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Superselection Rules for Philosophers

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Abstract The overaraching goal of this paper is to elucidate the nature of superselection rules in a manner that is accessible to philosophers of science and that brings out the connections between superselection and some of the most fundamental interpretational issues in quantum physics. The formalism of von Neumann algebras is used to characterize three different senses of superselection rules (dubbed, weak, strong, and very strong) and to provide useful necessary and sufficient conditions for each sense. It is then shown how the Haag-Kastler algebraic approach to quantum physics holds the promise of a uniform and comprehensive account of the origin of superselection rules. Some of the challenges that must be met before this promise can be kept are discussed. The focus then turns to the role of superselection rules in solutions to the measurement problem and the emergence of classical properties. It is claimed that the role for "hard" superselection rules is limited, but "soft" (a.k.a. environmental) superselection rules or N. P. Landsman's situational superselection rules may have a major role to play. Finally, an assessment is given of the recently revived attempts to deconstruct superselection rules.

1 Introduction

At the 1951 International Conference on Nuclear Physics and the Physics of Fundamental Particles, Eugene Wigner broached the idea that quantum mechanics (QM) must recognize situations in which the matrix elements of any observable taken between two states that belong to what would later be called different superselection sectors are zero. According to Arthur Wightman's recollection (Wightman 1995, p. 753), some members of the audience were "shocked," presumably because Wigner's proposal contradicts (as will be seen below) von Neumann's (1932) assumption—an assumption that had

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become accepted by the physics community—that every self-adjoint operator corresponds to an observable. But not everyone attending the session understood the implications of Wigner's idea well enough to be shocked, an impression that is buttressed by the session summary contained in the conference Proceedings. Wigner's idea, it was reported, led to new "conservation laws":

> But, it was asked, don't these conservation laws have physical content?
>
> J. R. Oppenheimer: I think that this is a matter of semantics. It depends on whether you think that you are discovering or describing something. (Orear et al. 1951, p. 110)

Evidently Edward Teller did not think much of substance was at issue, as indicated by the use of 'apropos' in the summary of Teller's remarks:

> Requiem was read by E. Teller who cited the apropos anecdote of a candidate for a doctor's in philosophy who made a statement presumed to be true. Upon being asked by a professor on the examining board, "In which universe?", he responded, "Which which?" (Orear et al. 1951, p. 110)

One member of the audience, Gian Carlo Wick, not only understood the implications of Wigner's idea but was so enthusiastic that he urged Wigner to publish his arguments and also volunteered to write the first draft (Wightman 1995, p. 753). The following year under the title of "The Intrinsic Parity of Elementary Particles," Wick, Wightman, and Wigner (1952) published the first formal description of superselection rules, together with a proof of a superselection rule for integer and half-integer angular momentum (later dubbed the univalence superselection rule or the fermion superselection rule) and the conjecture of a superselection rule for charge. The ideas, however, were not entirely novel. For example, David Bohm's 1951 text *Quantum Theory* contains an argument against the coherent superposition of states with integer and half-integer angular momentum (Bohm 1951, p. 389),[1]

[1] Bohm concludes that "a sensible theory could be made for orbital angular momenta, if the angular momenta were either all integral, or half-integral, but not if both were present together" (p. 390). The Preface of Bohm's book states that "Numerous discussions with students and faculty at Princeton University were helpful in clarifying the presentation"

but this point was not related to a limitation on observables. I suspect that a careful examination of the literature of the period will reveal many other precursors for the idea of superselection, but that is another project. But whatever this project reveals it is worth noting that in the early 1930s Wigner was already two-thirds of the way to the univalence selection rule. He had recognized that the time reversal operator R is anti-unitary and that $R^2 = \pm 1$, with $+1$ for integer and angular momentum states and -1 for half-integer states (see Wigner 1931, Ch. 20). It is then only a short proof to show, as was done in the W^3 paper, that the supposition of a coherent superposition integer and half-integer states leads to an absurdity.[2]

Skepticism about particular superselection rules, as well as superselection rules in general, was expressed in the physics literature as late as 1970.[3] On the whole, however, the physics community seems to have quickly accepted superselection rules as facts of quantum life. But for the most part superselection rules were treated as curious inconveniences—certainly worth noting, but once noted to be shoved aside to let quantum life proceed per usual. The rise of the algebraic approach to relativistic quantum field theory (QFT) put an entirely different complexion on superselection rules: they were not to be viewed as curious inconveniences but as part of the marrow of quantum fields. They are, for example, an essential part of the Doplicher-Haag-Roberts reconstruction of quantum fields from the algebra of observables.[4]

In the present paper I will not be concerned with such sophisticated matters. Nor will I comment on such recent developments such as the extension of the original notion of superselection rules to Bohm-Bell theories (see Colin et al. 2006) or the implications of superselection rules for quantum informa-

(p. v). Bohm was a colleague of Wightman and Wigner during the years 1947-1950 when he was an assistant professor at Princeton. It is interesting that Bohm's argument is from single-valuedness and that Wightman dubbed the superselection rule at issue the univalence rule.

[2] When it was realized that time reversal invariance might not be universally valid, a different proof using spatial rotations was given; see Hegerfeldt, Kraus, and Wigner (1968) and Section 12 below. Once again, all of the relevant considerations had been developed by Wigner in the 1930s.

[3] See Aharonov and Susskind (1967a, 1967b), Rohlnick (1967), and Mirman (1969, 1970), and Lubkin (1970). A defense of the validity of the superselection rule for charge was given by Wick, Wightman, and Wigner (1970). The recent revival of the attacks on superselection rules will be discussed in Section 12.

[4] See Doplicher et al. (1971, 1974). For an overview of the DHR program, see Halvorson (2007).

tion theory where it seems that the degree of entanglement is constrained by superselection rules (see Bartlett and Wiseman 2003) and that the standard notion of quantum nonlocality has to be modified when superselection rules are present (see Verstraete and Cirac 2003). My aim is to tackle the more modest and basic task of elucidating the nature of superselection rules in a form that does not compromise rigor (at least not fatally) but is accessible to philosophers of science who are familiar with the basics of QM. The connections between superselection and some of the most fundamental interpretational issues is quantum physics are emphasized.

The basic expository device used here is the nomenclature of von Neumann algebras. This apparatus reveals that the use of the term 'superselection rule' in the physics literature is ambiguous among three main senses. Section 2 exploits von Neumann algebras to state five formulations of superselection rules—SSRI-SSRV—which are provably equivalent and together constitute what I dub the weak sense of superselection rules. Pains are taken to clarify the often puzzling statements to the effect that (weak) superselection rules constitute a limitation on the superposition principle. Section 3 reviews an illuminating necessary and sufficient condition for the existence of a weak superselection rule—namely, the existence of what Jauch and Misra dub a supersymmetry. Section 4 distinguishes two further senses of superselection rules—SSRVI-SSRVII—dubbed strong and very strong rules because *very strong* ⇒ *strong* ⇒ *weak*, with the arrows not being reversible in general. The differences between the three grades of superselection rules are characterized in various ways. Very strong superselection rules are shown to be very strong indeed since they imply that the von Neumann algebra of observables is quite tame—it is simply a direct sum of the Type I factors algebras used in ordinary QM. In Section 5 a necessary and sufficient condition for the existence of very strong superselection rules is posed in terms of the hypothesis of commutativity of supersymmetries. An example of how this condition might fail is given. The issue is both delicate and pregnant with interpretational significance since it is related to the existence of paraparticles. The connection between superselection rules and gauge symmetries is sketched in Section 6.

Thus far the focus has been on the task of characterizing the different senses of superselection rules, without any attempt to explain the origin of superselection. Section 7 introduces the two main candidates for providing a systematic derivation of superselection rules, the group theoretic approach to quantization and the algebraic approach. The remainder of the paper concentrates on the latter approach, which is described in some detail in Sections 8-10. The story is well-known to the mathematical physicists and philosophers of physics who work with the algebraic formalism, but it only rarely finds its way in to main line physics texts or the philosophy of science literature. While generally approving, the version of the story told here also brings out challenges that have to be met before the algebraic approach can be said to offer a satisfying explanation of the origin of superselection rules.

Sections 11 and 12 turn to two controversial interpretational issues. The first concerns attempts to solve the measurement problem and to explain the emergence of classical properties by appeal to superselection rules. Extant attempts using "hard" superselection rules are found wanting, while alternative attempts using "soft" (or environmentally-induced) superselection rules or else Landsman's situational superselection rules are found to be more promising. The second issue concerns the recent revival of attempts to deconstruct superselection rules. My conviction is that superselection rules remain unscathed, but there are respectable arguments to the contrary.

Conclusions are presented in Section 13. The list of references represents only a small fraction of the literature, but it is intended to be representative enough that the interested reader can locate entry points to the relevant literature on major subtopics.

The reader is cautioned not to succumb to the illusion that the application of some high powered mathematics to the topic of superselection means that the topic itself is under rigorous control; in particular, it is important that the technical apparatus not be allowed to mask contingent physical assumptions. The situation was well summarized by Glance and Wightman (1989):

> The theoretical results currently available fall into two categories: rigorous results on approximate models and approximate results on realistic models (p. 204) ... If there is a moral to the story it is that the development of physics is often much untidier than would appear from the philosophy of science books (p. 205).[5]

Some of the untidiness that goes unrecounted in both physics and philosophy of science texts will be revealed in Sections 11 and 12.

2 Weak Superselection Rules

[5] I am grateful to a Referee for bringing these quotations to my attention.

The basic mathematical entity to be used here in elucidating the different senses of superselection rules is a von Neumann algebra $\mathfrak{M}$, a concrete C^*-algebra[6] of bounded linear operators acting on a Hilbert space[7] $\mathcal{H}$ that is closed in the weak topology[8] or, equivalently,[9] that has the property that $(\mathfrak{M}')' := \mathfrak{M}'' = \mathfrak{M}$, where "$'$" denotes the commutant.[10] When the discussion turns to the origin of superselection rules (Sections 7-10) is argued that it is best to view the relevant von Neumann algebra as a representation-dependent object that arises from a choice of state on a C^*-algebra. For present purposes the von Neumann algebra of observables may be thought of as being generated by the set $\mathcal{O}$ of (not necessarily bounded) self-adjoint operators on $\mathcal{H}$ that correspond to genuine physical observables in the intended sense of quantities that can, in principle, be measured. In more detail, $\mathfrak{M}(\mathcal{O}) = \mathcal{O}''$, where it is understood that a bounded B commutes with an unbounded self-adjoint $A \in \mathcal{O}$ just in case B commutes with every projector in the spectral resolution of A. Defining the algebra of observables generated by $\mathcal{O}$ in terms of the weak closure is physically reasonable since it guarantees that the desirable condition that any bounded function $f(O)$ of a self-adjoint $O \in \mathcal{O}$ is also in the algebra. Of course, the question of whether $\mathcal{O}$ is a proper subset of the self-adjoint operators and, if so, which proper subset is precisely the issue raised by superselection rules. This question is not being begged; rather an apparatus is being developed to characterize superselection rules and to derive necessary and sufficient conditions for the existence of such rules, the satisfaction of which will entail that $\mathcal{O}$ is a proper subset of the self-adjoint operators.

According to Strocchi and Wightman (1974), a superselection rule in the broadest sense for a quantum mechanical theory "can be defined as any

[6] A $*$-*algebra* is an algebra closed with respect to an involution $\mathcal{A} \ni A \mapsto A^* \in \mathcal{A}$ satisfying: $(A^*)^* = A$, $(A+B)^* = A^* + B^*$, $(cA)^* = \bar{c}A^*$ and $(AB)^* = B^*A^*$ for all $A, B \in \mathcal{A}$ and all complex c (where the overbar denotes the complex conjugate). A C^*-*algebra* is a $*$-algebra equipped with a norm, satisfying $\|A^*A\| = \|A\|^2$ and $\|AB\| \leq \|A\| \|B\|$ for all $A, B \in \mathcal{A}$, and is complete in the topology induced by that norm.

[7] It will be assumed that the Hilbert space is separable. This assumption is used explicitly or implicitly in some of the key theorems used below.

[8] A sequence of bounded operators $O_1, O_2, ...$ converges in the weak topology to O just in case $(\psi_1, O_j\psi_2)$ converges to $(\psi_1, O\psi_2)$ for all $\psi_1, \psi_2 \in \mathcal{H}$.

[9] The equivalence of these conditions is known as von Neumann's double commutant theorem.

[10] That is, if $\mathfrak{X} \subset \mathfrak{B}(\mathcal{H})$ (the algebra of bounded linear operators on $\mathcal{H}$), then the elements of $\mathfrak{X}'$ consists of all of those elements of $\mathfrak{B}(\mathcal{H})$ that commute with every element of $\mathfrak{X}$.

restriction on what is observable in the theory" (p. 2198). In terms of the above apparatus, this means that $\mathfrak{M}(\mathcal{O})$ is a proper subalgebra of $\mathfrak{B}(\mathcal{H})$ (the algebra of all bounded operators on $\mathcal{H}$) and, thus, $\mathfrak{M}(\mathcal{O})$ acts reducibly on $\mathcal{H}$, i.e. $\mathfrak{M}(\mathcal{O})$ leaves invariant a non-null proper subspace of $\mathcal{H}$ (for future reference label this notion SSRI).[11] By Schur's lemma SSRI is equivalent to the condition that $\mathfrak{M}(\mathcal{O})'$ is non-trivial in that does not consist of multiples of the identity operator (SSRII). If $\mathcal{H}_1$ is a subspace of $\mathcal{H}$ invariant under $\mathfrak{M}(\mathcal{O})$ then $\mathcal{H}_2 := \mathcal{H}_1^{\perp}$ is also invariant under $\mathfrak{M}(\mathcal{O})$. Thus, $\mathcal{H}$ has the direct sum decomposition $\mathcal{H}_1 \oplus \mathcal{H}_2$, and each element $A \in \mathfrak{M}(\mathcal{O})$ decomposes into a direct sum $A_1 \oplus A_2$. Of course, $\mathcal{H}_1$ and $\mathcal{H}_2$ may be further decomposable, but in order to avoid technical complications, I will only consider discrete superselection rules under which $\mathcal{H}$ decomposes into $\mathcal{H} = \oplus_j \mathcal{H}_j$ where j is a discrete index.[12]

A third notion of superselection rule (SSRIII) is usually stated in terms of a limitation on the superposition principle. Here is a sampling of attempted formulations of the limitation:

> A superselection rule acts by "effectively forbidding the superposition of states" (Bub 1997, p. 212); a superselection rule "exists whenever there are limitations to the superposition principle ... that is, whenever certain superpositions cannot be physically realizable (Cizneros et al. 1998, p. 238); a superselection rule implies that "some linear combination of states ... can never be physically realized" (d'Espagnat 1976, pp. 56-57); in the presence of a superselection rule "a vector of Hilbert space which has components in different [superselection sectors] cannot represent a physical state" (Roman 1965, p. 32); "Any statement that singles out certain rays [in Hilbert space] as not physically realizable is a superselection rule" (Streater and Wightman 1964, p. 5).

The uninitiated reader can be forgiven if she does not know quite what to make of these assertions. What the authors intend to say can be made clear with the help a bit of additional apparatus.

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[11] Reducibility is taken as the criterion of the existence of superselection rules by Emch and Piron (1963), although they work in terms of a lattice of propositions rather than an algebra of observables.

[12] Direct integral decompositions will be briefly discussed below.

A *state* ω on a von Neumann algebra $\mathfrak{M}$—or more generally on a C^*-algebra—is a normed positive linear functional $\omega : \mathfrak{M} \to \mathbb{C}$.[13] A *normal state* on $\mathfrak{M}$ is one that is countably additive on families of mutually orthogonal projection operators in $\mathfrak{M}$.[14] A *vector state* ω on $\mathfrak{M}$ is a state such that there is a $\psi \in \mathcal{H}$, $||\psi|| = 1$, with $\omega(A) = (\psi, A\psi)$ for all $A \in \mathfrak{M}$. All vector states are normal, but generally the converse is false. A *mixed state* ω is a state such that there are distinct states ω_1 and ω_2 with $\omega = \lambda\omega_1 + (1-\lambda)\omega_2$, $0 < \lambda < 1$. A *pure state* is a state that is not mixed.

The familiar version of ordinary QM without superselection rules is straightforward:

(i) the von Neumann algebra of observables $\mathfrak{M}$ is $\mathfrak{B}(\mathcal{H})$;

(ii) (a) the pure states on $\mathfrak{M}$ are identical with the vector states; and

(b) the mixed states correspond to non-trivial density operators ρ ($\rho^2 \neq \rho$) according to the trace prescription $\omega_\rho(A) := tr(A\rho)$ for all $A \in \mathfrak{M}$;

(iii) the superposition principle holds in following form:

(a) if ω_{ψ_1} and ω_{ψ_2} are vector states corresponding respectively to $\psi_1, \psi_2 \in \mathcal{H}$, then for any $\alpha_1, \alpha_2 \in \mathbb{C}$ with $|\alpha_1| + |\alpha_2| = 1$, there is a vector state $\omega_{\alpha_1\psi_1+\alpha_2\psi_2}$ corresponding to $\alpha_1\psi_1 + \alpha_2\psi_2$;

(b) if the vector states ω_{ψ_1} and ω_{ψ_2} are pure states, then so is $\omega_{\alpha_1\psi_1+\alpha_2\psi_2}$; and

(c) since by (ii)(a) the antecedent of (ii)(b) always holds, $\omega_{\alpha_1\psi_1+\alpha_2\psi_2}$ is a pure state.

In the typology of von Neumann algebras, the von Neumann algebra of observables in ordinary QM, $\mathfrak{B}(\mathcal{H})$, is a Type I factor.[15] Outside of ordinary QM more exotic types of von Neumann algebras are encountered. For example, in relativistic QFT the algebra of observables associated with local regions of spacetime are generically Type III.[16] Since the commutant $\mathfrak{M}'$ of a von Neumann algebra $\mathfrak{M}$ is the same type as $\mathfrak{M}$ itself, the commutant of

[13] That ω is positive means that for all $A \in \mathfrak{M}$, $\omega(A^*A) = 0$ implies that $A = 0$.

[14] $E \in \mathfrak{M}$ is a projector just in case it is self-adjoint and idempotent, i.e. $E^2 = E$. Two such projectors E and F are said to be orthogonal just in case they project onto orthogonal subspaces of $\mathcal{H}$, in which case $EF = FE = 0$.

[15] A factorial von Neumann algebra $\mathfrak{R}$ has a center $\mathcal{Z}(\mathfrak{R}) := \mathfrak{R} \cap \mathfrak{R}'$ that consists of multiples of the identity. A factorial $\mathfrak{R}$ is of Type I iff it contains minimal projectors. For a non-factorial $\mathfrak{R}$ the definition of a Type I is a bit more complicated; namely, $\mathfrak{R}$ contains an abelian projector whose central carrier is the identity I. That the projector $E \in \mathfrak{R}$ is abelian means that $E\,\mathfrak{R}E$ is an abelian algebra. The central carrier C_A of $A \in \mathfrak{R}$ is the meet of all projectors $F \in \mathfrak{R}$ such that $FE = E$.

[16] The distinguishing feature of Type III algebras is that they contain properly infinite

a Type III algebra is non-trivial, giving a SSRI. But Type III algebras have features that are inimical to the ordinary talk about superselection rules. For condition (ii)(a) fails utterly for Type III algebras since there are no normal pure states on such an algebra and, thus, no vector state on such an algebra is pure. Condition (iii)(a) continues to hold since it holds for all von Neumann algebras. Condition (iii)(b) is rendered vacuous for Type III algebras because its antecedent is never fulfilled, and by the same token (iii)(c) is rendered moot. In sum, Type III algebras entail not just a limitation on the superposition principle but a complete subversion of it.[17]

Typical discussions of superselection rules presuppose additional restrictions on the von Neumann algebra of observables that rule out such exotic cases and, thus, produce stronger senses of superselection rules that will be examined in subsequent sections. In particular, it is usually assumed that while (ii)(a) fails, it does not fail utterly since some vector states are pure and some are not; and more particularly, it is typically assumed that the superselection sectors are "coherent subspaces" in that the familiar superposition principle from ordinary QM holds within each sector. (We will see in Section 4 that this assumption can be proved for very strong superselection rules.) Under this assumption, a vector state corresponding to a vector belonging to one of the superselection sectors $\mathcal{H}_j$ of $\mathcal{H} = \oplus_j \mathcal{H}_j$ is pure state while a vector state corresponding to linear combinations of vectors from different sectors is mixed; so (iii)(b) and (iii)(c) sometime hold and sometime fail.[18] The failure of (iii)(b) can be expressed as follows. For any $\psi_m \in \mathcal{H}_m$ and $\psi_n \in \mathcal{H}_n$, $m \neq n$, with $||\psi_m|| = ||\psi_n|| = 1$, and any $\alpha, \beta \in \mathbb{C}$ with $|\alpha|^2 + |\beta|^2 = 1$, set $\varphi = \alpha\psi_m + \beta\psi_n$. Then $(\varphi, A\varphi) = |\alpha|^2(\psi_m, A\psi_m) + |\beta|^2(\psi_n, A\psi_n)$ for all $A \in \mathfrak{M}(\mathcal{O})$ and, thus, the vector state ω_φ is identical with the mixed state $|\alpha|^2\omega_{\psi_m} + |\beta|^2\omega_{\psi_n}$.

projectors and no finite projectors. For some of the implications for foundations issues, see Earman and Ruetsche (2007).

[17] For a general discussion of the superposition principle in the algebraic formulation of QM, see Horuzhy (1975).

[18] The habit, developed from working in ordinary QM, of identifying pure states with vector states is not easily broken. Witness the exposition of one of the discoverers of superselection rules: A superselection rule "means that there exist pure states [sic] described by a wave function $\psi = \alpha_1\psi_1 + \alpha_2\psi_2$ with $|\alpha_1|^2 + |\alpha_2|^2 = 1$, $||\psi_1|| = ||\psi_2|| = 1$, and $|(\psi_1, \psi_2)| = 0$ such that the relative phase of α_1 and α_2 is unobservable" (Glancre and Wightman 1989, p. 202). I have changed the notation to correct an obvious misprint. Wightman and Glance go on to say correctly that ψ may also be defined by a density matrix that describes a mixed state.

We are now in a position to assess the above quoted claims to the effect that in the presence of a superselection rule various Hilbert space vectors, or the rays they determine, are not "physically realizable." Streater and Wightman (1964, pp. 4-5) are quite clear what they mean: if ψ is a non-trivial linear combination of vectors belonging to different superselection sectors, then the ray determined by such a ψ is physically unrealizable precisely in the sense that the projector onto that ray is not an an observable, i.e. is not an element of $\mathfrak{M}$. This is, of course, correct; but left unqualified it tends to suggest the unwarranted conclusion that the state ω_ψ determined by ψ, or by any other vector in the ray corresponding to ψ, is physically impossible or physically unrealizable. That conclusion could only be correct to the extent that *no* mixed state is physically possible or physically realizable. As for superselection principles "forbidding the superposition of states," it is not that taking a linear combination of certain vectors to get another vector is not a legitimate operation or does not produce a possible state; it is simply that in the presence of a superselection rule, some vector states are pure states while convex linear combinations of them are not and, thus, these linear combinations do not have the properties we expect of genuine superpositions.[19]

Yet a fourth notion of superselection rule—SSRIV—explains the origin of the term. A conserved quantity in ordinary QM—say, energy for an isolated system—is said to induce a selection rule in that spontaneous transitions between states corresponding to different eigenvalues of energy are forbidden. But, of course, external perturbations acting on the system can engender transitions between selection sectors. A superselection rule is so-called because it absolutely forbids transitions between different superselection sectors $\mathcal{H}_j$ in the following sense: for any $A \in \mathfrak{M}(\mathcal{O})$ and any $\psi_m \in \mathcal{H}_m$ and $\psi_n \in \mathcal{H}_n$, $m \neq n$, , $(\psi_m, A\psi_n) = (\psi_n, A\psi_m) = 0$. *If* the dynamics of the system is a Hamiltonian dynamics and *if* the (time independent) Hamiltonian H for a system is a self-adjoint observable, then Schrödinger evolution will never evolve the state vector from one superselection sector to another and will always evolve a pure state to a pure state. For if $H \in \mathcal{O}$ then all of the projectors in the spectral resolution of H belong to $\mathfrak{M}(\mathcal{O})$ and, thus, $\mathfrak{M}(\mathcal{O}) \ni V_t := e^{-iHt}$ for all t. So $(\psi_m, V_t\psi_n) = 0$ for any $\psi_m \in \mathcal{H}_m$ and

[19] One of the few clear statements in the philosophical literature of the limitation imposed by superselection rules on the superposition principle comes from van Fraassen (1991, p. 186): "[T]he principle of superposition is curtailed: what looks mathematically like a superposition of pure states may actually represent a mixed state."

$\psi_n \in \mathcal{H}_n$, $m \neq n$. However, if H fails to be an observable, then Schrödinger evolution can cross superselection sectors and can evolve a pure state into a mixture. This fact is sometimes exploited in an attempt to solve the measurement problem (see Section 11). The prior issue concerns the assumption that the dynamics is Hamiltonian. For the case of very strong superselection rules this assumption can be given a convincing justification (see Section 11).

A fifth notion of superselection rule—SSRV—says that the relative phases of superpositions of vectors belonging to different superselection sectors of $\oplus_j \mathcal{H}_j$ are unobservable: for $\psi_m \in \mathcal{H}_m$ and $\psi_n \in \mathcal{H}_n$ define $\psi_\lambda^{mn} := \frac{1}{\sqrt{2}}(\psi_m + e^{i\lambda}\psi_n)$; then if $m \neq n$, for all $A \in \mathfrak{M}(\mathcal{O})$ and any $\lambda_1, \lambda_2 \in \mathbb{R}$, $(\psi_{\lambda_1}^{mn}, A\psi_{\lambda_2}^{mn}) = (\psi_{\lambda_2}^{mn}, A\psi_{\lambda_1}^{mn})$.

It is an easy exercise to show that all five notions of superselection rule—SSRI-SSRV—are equivalent, and the exercise is left to the reader. Individually and collectively they constitute what I will call the *weak sense of superselection rule* to contrast it with stronger senses to be discussed in Section 4.

It is natural to wonder whether the presence of superselection rules necessitates a change in Gleason's theorem which, for ordinary QM, shows that probability measures are generated by density operators (when $\dim(\mathcal{H}) > 2$). The answer is negative, for if Gleason's theorem is generalized in the appropriate way it holds for practically any von Neumann algebra. A von Neumann algebra $\mathfrak{M}$ is determined by its projectors $\mathcal{P}(\mathfrak{M})$ (i.e. $\mathcal{P}(\mathfrak{M})'' = \mathfrak{M}$). Take a quantum probability measure for $\mathfrak{M}$ to be a map $\mu : \mathcal{P}(\mathfrak{M}) \to [0,1]$ that is σ-additive on mutually orthogonal families of projectors.

> *Generalized Gleason's theorem.* Let $\mathfrak{M}$ be a von Neumann algebra acting on a separable $\mathcal{H}$, and let μ be a quantum probability measure on $\mathfrak{M}$. If $\mathfrak{M}$ does not contain any summands of Type I_2, there is a unique normal state ω on $\mathfrak{M}$ such that $\omega(E) = \mu(E)$ for all $E \in \mathcal{P}(\mathfrak{M})$.[20]

The connection to the usual trace prescription follows from

> *Lemma 1* (Bratelli and Robinson 1979, Theorem 2.4.21) Let ω be a state on a von Neumann algebra $\mathfrak{M}$ acting on $\mathcal{H}$. Then ω is

[20] See Hamhalter (2003, Ch. 5) for a proof. A von Neumann algebra is of Type I_n if the unit element can be written as the sum of n abelian projectors.

a normal state iff there is a density operator ρ acting on $\mathcal{H}$ such that $\omega(A) = Tr(\rho A)$ for all $A \in \mathfrak{M}$.

Thus, regardless of whether superselection rules are present, the familiar representation of probability measures by density operators is valid.

3 A Criterion for the Existence of Weak Superselection Rules

As defined by Jauch and Misra (1961), a *supersymmetry* is a unitary operator U that is not a multiple of the identity and that commutes with the set $\mathcal{O}$ of all observables. The existence of a supersymmetry provides a criterion for the existence of weak superselection rules.

> *Theorem 1.* (Jauch and Misra 1961) $\mathfrak{M}(\mathcal{O})$ acts reducibly if and only if there is supersymmetry U for $\mathcal{O}$.

The proof is straightforward. To show that the existence of a supersymmetry is sufficient for the reducibility of $\mathfrak{M}(\mathcal{O})$, assume that there is a non-trivial unitary $U \in \mathcal{O}'$. Now the commutant of a set of self-adjoint operators (bounded or unbounded) is a von Neumann algebra. Hence $\mathcal{O}'$ is a von Neumann algebra. So $\mathcal{O}' = \mathcal{O}''' = \mathfrak{M}(\mathcal{O})'$ and, therefore, $U \in \mathfrak{M}(\mathcal{O})'$. This shows that $\mathfrak{M}(\mathcal{O})'$ is non-trivial and, thus, that $\mathfrak{M}(\mathcal{O})$ acts reducibly. The existence of a supersymmetry is a necessary condition for reducibility since a von Neumann algebra is generated by its unitary elements (Kadison and Ringrose 1991, Theorem 4.1.7) and, thus, if $\mathfrak{M}(\mathcal{O})'$ is non-trivial it must contain a non-trivial unitary operator.

To connect the existence of a supersymmetry more directly to the original meaning of a weak superselection rule—a prohibition on transitions between superselection sectors—suppose in line with our focus on discrete superselection rules that the supersymmetry U has a pure point spectrum, and for ease of discussion suppose also that there are exactly two distinct eigenvalues. Since the eigenvalues of a unitary operator are complex numbers of modulus 1, $U = e^{i\phi_1}E_{\phi_1} + e^{i\phi_2}E_{\phi_2}$ where the E_{ϕ_j} are the projectors onto the corresponding eigenspaces. From the fact that U is a supersymmetry we have that for all $\psi_1, \psi_2 \in \mathcal{H}$ and all $A \in \mathfrak{M}(\mathcal{O})$, $(\psi_1, A\psi_2) = (U\psi_1, (UAU^*)U\psi_2) = (U\psi_1, AU\psi_2)$. Now choose ψ_1 and ψ_2 from the two eigenspaces of U. Then $(\psi_1, A\psi_2) = (\psi_1, A\psi_2)e^{i(\phi_2 - \phi_1)}$, which implies that $(\psi_1, A\psi_2) = 0$, i.e. a dichotomous SSRIV holds.

It may have occurred to the reader to ask why the Jauch-Misra definition of supersymmetry excludes the possibility that a supersymmetry can be an anti-unitary operator. A symmetry in QM is often defined as a mapping rays of $\mathcal{H}$ onto rays that preserves transition probabilities. According to a theorem of Wigner, every such ray mapping can be replaced by a vector mapping U that is either unitary or anti-unitary.[21] Jauch and Misra exclude the possibility of an anti-unitary supersymmetry with a simple argument that uses the fact that in non-relativistic QM and relativistic QFT alike, the time reversal transformation R is always anti-unitary. If U were an anti-unitary supersymmetry, then RU would also be a time reversal transformation, which is impossible because the product of two anti-unitary operators is a unitary operator (Jauch and Misra 1961, pp. 701-702).

In Section 6 it will be argued that a supersymmetry can justifiably be dubbed a gauge symmetry.

4 Strong and Very Strong Superselection Rules

In Section 2 we saw that SSRI-SSRV are all equivalent to the condition that the commutant $\mathfrak{M}'$ of the von Neumann algebra $\mathfrak{M}$ of observables is non-trivial. This is compatible with a situation where none of the elements of the superselection algebra $\mathfrak{M}'$ are elements of the algebra $\mathfrak{M}$ of observables or, to introduce some additional useful terminology, the center of $\mathfrak{M}$, $\mathcal{Z}(\mathfrak{M}) := \mathfrak{M} \cap \mathfrak{M}'$, is trivial in that it consists of multiples of the identity operator. The strong sense of superselection rule—SSRVI—asserts that this possibility does not obtain and $\mathcal{Z}(\mathfrak{M})$ is non-trivial.[22] Any Type III factor algebra provides an example that satisfies the conditions for a weak but not a strong superselection rule, while a Type III non-factor provides an example of a strong superselection rule. However, most writers would not cite the latter an as illustration of a superselection rule, which indicates that what they have in mind is a still stronger sense of superselection.

[21] For a rigorous proof, see Bargmann (1964). An anti-unitary U is anti-linear transformation, i.e. for all $\psi_1, \psi_2 \in \mathcal{H}$ and all $\alpha_1, \alpha_2 \in \mathbb{C}$, $U(\alpha_1\psi_1 + \alpha_2\psi_2) = \alpha_1^* U\psi_1 + \alpha_2^* U\psi_2$, such that $(\psi_1, \psi_2) = (U\psi_2, U\psi_1)$. A unitary U is linear, i.e. $U(\alpha_1\psi_1 + \alpha_2\psi_2) = \alpha_1 U\psi_1 + \alpha_2 U\psi_2$, and $(\psi_1, \psi_2) = (U\psi_1, U\psi_2)$.

[22] Sometimes a non-trivial center is taken as the definition of superselection rules; see, for example, Piron (1976). Speaking of the lattice of propositions Piron says: the system is classical if the center is whole lattice. "In the pure quantum case [ordinary QM] the center contains only 0 and I. In physics there exist a large number of intermediate cases where the center is strictly smaller than the whole lattice but contains nontrivial propositions. We shall then say that the system possesses superselection rules" (p. 29).

The very strong sense of superselection rules—SSRVII—requires not only that $\mathcal{Z}(\mathfrak{M})$ is non-trivial but also that the superselection algebra is a subalgebra of observables, i.e. $\mathfrak{M}' \subseteq \mathfrak{M}$ and, thus, that $\mathcal{Z}(\mathfrak{M}) = \mathfrak{M}'$. Note that requiring $\mathfrak{M}' \subseteq \mathfrak{M}$ is equivalent to requiring that $\mathfrak{M}'$ is abelian since abelianness holds just in case $\mathfrak{M}' \subseteq (\mathfrak{M}')' = \mathfrak{M}'' = \mathfrak{M}$. The abelianness of $\mathfrak{M}'$ is sometimes referred to as the *hypothesis of the commutativity of superselection rules* (see Wightman 1959 and 1995). From the point of view of weak and strong superselection rules, this is indeed an hypothesis in that it is an additional assumption that does not follow from SSRI-SSRVI. However, from the point of view of very strong superselection rules it is *not* an hypothesis but simply part of the defining condition of the very strong sense.[23] As will be seen below the commutativity of superselection rules implies that the von Neumann algebra of observables is rather tame—it is a Type I non-factor.[24]

Jauch and Misra have provided a necessary and sufficient condition for the commutativity of superselection rules.

> *Theorem 2.* (Jauch 1960; Jauch and Misra 1961) Let $\mathfrak{M}$ be a von Neumann algebra acting on $\mathcal{H}$. Then $\mathfrak{M}'$ is abelian if and only if $\mathfrak{M}$ contains an abelian algebra subalgebra maximal in $\mathfrak{B}(\mathcal{H})$.

The sufficiency of the condition is immediate: Suppose that there is a subalgebra $\mathfrak{A} \subseteq \mathfrak{M}$ that is abelian ($\mathfrak{A} \subseteq \mathfrak{A}'$) and maximal ($\mathfrak{A}' \subseteq \mathfrak{A}$). Then infer that $\mathfrak{M}' \subseteq \mathfrak{A}' = \mathfrak{A} \subseteq \mathfrak{M}$. The proof of the converse starts from the observation that the abelian subalgebras of $\mathfrak{M}$ are partially ordered by inclusion and then uses Zorn's lemma to conclude that there is an abelian subalgebra $\mathfrak{A}$ that is maximal abelian in $\mathfrak{M}$, i.e. $\mathfrak{A}' \cap \mathfrak{M} = \mathfrak{A}$. That $\mathfrak{A} \subseteq \mathfrak{M}$ implies $\mathfrak{M}' \subseteq \mathfrak{A}'$ so that $\mathcal{Z}(\mathfrak{M}) = \mathfrak{M} \cap \mathfrak{M}' \subseteq \mathfrak{M} \cap \mathfrak{A}' = \mathfrak{A}$. But the fact that $\mathfrak{M}'$ is abelian means that $\mathcal{Z}(\mathfrak{M}) = \mathfrak{M}'$ which leads to $\mathfrak{M}' \subseteq \mathfrak{A}$. Consequently, $\mathfrak{A}' \subseteq \mathfrak{M}'' = \mathfrak{M}$ so that $\mathfrak{A} = \mathfrak{M} \cap \mathfrak{A}' = \mathfrak{A}$.

Beltrametti and Cassinelli (1987, Sec. 5.2) take the existence of a complete set of commuting observables to be a *sine qua non* for QM. And the

[23]For examples where "superselection rule" is taken to mean what is called here a very strong rule, see Bogolubov et al. (1975) and Wan (1980). Indeed, most texts simply assume that superselection rules are of the very strong variety.

[24]There is a direct and easy way to see this. If $\mathfrak{M}'$ is abelian then the identity I is an abelian projector whose central carrier is I and, thus, $\mathfrak{M}'$ is Type I. But for any von Neumann algebra $\mathfrak{M}$, $\mathfrak{M}$ and $\mathfrak{M}'$ are the same type. The proof given below leads eventually to the same conclusion while showing some other useful things along the way.

existence of such a set they take to be equivalent to the existence of a maximal abelian von Neumann algebra of observables. Thus, for them the difference between strong and very strong superselection rules collapses. The opposing viewpoint is that the validity of the commutativity of superselection rules is a matter to be decided by nature and not by *a priori* reasoning. Later we will encounter examples where the commutativity of superselection rules might fail.

The differences between weak, strong, and very strong superselection rules can be characterized using what Jauch and Misra (1961) call the *core* $\mathfrak{C}(\mathfrak{M})$ of the algebra of observables $\mathfrak{M}$: $\mathfrak{C}(\mathfrak{M})$ is defined as the intersection of all of the maximal abelian algebras contained in $\mathfrak{M}$. In the presence of very strong superselection rules they show that $\mathfrak{C}(\mathfrak{M}) = \mathcal{Z}(\mathfrak{M}) = \mathfrak{M}'$. For weak superselection rules that are not strong rules, $\mathfrak{C}(\mathfrak{M}) = \mathcal{Z}(\mathfrak{M})$, but trivially so since both the core and the center are trivial. For strong superselection rules that are not very strong, $\mathfrak{C}(\mathfrak{M}) \neq \mathcal{Z}(\mathfrak{M})$ since $\mathfrak{C}(\mathfrak{M})$ is trivial while $\mathcal{Z}(\mathfrak{M})$ is not.

Another way to characterize the differences between the different strengths of superselection rules comes from the theory of central decompositions of von Neumann algebras. For any von Neumann algebra $\mathfrak{M}$ acting on a separable Hilbert space $\mathcal{H}$, its center $\mathcal{Z}(\mathfrak{M})$ determines essentially unique direct integral decompositions of $\mathcal{H}$ and $\mathfrak{M}$ respectively, $\mathcal{H} = \int_{\oplus} \mathcal{H}(\xi) d\mu(\xi)$ and $\mathfrak{M} = \int_{\oplus} \mathfrak{M}(\xi) d\mu(\xi)$, where μ is a central measure.[25] We are concerned with the case of discrete superselection rules, so it is fair to assume that the measure μ has discrete support, in which case the direct integral decompositions become direct sum decompositions $\mathcal{H} = \oplus_j \mathcal{H}_j$ and $\mathfrak{M} = \oplus_j \mathfrak{M}_j$, where the range of j may be finite or infinite. The discussion of superselection rules typically assumes that superselection sectors are "coherent subspaces" of the Hilbert space, indicating that the form of the superposition principle familiar from ordinary QM holds within the superselection sectors. Under this assumption the $\mathfrak{M}_j$ act irreducibly on their respective selection spaces, i.e. $\mathfrak{M}_j = \mathfrak{B}(\mathcal{H}_j)$ for all j. To see how this assumption fits with the different strengths of superselection rule, we can appeal to a basic result about central decompositions:

Theorem 3. (Kadison and Ringrose 1991, Theorem 14.2.4) When

[25] "Essentially unique" means unique up to measure zero.

the central decompositions of $\mathcal{H}$ and $\mathfrak{M}$ are discrete, the $\mathfrak{M}_j$ are $\mathfrak{B}(\mathcal{H}_j)$ iff $\mathcal{Z}(\mathfrak{M})$ is maximal abelian in $\mathfrak{M}'$, i.e. $\mathcal{Z}(\mathfrak{M})' \cap \mathfrak{M}' = \mathcal{Z}(\mathfrak{M})$.

In the presence of very strong superselection rules, $\mathcal{Z}(\mathfrak{M})$ is maximal abelian in $\mathfrak{M}'$ since $\mathcal{Z}(\mathfrak{M}) = \mathfrak{M}'$. Hence, by Theorem 3, $\mathfrak{M} = \oplus_j \mathfrak{B}(\mathcal{H}_j)$. The von Neumann algebra of observables is Type I but non-factorial, which is not very exotic at least as compared with Type III algebras. All of the projectors E_j from $\mathcal{H}$ to the sectors $\mathcal{H}_j$ belong to $\mathcal{Z}(\mathfrak{M})$, making it possible to define *superselection operators* in the sense of self-adjoint operators of the form $T = \sum_{(all)\ j} r_j E_j$, $r_j \in \mathbb{R}$. When there are only a finite number of superselection sectors, T is bounded and $T \in \mathcal{Z}(\mathfrak{M})$. When there are a countably infinite number of superselection sectors T may fail to be bounded, in which case $T \notin \mathcal{Z}(\mathfrak{M})$; but one can still say that T is affiliated with $\mathcal{Z}(\mathfrak{M})$ in the sense that all of its spectral projectors belong to $\mathcal{Z}(\mathfrak{M})$. Finally, it is worth noting that when $\mathfrak{M} = \oplus_j \mathfrak{B}(\mathcal{H}_j)$, Lemma 1 can be strengthened. In general, every normal state on a von Neumann algebra is generated by a density operator. But when $\mathfrak{M}$ is of the form required by very strong superselection rules, then for any normal state ω on $\mathfrak{M}$ there is a unique density operator ρ that is reduced by all the $\mathcal{H}_j$ such that $\omega(A) = tr(\rho A)$ for all $A \in \mathfrak{M}$.

In the case of a weak but not strong superselection rule $\mathcal{Z}(\mathfrak{M})$ is trivial and obviously not maximal abelian in $\mathfrak{M}'$ and, thus, $\mathfrak{M} \neq \oplus_j \mathfrak{B}(\mathcal{H}_j)$. $\mathfrak{M}$ is a proper subalgebra of $\oplus_j \mathfrak{B}(\mathcal{H}_j)$ that fails to contain any of the projectors E_j from $\mathcal{H}$ to $\mathcal{H}_j$ (since any E_j it contained would be in $\mathcal{Z}(\mathfrak{M})$, contradicting the triviality of $\mathcal{Z}(\mathfrak{M})$) and, thus, there is no non-trivial superselection operator affiliated with $\mathcal{Z}(\mathfrak{M})$. An example of how this situation might arise is discussed below in Section 9.

In the case of a strong but not very strong superselection rule, $\mathcal{Z}(\mathfrak{M})$ is non-trivial but is properly contained in $\mathfrak{M}'$. This does not permit us to infer in the abstract whether or not $\mathcal{Z}(\mathfrak{M})$ is maximal abelian in $\mathfrak{M}'$. But for the desired decomposition of $\mathfrak{M}$ we can reason as follows. Suppose for *reductio* that $\mathcal{Z}(\mathfrak{M})$ is maximal abelian in $\mathfrak{M}'$ and, thus, that $\mathfrak{M} = \oplus_j \mathfrak{B}(\mathcal{H}_j)$. Then $\mathfrak{M}$ contains a maximal abelian algebra. To form such an algebra, note that $\mathfrak{M}$ contains the projectors from $\mathcal{H}$ onto each of the $\mathcal{H}_j$, plus the one-dimensional projectors that project onto an orthonormal basis for each $\mathcal{H}_j$. The abelian algebra generated by these projections is maximal abelian. Thus, by Theorem 2, $\mathfrak{M}'$ is abelian and a very strong superselection rule obtains, contrary to assumption. The upshot is that in the presence of a strong but not very strong superselection rule $\mathfrak{M} \neq \oplus_j \mathfrak{B}(\mathcal{H}_j)$. As with the previous case of a weak but not strong superselection rule, $\mathfrak{M}$ is a proper subalgebra of $\oplus_j \mathfrak{B}(\mathcal{H}_j)$. But the case of a strong but not very strong superselection rule is differentiated from the previous case in that $\mathfrak{M}$ contains some but not all of the projectors E_j from $\mathcal{H}$ to $\mathcal{H}_j$. There are truncated superselection operators $T = \sum_{(not\ all)\ j} r_j E_j$ (where the range of j is now restricted to those E_j that lie in $\mathfrak{M}$) affiliated with $\mathcal{Z}(\mathfrak{M})$. This situation seems rather artificial, leading one to wonder whether such cases can arise in a physically natural way and, indeed, whether as a matter of fact or a matter of principle all (discrete) superselection rules are of the very strong variety, giving rise to the preferred structure $\mathcal{H} = \oplus_j \mathcal{H}_j$ and $\mathfrak{M} = \oplus_j \mathfrak{B}(\mathcal{H}_j)$. A possible example where a superselection rule is not a very strong rule is provided in the following section.

5 A Criterion for Very Strong Superselection Rules

The set $\mathcal{U}$ of non-trivial unitary elements of $\mathcal{O}'$ is the set of supersymmetries. The *hypothesis of commutativity of supersymmetries* says that $\mathcal{U}$ is abelian, i.e. $\mathcal{U} \subseteq \mathcal{U}'$.

> *Theorem 4.* If $\mathcal{U}$ is not empty then a very strong superselection rule obtains if and only if the hypothesis of commutativity of supersymmetries holds.

Suppose that $\mathcal{U}$ is not empty. Then by Theorem 1 a weak superselection rule obtains and $\mathfrak{M}(\mathcal{O})'$ is non-trivial. We have seen that $\mathcal{O}' = \mathcal{O}''' = \mathfrak{M}(\mathcal{O})'$ (see Theorem 1) and, therefore, $\mathcal{U}$ comprises the non-trivial unitaries in $\mathfrak{M}(\mathcal{O})'$. From $\mathcal{U} \subseteq \mathcal{U}'$ infer that $\mathcal{U}'' \subseteq \mathcal{U}'$. Furthermore, $\mathfrak{M}' = \mathcal{U}''$ since a von Neumann algebra is generated by its unitaries. Thus, $\mathfrak{M}' \subseteq \mathcal{U}'$ and consequently $\mathcal{U}'' \subseteq \mathfrak{M}''$, i.e. $\mathfrak{M}' \subseteq \mathfrak{M}$ and a strong superselection rule obtains. The argument can be reversed using the fact that the commutant of a set of unitary operators is a von Neumann algebra and, hence, $\mathcal{U}' = \mathcal{U}'''$, which follows from the more general fact that if $\mathcal{F}$ is a set of bounded operators acting on a Hilbert space, then $\mathcal{F}'$ is weakly closed.

A possible case where the hypothesis of commutativity of supersymmetries is violated is discussed by van Fraassen (1991, Ch. 11). The group of permutations of N particles is represented by a group of unitary operators. In one of its formulations, the postulate of Permutation Invariance asserts, in effect, that each such permutation operator is a supersymmetry (van Fraassen 1991, p. 397). But these operators do not commute for $N \geq 3$. However, the case is not straightforward as it may seem. In the following section a justification will be given for counting supersymmetries as gauge symmetries. In the present case these gauge symmetries form a group. If it is not demanded that this gauge group has an action on the Hilbert space, there is a physically equivalent way of reformulating the problem in which the commutativity of superselection rules is preserved (see Hartle and Taylor 1969 and Giulini 2003). The details are too technical to review here, but they hold more than a little physical interest since they are crucial in deciding whether there can be paraparticles which do not obey either Fermi or bosonic statistics.[26]

6 Superselection Rules and Gauge Symmetries

Suppose that a supersymmetry (as defined in Section 3) can be labeled a gauge symmetry. Then Theorem 1 says that weak superselection rules correspond to gauge symmetries, and Theorem 4 says that very strong superselection rules hold just in case the gauge symmetries are commutative. The application of the rubric of gauge is justified if it is agreed that the basic notion of gauge freedom is rooted in the redundancy of the descriptive apparatus of a theory; that is, a theory displays gauge freedom when the state descriptions correspond many-one to the same physical state, in which case the gauge transformations shuttle between the equivalent descriptions (see Henneaux and Teitelboim 1992). In the present context, this can be made precise as follows.

In ordinary QM the vectors ψ and $e^{i\phi}\psi$ are gauge equivalent in that they correspond to the same physical state in the sense of an expectation value functional ω on $\mathfrak{B}(\mathcal{H})$. In standard texts on ordinary QM this gauge freedom is acknowledged by the dictum that physical states correspond to rays in $\mathcal{H}$.[27] In the presence of a superselection rule there is an extra gauge freedom. A supersymmetry/gauge operator has the form $U = \sum_j e^{i\phi_j} E_j$, where per usual

[26] If it is demanded that the gauge group have an action on the Hilbert space, then commutativity of superselection rules fails and parastatistics are precluded; see Galindo et al. (1962).

[27] Or in more sophisticated language, by the dictum that the state space is the projective Hilbert space.

the E_j are the projectors onto the superselection sectors of $\mathcal{H} = \oplus_j \mathcal{H}_j$. Any two rays generated by the state vectors $\psi = \sum_j c_j \psi_j$, with $\psi_j \in \mathcal{H}_j$, and $U\psi$ are gauge equivalent in the sense that they correspond to the same physical state on the algebra of observables $\mathfrak{M} \subseteq \oplus_j \mathfrak{B}(\mathcal{H}_j)$; namely, the mixed state expectation value functional $\omega = \omega_\psi = \omega_{U\psi} = \sum_j |c_j|^2 \omega_{\psi_j}$.

Does gauge invariance provide the basis for a general explanation of superselection rules? A potential explanatory scheme would run as follows. Find that $U = \sum_j e^{i\phi_j} E_j$, where the E_j are the projectors onto the $\mathcal{H}_j$ of $\mathcal{H} = \oplus_j \mathcal{H}_j$, is a supersymmetry/gauge transformation. On the grounds that a genuine observable must be a gauge invariant quantity, conclude that the algebra of observables $\mathfrak{M}$ must satisfy $U\mathfrak{M}U^{-1} = \mathfrak{M}$ and, therefore, that $\mathfrak{M} \subseteq \oplus_j \mathfrak{B}(\mathcal{H}_j)$. The hitch is in the first step: without knowing what the observables are, how do we "find that" some U is a supersymmetry/gauge transformation? There may be persuasive considerations that some particular U (say, spatial rotation by 2π) must be a supersymmetry/gauge transformation, but there is no obvious method that can decide the issue for arbitrary U without first getting an antecedent grip on what $\mathfrak{M}$ is.

In classical theories one is forced to recognize gauge freedom in order to avoid indeterminism. For instance, the initial value problem for Maxwell's equations for classical electromagnetism does not have a unique solution if the equations are written in terms of the electromagnetic potentials and if these quantities are regarded as genuine physical magnitudes. The standard reaction is to restore determinism by demoting the potentials from genuine magnitudes to gauge dependent quantities that correspond many-one to the genuine magnitudes, the electric and magnetic field vectors, for which there is a well-posed initial value problem. Similarly, in general relativity theory one is forced to treat diffeomorphism invariance as a gauge symmetry if the initial value problem for Einstein's field equations is to have a unique solution even locally in time. It might seem that the cudgel of determinism is not available for quantum theories. For if the Hamiltonian for the system is (essentially) self-adjoint then the time evolute of every vector in the Hilbert space—regardless of whether or not it has components in different superselection sectors—is uniquely defined for all time; and if the Hamiltonian is not (essentially) self-adjoint there is no quantum dynamics—unless the boundary conditions for the problem pick out a particular self-adjoint extension, reverting to the previous case. In fact, however, in the presence of a very strong superselection rule there is gauge freedom in the definition of the time evolution operator. This operator has the form $V_t = \sum_j e^{-iH_j t} E_j$ where H_j is the Hamiltonian for the sector $\mathcal{H}_j$. The H_j are defined only up to an additive real constant c_j so that $\tilde{V}_t = \sum_j e^{-iH_j t} e^{ic_j t} E_j$ is an equally valid evolution operator. Needless to say, V_t and $\tilde{V}_t$ evolve vectors corresponding to the same physical state (expectation value functional) to vectors also corresponding to the same physical state.

7 The Origins of Superselection

The above discussion was based on the notion of the von Neumann algebra $\mathfrak{M}$ of observables generated by the set $\mathcal{O}$ of self-adjoint operators on $\mathcal{H}$ that correspond to genuine physical observables. No attempt was made to say directly what $\mathcal{O}$ and $\mathfrak{M}(\mathcal{O})$ are; rather $\mathcal{O}$ was taken as given, $\mathfrak{M}(\mathcal{O})$ was constructed as $\mathcal{O}''$, and then superselection rules of various strengths were characterized in terms of features of $\mathfrak{M}(\mathcal{O})$. In the absence of a systematic method for getting a fix on $\mathcal{O}$ and $\mathfrak{M}(\mathcal{O})$, superselection rules can be established either by appeal to empirical considerations or by group-theoretic/symmetry considerations. As an example of the former, in the early days of superselection rules it was noted that no one has ever succeeded in forming a coherent superposition of states of different charges. While such failure provides some evidence for a superselection rule for charge, it can never be definitive—perhaps the failure to form a coherent superposition is due to our lack of ingenuity or the crudeness of our experimental techniques. As an example of the latter, the superselection rule for integer and half-integer angular momentum was initially established using symmetry arguments concerning time reversal invariance (Wick et al. 1951). The argument uses SSRIII and proceeds via a *reductio* strategy. Suppose that a state ψ_I of integer angular momentum could be coherently superposed with a state $\psi_{I/2}$ of half-integer angular momentum. Then, assuming that time reversal R is a valid symmetry, R is an anti-unitary operator and the application of R^2 to the superposition of ψ_I and $\psi_{I/2}$ should produce the same physical state. But it doesn't because the relative phases of ψ_I and $\psi_{I/2}$ are changed since $R^2 = +1$ for states of integer angular momentum and -1 for states of half-integer angular momentum. When it was realized that time reversal might not be a valid symmetry, the univalence superselection rule was reestablished on the basis of rotational invariance (see Section 12).

Symmetry considerations can be definitive in a way that brute empirical considerations can never be, but it is not clear *a priori* that they capture all superselection rules. A guarantee that symmetry considerations do capture all superselection rules would flow from the doctrine, originally due to Hermann Weyl, that quantization is to be pursued by choosing an appropriate symmetry group and then determining the unitary representations of that group. This doctrine has been pursued by G. W. Mackey with some success, but it has never been brought to fruition (see Mackey 1998 for an overview). For cases that do fall under the doctrine, the derivation of the superselection rules associated with the relevant symmetry group is discussed Divakaran (1994).[28]

The other approach to quantum theory that promises to provide a uniform and comprehensive account of superselection rule—and the one that will the followed here—is due to Haag and Kastler (1964). In this approach the basic object is neither a symmetry group nor a concrete von Neumann algebra of operators acting on a Hilbert space $\mathcal{H}$ but an abstract C^*-algebra $\mathcal{A}$ of operators whose definition does not use or mention Hilbert space. Contact with the Hilbert space apparatus is made by taking a representation of $\mathcal{A}$, i.e. a *-homomorphism $\pi : \mathcal{A} \rightarrow \mathfrak{B}(\mathcal{H})$ for some Hilbert space $\mathcal{H}$. The von Neumann algebra associated with π is then $\pi(\mathcal{A})''$. Thus, from the point of view of the Haag-Kastler approach, von Neumann algebras are representation-dependent objects. How the choices of $\mathcal{A}$ and π are made will be discussed below. But first I want to indicate how the algebraic approach can illuminate superselection rules, especially those of the very strong variety.

8 The Algebraic Approach[29]

[28] An accessible introduction can be found in Giulini (2003). What makes the analysis complicated is that for many of the symmetry groups encountered in physics, the faithful unitary representations are projective (a.k.a. ray representations or representations up to a phase factor). In a projective unitary representation of a group G, $U(g_1)U(g_2) = \gamma(g_1, g_2)U(g_1, g_2)$, $g_1, g_2 \in G$, where the $U(g)$ are unitary operators and $\gamma : G \text{ x } G \rightarrow U(1)$ (the complex numbers of unit modulus). The multipliers γ must satisfy the cocycle condition $\gamma(g_1, g_2)\gamma(g_1 g_2, g_3) = \gamma(g_1, g_2 g_3)\gamma(g_2, g_3)$. The multipliers γ and γ' are said to be similar just in case there is an $f : G \rightarrow U(1)$ such that $f(e) = 1$ (e being the identity element of G) and $\gamma'(g_1, g_2) = \gamma(g_1, g_2)f(g_1 g_2)f(g_1)^{-1}f(g_2)^{-1}$ for all $g_1, g_2 \in G$. The classification of non-similar multipliers is one of the keys to the superselection rule associated with G.

[29] Only rudimentary aspects of the algebraic approach to superselection rules are treated here. For more advanced information, see Roberts and Roepstroff (1969), Kastler (1990), and Landsman (1991).

For ease of presentation I concentrate on the case of dichotomous superselection rules. According to the algebraic approach all of the information about the structure of observables of the system of interest is encoded in a C^*-algebra $\mathcal{A}$, and so all relevant representations will be representations of the relevant $\mathcal{A}$. A state ω on $\mathcal{A}$ is said to be *normal* with respect to a representation $\pi, \mathcal{H}_\pi$ of $\mathcal{A}$ if there is a density operator ρ acting on $\mathcal{H}_\pi$ such that $\omega(A) = Tr(\rho\pi(A))$ for all $A \in \mathcal{A}$. Two representations π_1 and π_2 are said to be *disjoint* just in case the set of π_1-normal states is disjoint from the set of π_2-normal states. Coherent superpositions of two states that are normal with respect to disjoint representations are precluded in the following sense. Let ω_i, $i = 1, 2$, be any π_i-normal states on $\mathcal{A}$, and let π be any representation of $\mathcal{A}$. If there are vectors $\varphi_1, \varphi_2 \in \mathcal{H}_\pi$ such that $\omega_i(A) = (\varphi_i, \pi(A)\varphi_i)$ for all $A \in \mathcal{A}$, then the vector state ω_Φ corresponding to the superposition $\mathcal{H}_\pi \ni \Phi = \alpha\varphi_1 + \beta\varphi_2$, $|\alpha|^2 + |\beta|^2 = 1$, is the mixed state $|\alpha|^2\omega_{\varphi_1} + |\beta|^2\omega_{\varphi_2}$. The converse is also true: if coherent superpositions of a π_1-normal state and a π_2-normal state is precluded, then π_1 and π_2 are disjoint.[30] Together these two results point to disjointness of representations as the source of the limitation on the superposition principle.

Respecting the fact that coherent supposition of states on disjoint representations $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ of the algebra $\mathcal{A}$ are not possible, the von Neumann algebras $\pi_1(\mathcal{A})''$ and $\pi_2(\mathcal{A})''$ affiliated with the representations can be combined in a direct sum $\pi_1(\mathcal{A})'' \oplus \pi_2(\mathcal{A})''$ acting on $\mathcal{H}_1 \oplus \mathcal{H}_2$. Alternatively, the relevant von Neumann algebra can be taken to be $\pi_{12}(\mathcal{A})''$ where $\pi_{12} := \pi_1 \oplus \pi_2$ is the direct sum representation acting on $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$. For arbitrary representations π_1 and π_2 it is always the case that $\pi_{12}(\mathcal{A})'' \subseteq \pi_1(\mathcal{A})'' \oplus \pi_2(\mathcal{A})''$. In general, however, the inclusion is proper; in particular, $\pi_{12}(\mathcal{A})'$ contains the projectors E_j from $\mathcal{H}$ to $\mathcal{H}_j$, $j = 1, 2$, but it is not guaranteed that $\pi_{12}(\mathcal{A})''$ contains the E_j. But for disjoint representations the inclusion is in fact identity:

> *Theorem 5.* (Kadison and Ringrose 1991, Theorem 10.3.5) The following conditions are equivalent:
>
> (i) $\pi_{12}(\mathcal{A})'' = \pi_1(\mathcal{A})'' \oplus \pi_2(\mathcal{A})''$
>
> (ii) the representations π_1 and π_2 are disjoint.
>
> These conditions hold if and only if $E_j \in \pi_{12}(\mathcal{A})''$ and, thus, $E_j \in \pi_{12}(\mathcal{A})'' \cap \pi_{12}(\mathcal{A})' = \mathcal{Z}(\pi_{12}(\mathcal{A})'')$.

[30] These results come from a slight extension of Theorem 6.1 of Araki (1999).

For very strong superselection rules we also want the superselection sectors to be coherent subspaces and, thus, π_1 and π_2 should be irreducible. For irreducible representations π_1 and π_2, disjointness of representations coincides with *unitary inequivalence*: there is no unitary map $U : \mathcal{H}_1 \to \mathcal{H}_2$ such that $\pi_2(A) = U\pi_1(A)U^{-1}$ for all $A \in \mathcal{A}$.[31] Thus, from the point of view of the algebraic approach, very strong superselection rules arise from irreducible but unitarily inequivalent representations of the relevant C^*-algebra. Of course, for this dictum to be illuminating it must be accompanied by an account of how the physically relevant representations arise. This issue will be taken up in the following section. In the remainder of this section I will comment on the prior issue of how the relevant C^*-algebras are obtained, and will take up a challenge to the algebraic explanation of the origin of superselection rules.

There is no extant general account of how to characterize the physically relevant C^*-algebras much less how to explicitly construct them. But there is a large body of detailed work on the characterization and/or construction of the algebras for particular cases. And all that can be said without going into details, which would be inappropriate here, is that in all of the cases of which I am aware the physical motivations underlying the constructions are convincing. For an accessible example from QFT, the reader may wish to consider the construction of the algebra of symplectically smeared fields for the case of a Klein-Gordon field in Wald (1994). Here I will confine my remarks to the humbler case of non-relativistic QM where the algebraic approach might seem to run into problems in accounting for superselection rules.

The relevant algebra for this humble case would seem to be the canonical commutation relation (CCR) algebra. Because the CCR cannot be satisfied by bounded operators, in order to construct a C^*-algebra that encodes the CCR it is necessary to use the Weyl version of the CCR which exponentiates the momentum P and position Q operators to form one-parameter unitary groups $S(s) := e^{iPs}$ and $T(t) := e^{iQt}$, $s, t \in \mathbb{R}$. The CCR are then encoded in the commutation condition $S(s)T(t) = e^{ist}T(t)S(s)$.[32] This concrete form of Weyl CCR algebra can be replaced by an abstract CCR C^*-algebra.[33]

[31] An equivalent characterization of disjoint representations can be given using this concept; namely, two representations are disjoint just in case they do not share any unitarily equivalent subrepresentations.

[32] The Weyl CCR entail the familiar CCR but not vice versa.

[33] On the advantages of the C^*-algebra approach is that it is supposed to be representa-

The Stone-von Neumann theorem shows that all irreducible strongly continuous representations of a finite dimensional Weyl CCR algebra are unitarily equivalent—in particular, they are all unitarily equivalent to the familiar Schrödinger representation (see Reed and Simon 1980, Theorem VIII.14).[34] It would seem then that all of the physically acceptable irreducible representations are unitarily equivalent and, thus, from the point of view of the algebraic approach, there can be no very strong superselection rules in non-relativistic QM—which might seem to constitute a *reductio* of the algebraic approach to superselection since there are plausible candidates for very strong superselection rules in non-relativistic QM, the superselection rule for integer and half-integer angular momentum being one.[35]

The obvious response comes in parts. First, for a system consisting of a finite number of spinless point particles a finite dimensional Weyl CCR algebra is the appropriate C^*-algebra, and since there are no unitarily inequivalent irreducible representations in the offing for this algebra there can be no (very strong) superselection rules—which seems correct. Second, if additional degrees of freedom are added, the relevant algebra will be not be the Weyl CCR algebra but some other algebra which may admit unitarily inequivalent representations. And contrary to folklore, neither an infinite number of particles nor an infinite number of degrees of freedom is necessary for the existence of unitarily inequivalent representations. For example, consider the mystery algebra ?-algebra generated from the elements A_x, A_y, A_z satisfying the commutation relations $[A_x, A_y] = i\hbar A_z$, $[A_y, A_z] = i\hbar A_x$, $[A_z, A_x] = i\hbar A_y$ and the self-adjointness conditions $A_x^* = A_x$, etc. A consequence of these conditions is that $A^2 := A_x^2 + A_y^2 + A_z^2 = \hbar^2 a(a+1)$, where a may take on integer and half-integer values. To make a C^*-algebra it is necessary to exponentiate the generators A_x, A_y, A_z along the lines of the Weyl version of the CCR algebra. For each value of a there are irreducible representations of

tion independent. It is somewhat embarrassing, therefore, that typical constructions of the C^*-algebra version of the concrete Weyl CCR algebra use Hilbert space representations. However, the embarrassment is overcome by showing that the C^*-algebra is independent of the representation used in the construction in the sense that unitarily equivalent representations lead to the same C^*-algebra. For a discussion of these matters, see Baez et al. (1992, Ch. 5).

[34] The Stone-von Neumann theorem breaks down for an infinite number of degrees of freedom—as encountered in QFT and quantum statistical mechanics—and unitarily inequivalent representations of the Weyl CCR exist in abundance.

[35] For an overview of superselection rules in non-relativistic QM, see Cisneros et al. (1998).

the (exponentiated) ?-algebra acting on a $(2a+1)$-dimensional Hilbert space, and they are all unitarily equivalent. But for any two distinct values of a these representations are unitarily inequivalent.

Alert readers will recognize that the natural interpretation for "?" in the mystery algebra is angular momentum, with A_x, A_y, A_z acting as the generators of spatial rotations. (So as not to make this too obvious I used A and a rather than the more standard L and ℓ.) In that case the physically relevant C^*-algebra will not be the angular momentum algebra *per se* but the combination of this algebra and the Weyl CCR algebra with P_x, P_y, P_z interpreted as the generators of spatial translations. Spatial translations and rotations do not commute, and as a result there is no longer a superselection rule for every pair of distinct a values since the spatial translations "mix up" representations of the angular momentum algebra labeled by different a values. However, a dichotomous superselection rule—the univalence rule—survives. This is seen most easily from group theoretic considerations. The Weyl CCR algebra has a group structure, and the exponentiations of the angular momentum operators produces the group of spatial rotations $SO(3)$. Rotations of 2π do, of course, commute with translations; and the representations of 2π rotations are inequivalent for integer and half-integer angular momentum states (see Section 12).[36] On the other hand, it should be emphasized that it is not given *a priori* that ? is angular momentum. A toy example where A_x, A_y, A_z are *not* interpreted as generators of spatial rotations and where the ?-algebra *is* the total algebra of observables is given in exercise XIII.15 of Messiah (1962, pp. 579-580). In such a case there is a superselection rule for each pair of distinct a values—or so the algebraic approach says.

The general challenge to the algebraic approach is to show in detail how every known superselection rule can be reproduced and, it can be hoped, to show how new superselection rules can be predicted without, it can again be hoped, predicting "too many" such rules. I will return to this challenge in Section 10.

9 More Representation Theory

For any state ω on a C^*-algebra $\mathcal{A}$ there is a representation $(\pi_\omega, \mathcal{H}_\omega, \Omega_\omega)$ called the GNS representation determined by ω. $\Omega_\omega \in \mathcal{H}_\omega$ is a cyclic vector,

[36] It is natural to wonder whether the univalence superselection rule survives when the rotation and translation groups are imbedded in an even larger group—for example, the Galilean or the Poincaré group. The answer is yes; see below.

i.e. $\{\pi_\omega(\mathcal{A})\mathcal{H}_\omega\}$ is dense in $\mathcal{H}_\omega$, and $\omega(A) = (\Omega_\omega, \pi_\omega(A)\Omega_\omega)$ for all $A \in \mathcal{A}$. π_ω is the unique, up to unitary equivalence, cyclic representation that preserves expectation values of the state ω. Since every representation of $\mathcal{A}$ is a direct sum of cyclic representations, the GNS representations can be regarded as the fundamental ones.

Recall our working assumption that the superselection sectors are coherent subspaces. So in the case of dichotomous superselection rules we want the representations π_1 and π_2 that produce the von Neumann algebras of observables acting on the selection sectors to be irreducible. A basic fact about GNS representations is that π_ω is irreducible iff ω is a pure state. So choose pure states ω_1 and ω_2 and set $\pi_1 = \pi_{\omega_1}$ and $\pi_2 = \pi_{\omega_2}$. And for a very strong superselection rule we want π_{ω_1} and π_{ω_2} to be disjoint (= unitarily inequivalent since they are irreducible). Now consider the result of taking the GNS representation of a mixture of disjoint pure states.

> *Theorem 6.* Let ω be the mixed state $\frac{1}{2}(\omega_1 + \omega_2)$ where ω_1 and ω_2 are distinct pure states. Then the following are equivalent:
>
> (i) the GNS representations π_{ω_1} and π_{ω_2} are disjoint (= unitarily inequivalent).
>
> (ii) $\pi_\omega(\mathcal{A})'' = \pi_{\omega_1}(\mathcal{A})'' \oplus \pi_{\omega_2}(\mathcal{A})''$, $\mathcal{H}_{\pi_\omega} = \mathcal{H}_{\pi_{\omega_1}} \oplus \mathcal{H}_{\pi_{\omega_2}}$, and the projectors E_j, $j = 1, 2$, from $\mathcal{H}_{\pi_\omega}$ onto $\mathcal{H}_{\pi_{\omega_j}}$ are in $\pi_\omega(\mathcal{A})'' \cap \pi_\omega(\mathcal{A})'$.

All of the ingredients of the proof are in contained in Kadison and Ringrose (1991, Theorem 10.3.5) and Bratelli and Robinson (1971, Lemma 4.2.8). Thus, *if* the relevant von Neumann algebra of observables is $\pi_\omega(\mathcal{A})''$, where ω is a mixture of disjoint pure states, then there is a very strong superselection rule.[37] The following section will discuss whether this result can be used as a basis for an explanation of the origin of superselection.

To drive home the point about the importance of unitarily inequivalent representations, start with the case of ordinary QM where the von Neumann algebra of observables is $\mathfrak{B}(\mathcal{H})$. Choose vectors $\psi_1, \psi_2 \in \mathcal{H}$ such that $(\psi_1, \psi_2) = 0$. Construct the density operator $\varrho = \frac{1}{2}E_{\psi_1} + \frac{1}{2}E_{\psi_2}$ where E_{ψ_j} projects onto the ray spanned by ψ_j. By the trace prescription ϱ determines

[37] Here the notion is disjointness is extended to states by saying that two states are disjoint just in case they determine disjoint GNS representations.

a state ω_ϱ on $\mathfrak{B}(\mathcal{H})$ given by $\omega_\varrho(A) := tr(\varrho A)$ for $A \in \mathfrak{B}(\mathcal{H})$; and, of course, there are vector states ω_{ψ_j} corresponding to the ψ_j. Take the GNS representations π_{ψ_1}, π_{ψ_2}, and π_{ω_ρ} of $\mathfrak{B}(\mathcal{H})$ determined by these three states. π_{ψ_1} and π_{ψ_2} are irreducible unitarily equivalent representations. π_{ω_ρ} is a reducible representation that is a subrepresentation of the direct sum representation $\pi_{\psi_1} \oplus \pi_{\psi_2}$, and $\pi_{\omega_\varrho}(\mathfrak{B}(\mathcal{H}))'' \subset \pi_{\psi_1}(\mathfrak{B}(\mathcal{H}))'' \oplus \pi_{\psi_2}(\mathfrak{B}(\mathcal{H}))''$ where the inclusion is proper; in particular, the projectors from $\mathcal{H}_{\pi_{\psi_1}} \oplus \mathcal{H}_{\pi_{\psi_2}}$ onto the selection sectors are not in $\pi_{\omega_\varrho}(\mathfrak{B}(\mathcal{H}))'' \cap \pi_{\omega_\varrho}(\mathfrak{B}(\mathcal{H}))'$. So by taking the GNS representation of a mixed state in ordinary QM we get an artificial example of a weak superselection rule that is not a strong superselection rule—artificial because $\pi_{\omega_\varrho}(\mathfrak{B}(\mathcal{H}))''$ is not the "correct" von Neumann algebra of observables. What goes into picking the correct algebra is the topic of the next section.

10 An Explanation of the Origin of Superselection Rules?

The preceding two sections show how the representation theory for C^*-algebras can clarify the mathematical structure of superselection rules. But does this representation theory offer a satisfying explanation of the origin of superselection? Strocchi and Wightman (1974) opine that in the Haag-Kastler algebraic approach, each quantum theory "predicts its own superselection rules" (p. 2198). Here is one explication of what they mean.

Step 1. Choose the relevant C^*-algebra $\mathcal{A}$ for the system of interest.

Step 2. Identify the class of physically admissible states $\mathcal{S}$ on $\mathcal{A}$. Examples of proposed criteria of admissibility: (a) For a non-interacting scalar quantum field it has been proposed that admissible states must satisfy the Hadamard condition that guarantees that an expectation value for the renormalized stress-energy tensor of the field can be defined (see Wald 1994). (b) For quantum fields in general it is widely assumed that an admissible state satisfies the spectrum condition (energy-momentum operator has a spectrum confined to the future light cone). (c) The Doplicher-Haag-Roberts selection criterion requires that an admissible state differs from the vacuum only in a bounded region of spacetime.

It is at the first two steps that the algebraic approach can join forces with the group-theoretic/symmetry approach to superselection since the choice of $\mathcal{S}$ and $\mathcal{A}$ may be influenced by symmetry considerations. And, of course, many other considerations may influence the choice. But once Steps 1-2 are completed the superselection sectors are automatically fixed by the next three steps.[38]

Step 3. The pure states among $\mathcal{S}$ determine the admissible irreducible representations of $\mathcal{A}$. Partition these representations into unitary equivalence classes.

Step 4. For present purposes assume that there are a countable number of unitary equivalence classes. Choose one representation π_j, $j = 1, 2, ...$, from each class, and form the direct sum representation $\oplus_j \pi_j$.

Step 5. Posit that the relevant von Neumann algebra of observables is $(\oplus_j \pi_j)'' = \oplus_j \pi_j(\mathcal{A})'' = \oplus_j \mathfrak{B}(\mathcal{H}_{\pi_j})$ acting on $\oplus_j \mathcal{H}_{\pi_j}$.[39] Then, as follows from the above discussion, there is a very strong superselection rule.

If the resulting superselection rule does not accord with intuitions, the algebra of Step 1 or the admissibility requirements of Step 2 can be adjusted. Such a move threatens circularity unless independent motivations can be found for the adjustments. But there can be no objection to using a reflective equilibrium in making a joint judgment of what counts as a genuine superselection rule and of how to implement Steps 1 and 2.

It is tempting to venture a justification for the Posit of Step 5:

Step 6. The justification/explanation of the Posit of Step 5 is illustrated for the dichotomous case and can be generalized to the countable case. Let ω_j, $j = 1, 2$, be the disjoint pure states that give rise to the π_j of Step 4. Make the further posit that the state ω of the system is a mixed state obtained from a convex linear combination of the ω_j. Then the von Neumann algebra of observables is $\pi_\omega(\mathcal{A})''$, and by Theorem 6 this is algebra is $\pi_{\omega_1}(\mathcal{A})'' \oplus \pi_{\omega_2}(\mathcal{A})'' = \mathfrak{B}(\mathcal{H}_{\pi_{\omega_1}}) \oplus \mathfrak{B}(\mathcal{H}_{\pi_{\omega_2}})$.[40]

This explanation has an as-if quality, and as-if explanations are not explanations. The as-if quality arises from the fact that the best one can say for the mixed state ω is that it is as if the actual state of the system were ω. For in the first place, the choice of the ω_j in the mixture that defines ω involves a

[38] "[W]e will simply use the phrase physically admissible to indicate a state selected according to some appropriate criteria. The important point is that, once the definition of physically admissible has been fixed, the superselection sectors of a Haag-Kastler [local quantum field theory] are given by the unitarily inequivalent physically admissible representations of the quasilocal algebra" (Strocchi and Wightman 1974, p. 2198).

[39] Note that if π and $\hat{\pi}$ belong to the same unitary equivalence class, then $\pi(\mathcal{A})''$ and $\hat{\pi}(\mathcal{A})''$ are $*$-isomorphic. Thus, the algebras of observables $\oplus_j \pi_j(\mathcal{A})''$ and $\oplus_j \hat{\pi}_j(\mathcal{A})''$ are the same.

[40] Strocchi and Wightman (1974) do *not* take Step 6.

large degree of arbitrariness since different representatives $\hat{\pi}_j$ of the unitary equivalence classes and the states $\hat{\omega}_j$ that give rise to them could have been chosen; the von Neumann algebra of observables would be unaffected but the state $\hat{\omega}$ would be different. And in the second place, even when a choice of the ω_j is made, different convex linear combinations will give rise to the same von Neumann algebra of observables. In addition, the most plausible interpretation of the mixed state ω is an ignorance interpretation—the weights of the linear combination of pure states that defines ω represent our ignorance of which pure state actually obtains. Is it at all plausible that superselection rules arise because of our ignorance of the actual state of the system? And do the superselection rules cease to apply when our ignorance is cured?

Failing a convincing *a priori* justification, the Posit of Step 5 is open to various challenges. The first derives from the fact that it accommodates only very strong superselection rules. This can be taken as a virtue since it is typically assumed that superselection rules are of the very strong variety. On the other hand, traditional arguments for superselection often only demonstrate weak superselection in the sense that some pure states cannot be coherently superposed, and it is conceivable that this is as strong as superselection gets. If conceivability is matched by genuine physical possibility, then the proponent of the algebraic approach can backpedal to a weakened version of the Posit that asserts only that the von Neumann algebra of observables for the system in question is a subalgebra of $\oplus_j \pi_j(\mathcal{A})''$ acting on $\oplus_j \mathcal{H}_{\pi_j}$. But then the promise of the algebraic approach to provide a uniform and comprehensive account of the origin of superselection is compromised since additional considerations would need to be brought in to explain which subalgebra of $\oplus_j \pi_j(\mathcal{A})''$ is appropriate to use. The more sanguine response would seek to maintain the undiluted Posit. Here is a heuristic argument in support. The algebra of observables would be a proper subalgebra of $\oplus_j \pi_j(\mathcal{A})''$ if some of the projectors E_j from $\oplus_j \mathcal{H}_{\pi_j}$ onto the sectors $\mathcal{H}_{\pi_j}$ are not properly regarded as genuine observables—or relatedly, if no self-adjoint operator of the form $T = \sum_{(all)\ j} r_j E_j$, $r_j \in \mathbb{R}$, is properly regarded as an observable—in which case there would either be a weak or a strong superselection rule but not a very strong rule. But (the argument would go) if unitarily inequivalent representations are going to contribute to a genuine superselection rule—even a weak one—they must be distinguished by some feature that makes for a real physical difference, and this difference can be expected to provide a basis for regarding all of the E_j as in-principle measurable and, thus, elements of the observable algebra. This heuristic argument has some force, but it assumes what needs to be demonstrated; namely, in every case where extant arguments establish only a weak superselection rule, there is a physically motivated C^*-algebra $\mathcal{A}$ whose admissible unitarily inequivalent representations ground (by the Posit) the very strong version of the established weak superselection rule.

Even if all superselection rules are of the very strong variety, there is a second challenge to the Posit which questions whether all physically admissible unitarily inequivalent representations contribute to superselection rules. It is known, for example, that non-interacting relativistic scalar fields of different masses correspond to unitarily inequivalent representations of the Weyl CCR algebra (see Reed and Simon 1975, Theorem X.46). Presumably all of these representations are physically admissible, but there is no superselection rule for mass in relativistic QFT—or at least the heuristic argument for a superselection rule for mass in non-relativistic QM does not apply in the relativistic case. For a spinless particle obeying a Galilean invariant Schrödinger equation, the argument for a mass superselection rule proceeds in *reductio* fashion. Assume that there can be a linear superposition of states of different masses. Perform a sequence of transformations consisting of a spatial translation, a Galilean velocity boost, the opposite spatial translation and, finally, the opposite velocity boost. The resulting state should be the same as the starting state, but it is not because there has been a change in the relative phases of the two mass states (see Kaempffer 1965, Appendix 7).[41] This argument no longer applies when the Klein-Gordon equation is substituted for the Schrödinger equation and Poincaré invariance is substituted for Galilean invariance (see again Kaempffer 1965, Appendix 7).

One way of deflecting the potential counterexample to the Posit is to claim that inequivalent representations of the Klein-Gordon field with different masses are representations of different systems, not representations of different ways of being of the same system. But such a response does not mesh well with the algebraic approach since different systems should implicate different C^*-algebras. Another way of deflecting the example is to claim that it is not appropriate for testing the Posit; since mass can have a continuum of values, a superselection rule for mass would have to be of the

[41] This argument has been labeled heuristic because to bring it into the formalism described above would require the introduction of a mass operator for non-relativistic QM. For how this can be done and the implications for the status of a superselection rule for mass, see Giulini (1996).

continuous variety, for which no apparatus has been provided.[42] But perhaps a better strategy for the advocate of the algebraic approach is not to be defensive but to take the offensive and declare victory: declare that the unitary inequivalence of representations with different mass values proves that there *is* a superselection rule for mass in the case of a Klein-Gordon field; and more generally, claim that the superiority of the algebraic approach is manifested in the fact that it is able to reveal superselection rules that are not captured—at least not in any obvious way—by traditional methods which emphasize group-theoretic/symmetry considerations.

In sum, one of the most attractive features of the algebraic approach is that it offers a systematic method for deriving superselection rules. But whether this method can succeed without supplementation of considerations from outside the approach remains to be seen.

11 Superselection, the Measurement Problem, and Classicality

The father of superselection rules, Eugene Wigner, was not sanguine about the ability of such rules to resolve the most troubling interpretational issues of QM.

> [T]he so-called superselection rules do limit the absolute generality of the rule of superposition—they limit it, however, just enough to impair the mathematical beauty of the general, single and uniform Hilbert space as a frame for the description of all quantum mechanical states. They do not seem to alleviate significantly the conceptual question raised. (Wigner 1973, p. 370)

The conceptual question referred to is how to interpret a linear combination of classical states (e.g. "live cat" and "dead cat"). Other authors are more sanguine; indeed, some see superselection rules as the key to a solution of the measurement problem and, more generally, to an explanation of how the classical world we perceive can be reconciled with quantum theory. It is easy to understand the initial attraction of this idea since (very strong) superselection rules provide two essential necessary conditions for classical observables: a self-adjoint operator in the center $\mathcal{Z}(\mathfrak{M})$ of the von Neumann algebra $\mathfrak{M}$ of observables is an observable that commutes with all observables, and any linear combination of vector states corresponding to different eigenvalues of

[42] Some remarks on the problem with extending the formalism to include continuous superselection rules are given in Section 13.

such an observable is equivalent to a mixture over these states. Moreover, in the case of very strong superselection rules this mixed state lends itself to an ignorance interpretation since the decomposition into disjoint pure states is unique.[43] (Some authors take these conditions to be jointly sufficient for classicality; see below). But how to guarantee that the classical observables we care about are subject to superselection rules? Before turning to this issue, a comment on the relationship between superselection rules and dynamics is in order.

Given the assumption, common to all approaches to quantum theory, that the dynamics is given by an automorphism of the algebra of observables, superselection rules belong to the kinematic structure of observables; for the assumption implies that superselection rules must be present *ab initio* if they are to be present at all. In general, however, there is no guarantee that a dynamical automorphism can unitarily implemented by the familiar Hamiltonian dynamics. But such implementation is guaranteed for the case of very strong superselection rules, as follows from

> *Lemma 2* (Emch 1972, p. 157) Let $\mathfrak{M}$ be a von Neumann algebra acting on $\mathcal{H}$. If $\mathfrak{M}$ has an abelian commutant (as is the care for a very strong superselection rule) then every automorphism α of $\mathfrak{M}$ is unitarily implementable in the sense that there is a unitary $V : \mathcal{H} \to \mathcal{H}$ such that $\alpha(A) = VAV^{-1}$ for all $A \in \mathfrak{M}$.

This result generalizes to the case where α_t is a strongly continuous one-parameter group of automorphisms of $\mathfrak{M}$, in which case α_t is implemented by a strongly continuous unitary group V_t. By Stone's theorem, the generator of the latter is a self-adjoint operator H, the Hamiltonian. Note, however, that it does not follow that H is affiliated with $\mathfrak{M}$ or that $V_t \in \mathfrak{M}$, a possibility that will be exploited below.

One way to guarantee that superselection rules are present in a form that ensures classicality of macro-observables is the time honored method of theft over honest toil. To illustrate, suppose that the Hilbert space for a composite system consisting of an object system (o) and a macroscopic measuring apparatus (M) is the tensor product space $\mathcal{H}_o \otimes \mathcal{H}_M$. The classical observable we care about for M is the pointer observable P whose eigenvalues $p_0, p_1, ...$ correspond to the macroscopically distinct pointer positions on the

[43] The importance of the uniqueness is emphasized by Landsman (1995).

dial of the apparatus. We can try to guarantee classicality by postulating that $\mathcal{H}_M = \oplus_j \mathcal{H}_j$ where the superselection sectors $\mathcal{H}_j$ are the eigenspaces of P. But this postulation does not mesh well with the usual story about measurement, which has it that a measuring apparatus functions by interacting with the object system so as to establish a correlation between the target observable of the object system and the pointer positions of the apparatus. For an ideal non-disturbing measurement this means that if φ_j and ϑ_k are respectively the eigenstates of the target observable[44] and the pointer observable P, then the evolution of the composite system is such as to take the initial state $\varphi_j \otimes \vartheta_0$ (ϑ_0 being the ready state corresponding to the null pointer position p_0) to $\varphi_j \otimes \vartheta_j$ in a finite time. Thus, if the object-apparatus system state vector is initially $\sum_k \alpha_k \varphi_k \otimes \vartheta_0$, it evolves in a finite time to $\sum_k \alpha_k \varphi_k \otimes \vartheta_k$. By the postulated superselection structure, this latter state has exactly the interpretation needed to resolve the measurement problem since it is equivalent to a mixture over the correlated object-apparatus states $\varphi_k \otimes \vartheta_k$ with mixture weights $|\alpha_k|^2$, and these mixture weights lend themselves to an ignorance interpretation.

Unfortunately, this story stumbles at the first step when the Hamiltonian H is an observable, for then the state vector cannot evolve from one superselection sector to another. Thus, an advocate of the superselection account of measurement must exploit the loophole mentioned above that $H \notin \mathcal{O}$. This is done explicitly by Wan (1980). The general sentiment in the philosophical community is that the price is too high (see Hughes 1989, Sec. 9.7 and Thalos 1998).[45]

The price might be worth paying if there were any plausible way to get the macroscopic superselection rules via honest toil rather than theft. According to the algebraic approach the toil would take the form of showing how the values of the superselected quantity corresponds to unitarily inequivalent representations of the appropriate C^*-algebra of observables. An

[44] For sake of simplicity the object observable is assumed to have a pure discrete and non-degenerate spectrum.

[45] There is also another worry. The superselection sectors for the measurement apparatus are not just superselection sectors for pointer position but for all the other quantities that are subject to (very strong) superselection rules. Thus, as the state vector moves from a sector corresponding to the null pointer position p_0 to a sector corresponding to, say, pointer position p_{15}, the values of all of the other superselected quantities must change, which is contrary to everything that is believed about superselection

example of how this might happen for a literally infinite quantum system was sketched by Bub (1988). In this example the measurement apparatus is supposed to consist of a doubly infinite linear array of spin $\frac{1}{2}$ particles. The macro-observable of "pointer up (down)," corresponding to all but a finite number of the particles having spin up (down), obeys a superselection rule because the values "up" and "down" characterize unitarily inequivalent irreducible representations of the infinite dimensional spin $\frac{1}{2}$ algebra.[46] Of course, treating the apparatus as an infinite system involves an idealization, and the macro-superselection rule that rests on this idealization could be dismissed as an artifact of the idealization. To the contrary, Bub (1988) takes the idealization as essential in that the determinatness of maco-properties requires it. I am against putting any ontological weight on the idealization at issue for two reasons. First, the claim of essentiality is undermined by other solutions to the measurement problem and other accounts of the emergence of classical properties that do not rely on the idealization of infinite systems. The many-worlds interpretation and the modal interpretation are two such viable candidates, and a third involving superselection rules of quite a different stripe will be discussed momentarily. Second, there is an obvious distinction between harmless and pernicious idealizations that comes into play in the present case. The distinction is purpose-relative, but that is as it should be. For purposes of predicting with some stated accuracy the recoil of a particle that hits a very massive wall, it can be useful but harmless to idealize the wall as having an infinite mass if the calculation is simplified by the idealization without compromising the required accuracy of prediction. But for a problem that calls for a prediction of the presence/absence of a dichotomous feature that is absent for all finite values of a parameter and present only in the infinite limit, the infinite idealization is pernicious since it yields a prediction that is contrary to the prediction for any actual finite system. This is precisely the situation with the presence/absence of the macro-superselection rule for "pointer up/down."[47]

In addition, the idealization of an infinite number of particles makes the problem of dynamics even more difficult. Bub designs a unitary dynamics that establishes in a finite time a perfect correlation between the infinite

[46] See Sewell (2002, Sec. 2.3) and Landsman (2007, Sec. 6.4). Bub's example does not fit well with the assumptions made here; in particular, the Hilbert space is non-separable and the commutant of the von Neumann algebra of observables is non-abelian.

[47] See Robinson (1990, 1994) for more about the status the idealization of infinite systems as well as superselection solutions to the measurement problem.

spin system pointer position and the object observable. But his dynamics is not an automorphism of the algebra of observables, and in fact it maps elements of the algebra outside the algebra (Landsman 1995, p. 54, note 16). Hepp's (1972) dynamics for a similar model is an automorphism of the algebra of observables, but in his model the pointer position registers only in the infinite $t \to \infty$ limit. This idealization is also pernicious for reasons that are similar to those that make the idealization of an infinite number of particles pernicious (see Landsman 1995, pp. 55-57).

This is hardly the end of the story. We saw above that an infinite number of degrees of freedom is not necessary for unitarily inequivalent representations. So perhaps superselection rules for macroscopic quantities need not be based on illicit infinite idealizations. Or if an infinite number of degrees of freedom are needed for such superselection rules, they can be had without idealization in the case of relativistic QFT. But while the possibility remains open, there is not a single extant, physically interesting example of how "pointer up (down)," or "live (dead) cat," etc. correspond to unitarily inequivalent representations of the relevant C^*-algebra of observables.[48] This may reflect our ignorance of how the macroscopic supervenes on the microscopic, or it may indicate that the classicality of the world we observe is not built into the structure of observables but is something that emerges under appropriate circumstances.

The decoherence approach attempts to implement the latter option. The goal is to show how, without assuming *ab initio* a superselection structure for the pointer observable of the measurement apparatus, the coupling of object-apparatus system to an environment induces "effective superselection rules" (see Zurek 1982). When the environment is taken into account, the initial object-apparatus-environment state vector $\sum_k \alpha_k \varphi_k \otimes \vartheta_0 \otimes \varepsilon_o$ evolves in time t to $\sum_k \alpha_k \varphi_k \otimes \vartheta_k \otimes \varepsilon_k(t)$. Tracing out over the environmental degrees of freedom produces a reduced density matrix for the object-apparatus system that is approximately $\rho^{oM}(t) \simeq \sum_k |\alpha_k|^2 E_{\varphi_k} \oplus E_{\vartheta_k}$—approximately, because the off-diagonal elements, which are proportional to $|(\varepsilon_m(t), \varepsilon_n(t))|$, are not necessarily zero. In toy models $|(\varepsilon_m(t), \varepsilon_n(t))|$ is shown to rapidly approach zero, which the advocates of decoherence program take to mean that ρ^{oM} "can be thought of as describing the apparatus in a definite [but unknown]

[48] At least not if the C^*-algebra is the total algebra of observables. But if the relevant algebra is some subalgebra of the total algebra, the situation changes; see below.

state" (Zurek 1982, p. 1863). Sometimes this claim seems to have a FAPP interpretation while other times a literal reading seems to be intended.[49] Even when the environmental states $\varepsilon_k(t)$ are exactly orthogonal, the literal reading would require a semantics for value assignments that breaks the eigenvalue-eigenvector link, which asserts that in a given state $\psi \in \mathcal{H}$ an observable has a definite value if and only if ψ is an eigenstate of the operator corresponding to said observable. The value assignments provided by some version of the so-called modal interpretation of QM (see Vermaas 1999) seems tailor made to complete the decoherence approach. The modal interpretation preserves the 'if' part of the eigenvalue-eigenvector link but rejects the 'only if' part. Leaving aside many nuances, the 'only if' part is replaced by the rule that when a quantum (sub)system is described by a reduced density operator ρ, the observables determinate on that (sub)system are those that commute with all of the projectors onto an eigenbasis of ρ. The viability of such a semantics is not our concern; rather, the issue before us is the justification for speaking of an environmentally-induced superselection rule.

The decoherence program acknowledges that $\sum_k \alpha_k \varphi_k \otimes \vartheta_k \otimes \varepsilon_k(t)$ is a coherent superposition and, thus, that there is no hard superselection rule for pointer position. In the best case scenario where the environmental states $\varepsilon_k(t)$ become exactly orthogonal at some finite t, the "monitoring" of the object-apparatus system by the environment makes it *as if* there were a superselection rule for pointer position in the sense that no measurement on the object-apparatus subsystem alone will reveal interference effects between the ϑ_k. While "environmentally-induced superselection rule" is not an abuse of language, such a rule is a far cry from original sense of superselection rule: the presence or absence of the former is relative to the state of the (sub)system while the latter depends not on the state but only on the kinematic structure of the observables. Moreover, the modal semantics (or some suitable substitute) does as much of the work in explaining classicality as does the environmentally-induced superselection rule.

An alternative account that uses what might be termed situational superselection rules instead of soft environmentally-induced superselection rules has been proposed by Landsman (1991, 1995). In this scheme, unlike the decoherence approach, the modal semantics does no explanatory work. Indeed, Landsman rejects the modal semantics in favor of the rule that all and only those observables whose operators are self-adjoint and are super-

[49] FAPP is John Bell's acronym for "for all practical purposes."

selected in the sense that they belong to the center of the von Neumann algebra of observables have definite values.[50] How does this superselection semantics help to explain the classical world we perceive? If it were safe to assume that the center $\mathcal{Z}(\mathfrak{M})$ for the von Neumann algebra of observables $\mathfrak{M}$ for the object-apparatus-environment contained the macroscopic observables that we perceive as value-definite, the explanation would have a clear path to success. But the above discussion shows that this assumption is both dubious in itself and difficult to mesh with a plausible dynamics. Landsman's insight is to note that an embodied observer O is localized and, therefore, unable to measure correlations with systems beyond some characteristic distance. This means that the effective non Neumann algebra of observables $\mathfrak{M}_O$ for such an observer is a proper subalgebra of $\mathfrak{M}$, and it is plausible, Landsman argues, that $\mathcal{Z}(\mathfrak{M}_O)$ contains observables such as pointer position.

This explanation needs to be examined in more detail. There are two options for applying the superselection semantics. One option is to give pride of place to a particular algebra—presumably the full von Neumann algebra of observables $\mathfrak{M}$ for the object-apparatus-environment—in deciding value-definiteness. Then, supposing that $\mathcal{Z}(\mathfrak{M})$ does not contain the observable for pointer position, the value-definiteness that a localized observer perceives for this observable is a kind of illusion. I find this result not uncongenial, but I would want the illusion to be well-founded and not a delusion. It is hard to see how this can be the case on the chosen option. That the center of localized Jane's effective algebra $\mathfrak{M}_J$ of observables contains the observable for pointer position explains why she cannot measure anything that reveals that the pointer does not have a definite position. But it does not explain why she has the positive (mis)impression that the pointer does have a definite position much less why she has the positive impression that it is pointing (say) up when (by hypothesis) it in fact *never* has a definite position. Explicitly including the Jane in the object-apparatus-environment system only deepens the mystery. Supposing that the center $\mathcal{Z}(\mathfrak{M})$ of the full von Neumann algebra of observables does not contain the observable for Jane's-brain-registers-pointer-up (down), the first option for applying the superselection semantics implies that this observable never has a definite value. So to explain Jane's perception that the pointer does have a definite position, (say) up, requires either that her perception does not supervene on her brain

[50] Or somewhat more liberally, a self-adjoint A can be deemed to have a definite value if A is affiliated with the center of the algebra of observables.

states or else that the supervenience does not work in any straightforward way. The former seems to involve a return of Cartesian dualism while the latter seems to threaten to make Jane's perception a delusion.

The second option for applying the superselection semantics is to refuse to give pride of place to any particular non Neumann algebra of observables and instead relativize value-definiteness to an algebra. Thus, relative to one algebra pointer position may enjoy value-definiteness but lack it relative to another algebra. This relativism is no cause for alarm if the goal is to explain why observers such as us perceive a classical world and if three conditions can be established: (i) there is a definite procedure for associating with any observer O such as us a von Neumann algebra of observables $\mathfrak{M}_O$, (ii) for all O, $\mathcal{Z}(\mathfrak{M}_O)$ contains all of the macroscopic observables that are part of the classical world picture, and (iii) for all O, $\mathfrak{M}'_O$ is abelian, guaranteeing that a mixed state on $\mathfrak{M}_O$ has a unique decomposition into disjoint pure states. One worry about (i) is that more natural than an association of an algebra with an observer is an association of an algebra with a spacetime region that contains the measurement process used by the observer. That the observer is a finite embodied creature who interacts with the system for a finite time can be cashed in by the requirement that the associated spacetime region is an open bounded set. But now (iii) comes under pressure for local relativistic QFT since, as mentioned above, for such regions the associated algebra is typically Type III, not a Type I non-factorial algebra as required by (iii).

Although there are problems to be resolved, Landsman's approach is sufficiently attractive as an alternative to the decoherence/modal approach that it deserves more attention than it has received in the philosophical literature (FAPP, none). There is no quick summary of this section, except to say that if "superselection rule" is understood in its original sense, then Wigner's downbeat assessment of the ability of superselection rules to alleviate the conceptual problems of quantum theory seems justified. But environmentally-based superselection rules—which are not superselection rules—or Landsman's situational superselection rules—which are superselection rules for a suitable subalgebra of the von Neumann algebra of observables—may well provide an important ingredient in an effective alleviation.

12 Deconstructing Superselection Rules: Reference Frames and All That

Contrary to the impression that the reader may have gotten from the preceding sections, questions about the existence of superselection rules in general and of particular superselection rules are not settled matters in physics. The reason that the debate is ongoing (and is never likely to be definitively settled) is that it implicates general methodological issues about the nature of theorizing in physics as well as fundamental issues in the foundations of physics. My positions on these issues place me on the pro-superselection rules side of the debate. But equally respectable positions militate in favor of the anti-superselection rules side. While not trying to hide my prejudices, I will endeavor to present both sides. An even-handed treatment is hard to achieve since the argumentation often takes the form of a burden-of-proof dispute whose outcome is in the eye of the beholder.

The early critics of superselection rules claimed, for example, that the proof of the superselection rule for integer and half-integer angular momentum is valid only if rotation by 2π is rotation of the entire system, including any measuring device that is used to probe the object system. But then (the criticism goes) the proof is physically irrelevant since "any meaningful distinction between 2π rotations and other rotations must refer to the relative rotation between one system and another" (Aharonov and Susskind 1967b, p. 1237). And once the relative nature of the rotation is made clear then 2π rotations do have observable consequences (see Werner at al. 1975 and Klein and Opat 1976).

This kind of criticism has been revived recently by two groups, one pursuing the decoherence program and the other focusing on quantum information. The former group finds congenial the conclusion of the early critics that there are no absolute superselection rules, but it also accepts the challenge of showing how effective superselection rules can emerge through a coupling of the system to an environment or a measuring device (see Giulini et al. 1995). The idea that all superselection rules are environmentally induced is an interesting one, but I will have nothing further to say about it here, and I will concentrate on the Aharonov-Susskind criticism of superselection rules and the more nuanced version put forward by the quantum information group.

The Aharonov-Susskind criticism produces in philosophers of science a sense of *deja vu* because it reawakens debates about whether meaningful statements about physical systems must be couched in terms of relational quantities. Without rehearsing this debate (which goes back at least as far as the 17th century and the Newton-Leibniz dispute over absolute vs. relational theories of space and time), one point of relevance here stands out; namely, verifiability/falsifiability are not good criteria of physical meaningfulness[51], so even if it granted that all experimentally verifiable/falsifiable assertions must be formulated in terms of relational quantities (e.g. rotation of a system relative to a measuring instrument), it does not follow that a theory of the phenomena at issue must employ only relational quantities if it is to be physically meaningful. To take the most obvious example, conservation of linear momentum in Newtonian physics derives from spatial translation invariance, where the translation is a translation of the entire system—not a relative translation of the system with respect to a measuring instrument. Naturally, the testing of conservation of linear momentum must use a measuring instrument, and what is measured is a relative quantity that relates the object system to the measuring instrument; but this does not imply that the conservation law for linear momentum does not have an exact validity or that such validity as it possesses must emerge from the interaction of the system with an environment or a measuring device.

If it were correct, the Aharonov-Susskind critique would not only overturn superselection rules but it also threatens to undermine the standard analysis of familiar symmetries in QM. To take a case in point, the unitary representation of the rotation group $SO(3)$ is a projective representation where each element $g \in SO(3)$ is represented by an equivalence class $\tilde{U}(g)$ of unitary operators that differ by a phase factor. For a spinless particle a "reduction of phase" can be performed by which a member $U(g)$ can be selected from each equivalence class so as to get a vector representation of $SU(2)$, the universal covering group of $SO(3)$. $SU(2)$ is a double covering, leaving a choice of two values, $+1$ or -1, for $U(2\pi)$; the conventional choice is $+1$, but this convention has no physical consequences. For a particle with spin, the reduction of phase has to be performed differently for the subspace of vectors corresponding to integer spin and the subspace corresponding to half-integer spin (see Hegerfeldt and Kraus 1968 and Hegerfeldt, Kraus, and Wigner 1968). The analysis shows that -1 must be selected for the latter, resulting in $U(2\pi) = E_+ - E_-$ where E_+ and E_- are respectively the projectors onto the subspaces of integer spin and half-integer spin states.[52] Thus,

[51] Or at least this is the general consensus in the philosophy of science community. But in the physics literature one occassionally sees falsifiability used as the touchstone of the physically meaningful.

[52] From the discussion in Section 8 it follows that it is more accurate to say that the univalence superselection rule results from invariance under the Euclidean group, the product of spatial translations and spatial translations $SO(3)$; for if $SO(3)$ were the relevant symmetry group, the superselection rules would be more fine grained. The univalence

the standard analysis of symmetries in QM leads inevitably to a non-trivial supersymmetry for particles with spin and, thus, a weak superselection rule. On the grounds that the difference between integer and half-integer spin is experimentally detectable, the projectors E_+ and E_- should count as observables, producing (at least) a strong superselection rule.

Since Aharonov and Susskind reject the univalence superselection rule they must also reject the analysis of symmetry outlined above, which presumably they would do on the same grounds as before, viz. that although the analysis of spatial rotation symmetry is formally correct it has no physical significance since rotation through an angle θ is physically meaningful only if θ is the angle of relative rotation of two systems. That one of the arguments for the superselection rule at issue can be deconstructed by deconstructing the standard analysis of symmetry and invariance in quantum theories is surely a fact, but the implications of this fact remain inscrutable until the critics of superselection have spelled out their alternative account of symmetry and invariance. A possible rejoinder would claim that in QM the rotation group is given by $SU(2)$ rather than $SO(3)$. This objection can perhaps be generalized to undermine any superselection rule based on a symmetry argument, as will be discussed below.

What then is the relevance of the experimental demonstration of the detectability of 2π rotations by Werner et al. (1975) and Klein and Opat (1976)? In a word, none. These experiments do *not* measure the relative phase of integer and half-integer states. What they show is that splitting a beam of neutrons and subjecting the two parts of the beam to a relative phase shift of an odd multiple of 2π results in a detectable Fresnel interference pattern when the beams are recombined. These are a beautiful experiments, but they concern the *dynamical* development of a system whose parts undergo *relative* rotations. The superselection rule for 2π rotations concerns the *kinematical* behavior of observables under *non-relative global* rotations. Weingard and Smith (1982) argue that the experimental results can be understood by positing that the two parts of the neutron beam keep track of their relative rotations by keeping track of their rotations with respect to their local spaces. If this reading is correct it would undercut the Aharonov-Susskind assumption that any meaningful notion of rotation must

superselection rule survives under relativization in the sense that it remains a valid superselection rule when the relevant symmetry group is the Poincaré group; see Divakaran (1994, Sec. 5.2).

be understood in terms of the relative rotation of two physically embodied systems; but needless to say, the Weingard-Smith reading is not the only possible one.

The recent revival of the Aharonov-Susskind critique of superselection by Bartlett et al. (2006, 2007) picks up on the notion of a "reference frame" that is implicit in the work of Aharonov and Susskind (1967a, b) and that is made more explicit in Mirman's critique of superselection (Mirman 1969, 1970, 1979). Their main claim is that superselection rules apply only when a reference frame is left unspecified because, they argue, by using an appropriate reference frame any superselection rule can be made to disappear (Bartlett et al. 2007, p. 575).[53] There is an uninteresting sense in which this claim is undoubtedly true. By Theorem 1 every superselection rule corresponds to a supersymmetry/gauge symmetry. So the claim at issue amounts to the assertion that the gauge freedom that expresses the a superselection rule can be killed by the choice of an appropriate reference frame. This is correct if a choice of a "reference frame" is identified with gauge fixing. Fix a "gauge frame" by fixing the phase angles ϕ_j in the supersymmetry/gauge transformation $U = \sum_j e^{i\phi_j} E_j$ that expresses the superselection rule and, presto, projectors onto rays crossing (former!) superselection sectors become "observables". But the "observables" produced by gauge fixing are no more genuine observables than those produced in classical electromagnetism by imposing on the electromagnetic potentials (say) the Lorentz gauge condition or those produced in general relativity theory by imposing on coordinate systems (say) the harmonic coordinate condition. If the essence of a superselection rule is a limitation on observables, then that essence has not been compromised.

Presumably, the response of the critics of superselection rules would be that what they have in mind is not gauge fixing in this trivial sense of choosing a mathematical convention but rather gauge fixing in some more substantive sense that involves the use of an actual or hypothetical physical system as a reference frame. Again there is a sense in which it is undoubtedly true that the use of such reference systems can kill any particular superselection rule. The point can be most clearly illustrated using the algebraic account of the origin of superselection rules. Start with a C^*-algebra $\mathcal{A}$ that, by the lights of

[53] I have no quarrel with the analysis of Bartlett et al. (2006, 2007) of the debate about the presence or absence of coherence in various experiments in quantum optics. But I am skeptical that this analysis extends to showing that all superselection can be undermined by introducing reference frames.

the algebraic account, grounds a superselection rule because $\mathcal{A}$ admits irreducible, unitarily inequivalent representations corresponding to the selection sectors of said superselection rule. By adding enough additional observables, the algebra can always be changed to an algebra $\widetilde{\mathcal{A}}$ that does not admit irreducible, unitarily inequivalent representations corresponding to the selection sectors of said rule. So if the introduction of reference frames means, or at least implicates, the unlimited introduction of additional observables, then any superselection rule can always be undermined by the use of appropriate frames. Of course, to guarantee that this trick will always work, the notion of a reference frame must be given an elastic meaning that can be stretched far beyond its standard meaning of an abstract or materially embodied congruence of timelike curves (the worldlines of the points of the frame) with respect to which physical processes can be spatiotemporally located. But this is merely a terminological issue. The substantive issue is which of the algebras—$\mathcal{A}$ vs. $\widetilde{\mathcal{A}}$—lends itself to the best account of the phenomena in the domain under examination, and settling this issue is not a straightforward empirical matter since presumably "best" is to be judged not only empirical adequacy but theoretical fruitfulness, simplicity, etc. The advocates of a superselection rule have to assume the initial burden of proof. Arguably, they have discharged this burden in some cases, an example being the Stocchi and Wightman (1974) proof of a charge superselection rule for QED. In such cases the burden of proof shifts to the naysayers on superselection.

The above discussion proceeded on the basis of the assumption that a "reference frame" can always be fully described within a quantum theory of the domain under investigation. (Bartlett et al. 2006, 2007 accept this assumption. It is unclear whether or not Aharonov and Susskind would follow suit.) This assumption can be challenged by the Copenhagean notion that the application of quantum theory requires the use of essentially classical reference systems which cannot be treated internally in quantum theory. I reject this notion, and I believe that everything that can be meaningfully said about the world can be said within the quantum theory. Admittedly this stance generates the still unsolved problem of explaining how the classical world perceived by us is a reflection of the quantum world, a problem for which superselection rules may or may not hold the key. But it is better to squarely face problems than to treat them with soothing nostrums, such as found in Bohr's philosophy of quantum mechanics.[54]

[54]For a recent reappraisal of the Copenhagen interpretation of QM, see Landsman (2007,

Sec. 3).

The Aharonov-Susskind criticism and its more recent descendants seek to undermine superselection rules by inflating the algebra of observables. A different way to undermine superselection rules proceeds by deflation rather than by inflation. According to the algebraic approach, the superselection algebra of observables $\oplus_j \pi_j(\mathcal{A})''$ acting on $\oplus_j \mathcal{H}_{\pi_j}$ results from taking the direct sum of the von Neumann algebras determined by (physically admissible) irreducible unitarily inequivalent representations $\pi_j(\mathcal{A})$ of the relevant C^*-algebra $\mathcal{A}$. Now we may be unaware of what the actual values of the superselected quantities are, but known or not these values pick out one of the von Neumann algebras $\pi_{@}(\mathcal{A})'' = \mathfrak{B}(\mathcal{H}_{\pi_{@}})$. And assuming an ignorance interpretation of a mixed state on $\oplus_j \pi_j(\mathcal{A})''$, the actual state of the system is a pure state corresponding to a ray in $\mathcal{H}_{\pi_{@}}$. And rejecting the idea (examined in the preceding section) that the Hamiltonian is not an observable, the temporal evolution of any vector in this ray keeps the vector in $\mathcal{H}_{\pi_{@}}$. Thus, insofar as describing what happens in the the actual world is the goal, all of the other superselection sectors are just extra baggage that does no real work.

This conclusion is very narrow minded. An ordinary selection rule where the selection sectors are defined, say, by different values of energy is not rendered moot by being told that in fact the actual world occupies a particular energy sector. Both ordinary selection rules and superselection rules concern the *possibility structure* of observables and states. Hard bitten positivists may respond that if the status of ordinary selection rules and of superselection rules turns on mere possibilities rather than actualities, it is of no importance to physics. On the contrary, theoretical physics is precisely the business of articulating the possibility structures. Physics differs from metaphysics by confining itself to articulating the kinematical and dynamical possibility structures for our world, and among the most important questions about these structures for quantum physics is whether superselection rules apply.

The ways of deconstructing superselection rules seem endless. A means to deconstruction that is perhaps more serious than the ones discussed above is based on the observation that a superselection rule that derives from symmetry group can be made to disappear when the group is enlarged (Giulini 2003). The discussion of the representations of the Galilean group and its central extension in Giulini (1996, 2003) is particularly effective in casting doubt on the claim that Galilean invariance provides the basis for a superselection rule for mass in ordinary QM. It is far from clear, however, how effectively the point generalizes to all superselection rules based on a symmetry. And in any case, as has been urged above, there is no convincing basis for thinking that all superselection must derive from group theoretic/symmetry considerations. If the algebraic approach is on the right track, the ubiquity of cases in non-relativistic QM and relativistic QFT where unitarily inequivalent representations are implicated gives currency to the notion that while some superselection rules may be vulnerable to deconstruction, superselection is a robust feature of both non-relativistic QM and relativistic QFT.

13 Conclusion

Three different senses of superselection rules—weak, strong, and very strong—were characterized using the nomenclature of von Neumann algebras. The relationships among these three senses can be elucidated using surprisingly elementary results (the only non-elementary result used here is Theorem 3) that serve to dispel the exotic air initially surrounding superselection. But the foundations issues that arise from attempting to understand the origins and implications of superselection rules are far from elementary, for these issues go directly to the heart of the meaning of quantization and the structure and interpretation of quantum observables. I approached these issues through the lens of the Haag-Kastler algebraic formulation of quantum theory, according to which unitarily inequivalent representations the relevant C^*-algebra hold the key to understanding the origin of superselection rules. The attractiveness of the algebraic approach is that it offers a uniform and general account of the origin of superselection rules. But as discussed in Section 10, the proponents of this account must show that it can stand up to various challenges. In addition, the group theoretic approach to quantization and superselection sectors needs to be given its due. As far as I am aware, there does not exist in the literature a detailed comparison between these two approaches to superselection; such a comparison should be possible since on both approaches superselection rules are bottomed on unitarily inequivalent representations—in the one case inequivalent representations of an algebra, in the other case inequivalent representations of a group. One obvious worry about the group theoretic approach is that it appears to be stymied when there is no non-trivial symmetry group in the offing.

I would also emphasize the need for more discussion of three matters. The first concerns the merits of the attack on superselection rules originated by Aharonov and Susskind and recently revived in various guises. I took a skeptical view of these attacks, but it must be admitted that the foundations issues raised by the attacks are far from settled. I also took a skeptical stance on the attempt to base hard macro-superselection rules on the idealization of infinite systems, but again issues about the role of idealizations in physics are not easily put to rest. Much more promising in this regard are Landsman's situational superselection rules, but the merits of his superselection semantics vs. modal semantics need further adjudication. The third concerns the desirability of lifting the limitation imposed above to discrete superselection rules. The obvious suggestion of treating continuous superselection rules in terms of the integral decompositions $\mathcal{H} = \int_{\oplus} \mathcal{H}(\xi) d\mu(\xi)$ and $\mathfrak{M} = \int_{\oplus} \mathfrak{M}(\xi) d\mu(\xi)$ runs into problems. For instance, if $\mathcal{H}$ is not separable then the $\mathcal{H}(\xi)$ are not subspaces of $\mathcal{H}$, and it is then not evident how to reformulate some of the core conditions for superselection rules. Moving to nonseparable Hilbert spaces is an option, but adopting it means that some of the fundamental results used in characterizing superselection rules—such as Theorem 3—are no longer available.[55]

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[55] For comments about continuous superselection rules, see Piron (1969).

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