
12. The details of the proof will be published elsewhere. I am grateful to David for allowing me to mention his result.
13. Arntzenius (2001) also rejects Albert's account of time reversal invariance. But his account of the time reversal operation is rather different from the one developed here.
14. Assertion (a), as well as (b) and (c) immediately following, apply to a particular inertial frame. But it is easy to verify that if the future and the past are mirror images of each other—in the sense that

$D(-t) = {}^{R}D(t)$ and $D(t) = {}^{R}D(-t)$-holds in one inertial frame, then it holds in all. This is a consequence of the way the relevant quantities transform under Galilean or Lorentz transformations.

15. See the subsection *Physical implications of time reversal invariance*.

16. I agree with Albert when he says that time translation invariance holds for "every single one of the candidates for a fundamental dynamical equation of the evolution of the world that anybody has taken seriously since the scientific revolution of the Renaissance" (p. 132, fn. 2). Recently evidence has surfaced from cosmology that the fine structure constant changes with time. However, the matter is highly speculative and will not be considered here.

17. It is hard to reconcile such cases with Liouville's theorem (conservation of volume in phase space) which holds for Hamiltonian mechanics. And any unconstrained Hamiltonian system which is time translation invariant is deterministic, both past and future.

18. "Since it is always possible [in the case of a time reversal invariant system] to construct a converse [i.e. a time reverse] description, positive and negative time supply equivalent descriptions, and it would be meaningless to ask which of the two descriptions is true" (1971, pp. 31–32).

19. There are attempts to demote the electromagnetic field to such an auxiliary status; see, for example, Wheeler and Feynman (1949).

20. The original detection of time reversal non-invariance was indirect: it provided direct evidence of CP-violation and then an appeal to the CPT theorem was made to get the conclusion that T must also be violated. More recently experimentalists have claimed to have measured direct violations of T-invariance; see Alvarez-Gaumé *et al.* (1999). The CPT theorem for quantum field theory is based on three main assumptions: that the fields are covariant under Lorentz transformations, that the fields at relatively spacelike points commute, and that the four-momentum lies in or on the forward light cone.

21. The leading options are brilliantly described in Albert's *Quantum Mechanics and Experience* (1992).

22. One cannot be too profligate in such assignments on pain of running into Kochen–Specker difficulties.

23. If one likes allowance can be made for the case where the initial and final Hilbert spaces $\mathcal{H}_i$ and $\mathcal{H}_f$ are different (i.e. non-isomorphic) spaces.

24. The notation $\tilde{R}$ is used to distinguish the reversal operator acting on density matrices from the reversal operator $\hat{R}$ acting on state vectors. As noted above, the involutional character of $\hat{R}$ implies not that $\hat{R}\hat{R} = \hat{I}$ but that $\hat{R}\hat{R} = \pm\hat{I}$.

25. Trivial example: Suppose that $\dim(\mathcal{H}) = n$. Let **S** act by mapping each unit $\psi \in \mathcal{H}$ to the density matrix $(1/n)\hat{\mathbf{I}}$. When the dynamics is given by unitary evolution, time reversal invariance is equivalent to the condition that all transition probabilities for a process and its time inverse are the same; see Section 2 above.

26. In line with the discussion of the preceding section, the initial state is time symmetric just in case the second fundamental form vanishes. If this is the case, then the past and future developments of the initial data will be temporal mirror images of one another.

27. For a general discussion of quantum field theory on curved spacetimes and the physics of black hole evaporation, see Wald (1994).

28. In Figure 5 the standard conventions for conformal diagrams are in effect; for example, ι^{o} denotes spacelike infinity and $\mathscr{I}^{+}$ denotes future null infinity (i.e. the terminus of outgoing light rays).

29. The phenomena of Hawking radiation and black hole evaporation are based on calculations in semi-classical quantum gravity which, it might be objected, is not a coherent theory but a collection of heuristics. Nevertheless, everyone working in quantum gravity—whether they pursue the string theory approach or the loop formulation of canonical quantum gravity—assume that Hawking radiation and black hole evaporation are real phenomena that have to be incorporated into the final theory. By contrast, the GRW collapse mechanism, which also produces pure-to-mixed state transitions of which Albert makes much, is not taken seriously by working physicists.

30. For a definition of global hyperbolicity, see Wald (1984). In a non-globally hyperbolic spacetime there is no Cauchy surface (= spacelike hypersurface which intersects every timelike curve without endpoint exactly once).

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