Chapter 19
Substantive General Covariance: Another Decade of Dispute

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19.1 Orthodoxy and a Recent Challenge

Whether Einstein's theory of general relativity (GR) satisfies a substantive principle deserving the name "general covariance" is a notoriously controversial matter. John Norton's masterful review of the matter, published in 1993, was aptly subtitled "eight decades of dispute" (Norton 1993). And yet, despite the continuing controversy, there has been broad agreement about a number of core issues. Two closely related theses are part of the orthodox position: (i) that general covariance does not distinguish general relativity from pre-relativistic theories when the latter are appropriately formulated and (ii) that general covariance, by itself, does not have any physical content.

The first of these theses is almost as old as GR itself. Einstein had sought a gravitational theory that was compatible with special relativity (SR). Soon after 1905 he came to believe that what was required was a generalization of SR's restricted relativity principle. According to SR, all inertial frames are on a par from the point of view of the fundamental laws. What Einstein sought was a theory according to which *all* frames are on a par. General covariance was supposed to implement this. A theory is general covariant if *the equations that express its laws are left form-invariant by smooth but otherwise arbitrary coordinate transformations*.[1] Since these coordinate transformations include transformations between coordinate systems adapted to frames in arbitrary relative motion, it would seem that there can be no privileged frames of reference in a general covariant theory.

This impression, however, is misleading. As Kretschmann famously pointed out, "by means of a purely mathematical reformulation of the equations representing the theory, and with, at most, mathematical complications connected with that reformulation" any physical theory can by made generally covariant, and

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[1] As discussed below, this is but one of a number of closely related properties that go by the name "general covariance".

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this "without modifying any of its content that can be tested by observation" (Kretschmann 1917, 575–6).[2] Generally covariant formulations of pre-relativistic theories (i.e., Newtonian and specially relativistic theories) are now utterly familiar in philosophical and foundational discussion. Even if it can be argued that, so formulated, the physical content of such a theory is somehow different from that of the "standard", non-covariant formulation (a claim I reject), it seems that general covariance cannot be what distinguishes GR from *generally covariant versions* of pre-relativistic theories.

The idea that general covariance per se has no physical content is reinforced when one considers the nature of the controversy dissected in Norton's review. In the conclusion to his paper, Norton claims that there are essentially three views on the question whether a 'principle of general covariance' plays a foundational role in GR (852–3). The third of these views straightforwardly rejects the idea that general covariance has any foundational role at all. The first view seeks to *supplement* general covariance with some other requirement. For example, GR might be distinguished from a rival theory T either because T's *simplest formulation* is not its generally covariant formulation, or because, when the generally covariant formulation of T is compared to (generally covariant) GR, it is seen that GR is the simpler, more elegant theory. Such an approach to identifying the 'principles' that distinguish GR faces a host of problems. But what is important for the current discussion is that, according to the approach, a theory's being generally covariant has nothing to do with its special status. Instead the generally covariant formulations of two theories to be compared merely make manifest the truly distinguishing characteristic, viz., some kind of simplicity.

The second point of view Norton mentions is associated with the so-called Anderson–Friedman programme (Anderson 1967, 73–88; Friedman 1983, 46–61). Here one distinguishes between two types of geometric object that can feature in the formulation of a spacetime theory. There are the truly dynamical objects on the one hand and, on the other, the *absolute objects*: very roughly, objects that do not vary from model to model of the theory. The programme also distinguishes between the *covariance group* of the theory (the group of transformations, defined on the theory's space of kinematically possible models, that leaves the space of dynamically possible models invariant) and the theory's *invariance group*. The latter is that subgroup of the covariance group that includes all and only automorphisms of the theory's absolute objects. One can then differentiate GR from pre-relativistic theories by noting that only GR satisfies a principle of general *invariance*: the invariance group of the theory should include the group of all smooth, but otherwise arbitrary coordinate transformations. For a specially relativistic theory, for example, the invariance group will be the Poincaré group, no matter whether the standard formulation of the theory is considered (in which case the covariance group will also be the Poincaré group) or whether a generally covariant formulation is considered.

[2] I gloss over the fact that the claim that "all physical observations consist in the determination of purely topological relations" formed part of Kretschmann's argument. See Norton (1993, 818), from where the translation is taken.

Anderson's definitions validate a sense in which the group of general coordinate transformations (or the diffeomorphism group) is a 'symmetry' group of GR but not of, say, SR (however the latter is formulated). In a generally covariant formulation of a specially relativistic theory, the transformations of a proper subgroup of the diffeomorphism group isomorphic to the Poincaré group have a special status: they leave the theory's absolute objects invariant. In GR, *every* diffeomorphism has this special status and so (it seems) diffeomorphisms in GR differ in status to diffeomorphisms in SR.

On closer inspection, things are not so clear-cut. A group gets to be a symmetry group on Anderson's view if it leaves the theory's absolute objects invariant. Arbitrary diffeomorphisms preserve the absolute objects of GR, and are thus symmetries, only because *GR has no absolute objects* and thus, trivially, *any* transformation preserves GR's absolute objects. It therefore seems that what really differentiates GR from pre-relativistic theories is its *lack of absolute objects*.[3] In particular, nothing in Anderson's approach suggests we should treat two diffeomorphically related models of GR differently from how we might treat two diffeomorphically related models of a generally covariant SR theory.

Thus, on any of the three views that Norton highlights, GR's general covariance has little to do with what distinguishes GR from pre-relativistic theories. Pre-relativistic theories can be given generally covariant formulations and the substantive principles just reviewed have little to do with general covariance per se. They might highlight various special features of GR. They might even highlight differences between the status of diffeomorphisms in GR and in SR. But they do not licence the claim that GR's general covariance is somehow more substantive than that of SR. All this, I claim, is orthodoxy. It has recently been challenged.

Amongst philosophers of physics, the challenge has been spearheaded by Earman (2006a; 2006b). He claims to be following physicists in distinguishing two kinds of general covariance: *merely formal* general covariance and *substantive* general covariance. Generally covariant formulations of pre-relativistic theories are supposed to satisfy only the former of the two. Note that Earman's substantive general covariance is quite distinct from Anderson's 'principle of general invariance' or any other of the notions just reviewed. It will clarify matters to introduce Earman's definition as just one of a number of versions of general covariance. It will also be helpful to introduce a number of 'toy' theories, whose satisfaction of the various versions of general covariance can then be assessed.

[3] I should note that whether GR does indeed lack absolute objects in the Anderson–Friedman sense is currently a live topic. In fact, it seems that $\sqrt{-g}$ counts as an absolute object (Pitts 2006; Giulini 2007; Sus 2008, Chapter 3).

19.2 Varieties of General Covariance

Our toy theories are all theories of the Klein–Gordon field. Their sole matter field will be a single, real scalar field. They differ, inter alia, in the geometric structure they posit.

The first theory is a specially relativistic theory written in standard form. It is defined by the equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} - \frac{\partial^2 \Phi}{\partial t^2} - m^2 \Phi = 0 \tag{SR1}$$

This equation is only satisfied by descriptions of Φ given with respect to inertial coordinate systems. I.e., if $\Phi(x)$ is a coordinate representation of our scalar field that satisfies equation SR1, then (in general) of those coordinate redescriptions obtained from $\Phi(x)$ via coordinate transformations, only those obtained via Poincaré transformations will also satisfy SR1.

Contrast this theory with generally relativistic Klein–Gordon theory, defined via the following equations:

$$\begin{gathered} g^{\mu\nu} \Phi_{;\nu\mu} - m^2 \Phi = 0 \\ G_{\mu\nu}(g) = \kappa T_{\mu\nu}(\Phi, g). \end{gathered} \tag{GR1}$$

Here the equations are intended to be read as identifying the values of the coordinate components of the objects involved. If all the components of the pair (g_{ab}, Φ) with respect to some coordinate chart $\{x\}$ satisfy these equations, then their components with respect to any chart smoothly related to $\{x\}$ will also do so in the region where the charts overlap. The theory might equally be specified via a set of equations relating the geometric objects themselves, rather than their coordinate components:

$$\begin{gathered} g^{ab} \nabla_a \nabla_b \Phi - m^2 \Phi = 0 \\ G_{ab}(g) = \kappa T_{ab}(\Phi, g) \end{gathered} \tag{GR2}$$

Now for the first two formulations of general covariance. A theory T is generally covariant iff:

GC1 the equations of motion/field equations of T transform in a generally covariant manner under an arbitrary coordinate transformation, or

GC2 the equations of motion/field equations of T relate "intrinsic, coordinate-free"[4] objects; they are true independently of coordinate systems.

GR1 and GR2, our two formulations of generally relativistic Klein–Gordon theory, satisfy GC1 and GC2 respectively. Our specially relativistic theory satisfies neither. But this is easily corrected via a Kretschmann-type move. We simply rewrite

[4] The terminology is Earman's (2006a, 446).

equation SR1 so that it holds good in arbitrary coordinates, making the role of the fixed metric of Minkowski spacetime explicit:

$$\eta^{\mu\nu}\Phi_{;\nu\mu} - m^2\Phi = 0. \tag{SR2}$$

Alternatively, rather than equating coordinate components, we can write down an equation referring directly to the geometric object fields themselves:

$$\eta^{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0. \tag{SR3}$$

It is clear that, appropriately formulated, our specially relativistic theory now satisfies GC1 and GC2.

So far we have focused on the transformation properties of a theory's equations. Let's consider models of the theories. Models of GR2 are triples of the form (M, g, Φ), where M is some 4-dimensional differentiable manifold, g is a Lorentzian metric on M and Φ is a scalar field on M. g and Φ must satisfy the equations GR2. Models of SR3 are likewise triples of the form (M, η, Φ), where M is some 4-dimensional differentiable manifold, η is now a flat, Minkowski metric on M and Φ is a scalar field on M. η and Φ must satisfy the equation SR3.

Our third formulation of general covariance is stated in terms of models. A theory T, with models of the form $(M, O_1, O_2, \ldots, O_N)$ is generally covariant iff

GC3 If $(M, O_1, O_2, \ldots, O_N)$ is a model of T, then so is $(M, d^*O_1, d^*O_2, \ldots, d^*O_N)$ for any diffeomorphism $d \in \text{Diff}(M)$.[5]

It is uncontroversial that generally relativistic theories, and hence our theory GR2, satisfy GC3.[6] What of our reformulations of SR1?

The orthodox (philosopher's) answer is that the theory specified via SR3 satisfies GC3 just as much as any generally relativistic theory. For suppose that (M, η, Φ) satisfies SR3. It follows from the fact that this is a tensor equation that $(M, d^*\eta, d^*\Phi)$ also satisfies SR3. I.e., if $\eta^{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0$ then $d^*\eta^{ab}\nabla'_a\nabla'_b d^*\Phi - m^2 d^*\Phi = 0$, where ∇' is the covariant derivative associated with $d^*\eta$. In their agenda-setting paper on the hole argument, Earman and Norton embraced this equivalence with respect to GC3 of appropriately formulated pre-relativistic theories and generally relativistic theories, arguing that the substantivalist was compelled to classify *all* "local spacetime theories" as indeterministic (Earman and Norton 1987, 524).

[5] This matches the definition given by Earman (1989, 47). Diff(M) is the group of M's automorphisms; i.e., the group of all invertible maps from M onto itself that preserve its differentiable structure. GC3 is the requirement that Diff(M) be a subgroup of T's *covariance group* in Anderson's sense.

[6] Uncontroversial, that is, amongst those who classify GR as a generally covariant theory. Maudlin's metrical essentialist (Maudlin 1988, 1990) denies that both $(M, O_1, O_2, \ldots, O_N)$ and $(M, d^*O_1, d^*O_2, \ldots, d^*O_N)$ represent genuine possibilities. But even the metrical essentialist can admit that $(M, O_1, O_2, \ldots, O_N)$ and $(M, d^*O_1, d^*O_2, \ldots, d^*O_N)$ are on a par as models of T. They should claim only that, *relative to the choice of* $(M, O_1, O_2, \ldots, O_N)$ as the representation of a genuine possibility, $(M, d^*O_1, d^*O_2, \ldots, d^*O_N)$ does not represent a possibility (compare Bartels 1996).

19.3 In Search of Substantive General Covariance

Let us return to our general relativistic theory, GR2. A key premise in Earman and Norton's argument is their claim that the substantivalist must interpret (M, g, Φ) and $(M, d^*g, d^*\Phi)$ as representations of *distinct* possibilities. Most commentators (relationalists and substantivalists alike) take the moral of the hole argument to be that (M, g, Φ) and $(M, d^*g, d^*\Phi)$ should be interpreted as representing the same physical state of affairs. This gives us our fourth version of general covariance, Earman's "substantive general covariance" (2006a, 447; 2006b, 4–5).

GC4 A theory T is generally covariant iff:

1. If $(M, O_1, \ldots, O_N)$ is a model of T, then so is $(M, d^*O_1, \ldots, d^*O_N)$ for any $d \in \text{Diff}(M)$.
2. $(M, O_1, \ldots, O_N)$ and $(M, d^*O_1, \ldots, d^*O_N)$ represent the same physical possibility.

In other words GC4 supplements GC3 with the requirement that $\text{Diff}(M)$ is a *gauge group* in the non-technical sense: diffeomorphisms relate distinct representations of one and the same situation.

Does the specially relativistic theory expressed by SR3 satisfy GC4? Not according to Earman. GC4 counts as "substantive" because:

> it is *not* automatically satisfied by a theory that is formally generally covariant, i.e., a theory whose equations of motion/field equations are written in generally covariant coordinate notation or, even better, in coordinate-free notation (Earman 2006a, 444).

Thus, for Earman, the substantive principle embodied in GC4 differentiates GR from pre-relativistic theories, even when these are formulated using generally covariant notation, along the lines of SR3. He is committed to denying that $\text{Diff}(M)$ is a gauge group with respect to the theory expressed by SR3. What justification does he offer?

19.4 When (Not) to See Gauge Freedom

According to Earman, the physics literature contains a "generally accepted apparatus that applies to a very broad range of spacetime theories and that serves to identify the gauge freedom of any theory in the class." This apparatus decrees that GR does satisfy GC4 whereas "formally generally covariant forms of special relativistic theories… need not satisfy substantive general covariance" (Earman 2006a, 445).

The "broad range" of spacetime theories Earman refers to are those whose field equations are derivable from an action principle. Suppose T's r equations of motion are derivable from an action $S = \int d^p x L(\mathbf{x}, \mathbf{u}, \mathbf{u}^{(n)})$. $\mathbf{x} = (x^1, \ldots, x^p)$ are the independent variables (the spacetime coordinates x, y, z, t in the cases we're considering) and $\mathbf{u} = (u^1, \ldots, u^r)$ are the dependent variables (e.g., $g^{\mu\nu}$ and Φ). The term $\mathbf{u}^{(n)}$ indicates that L can depend on the derivatives of $\mathbf{u}$ (with respect to the independent variables) up to some finite order n. A group $\mathscr{G}$ of transformations $g : (\mathbf{x}, \mathbf{u}) \mapsto (\mathbf{x}', \mathbf{u}')$ whose generators leave L form-invariant up to a divergence term is a *variational symmetry group* of S (Earman 2006a, 449–50).

Associated with this notion of a symmetry of S are the (generalized) Noether theorems. The one relevant to Earman's proposal is Noether's second theorem: if the parameters of $\mathscr{G}$ are s arbitrary functions of the independent variables, then there are s independent equations relating the r *Euler expressions*, the r variational derivatives $\delta L/\delta u^i$ of the Lagrangian with respect to each dependent variable. Imposing Hamilton's principle with respect to that variable (i.e., requiring that S is stationary with respect to arbitrary infinitesimal variations of that dependent variable that vanish on the boundary of integration) gives the Euler–Lagrange equation $\delta L/\delta u^i = 0$. Thus Noether's second theorem shows that the equations of motion are not independent and we have fewer independent equations of motion than field variables. When time is amongst the independent variables, this underdetermination manifests itself as apparent indeterminism. The physicist's standard move is to restore determinism by identifying solutions related by the variational symmetries. Thus Earman writes that "the applicability of Noether's second theorem is taken to signal the presence of gauge freedom" (Earman 2006b, 7) and proposes that, according to the physicists' apparatus, "variational symmetries containing arbitrary functions of the independent variables connect equivalent descriptions of the same physical situation, i.e., are gauge transformations." (Earman 2006a, 450).

Applied to our generally relativistic Klein–Gordon theory this gives us the expected result. What seems to me more suspect is Earman's application of the machinery to our specially relativistic Klein–Gordon theory. He notes that our formally generally covariant equation SR3 is derivable from the action:

$$S(\Phi, \eta) = \int \frac{1}{2}(\eta^{ab}\nabla_a \Phi \nabla_b \Phi + m^2 \Phi^2)\sqrt{-\eta}d^4x \qquad (19.1)$$

where Φ *but not* η is subject to Hamilton's principle. Earman concludes that while "the action admits the Poincaré group as a variational symmetries ... the apparatus sketched above renders the verdict that there is no non-trivial gauge freedom in the offing" (Earman 2006a, 452). In other words, Earman suggests that applying his apparatus to this theory yields the verdict that diffeomorphisms are *not* gauge transformations.

There are at least three reasons to be sceptical of the method used to reach this conclusion.

1. The criterion Earman claims to find in the physics literature tells us that *if* some group $\mathscr{G}$ is a variational symmetry to which Noether's 2nd theorem applies, then $\mathscr{G}$ is a gauge group. I.e., it tells us when to see gauge freedom. But to draw the conclusion he does concerning SR3, Earman needs the converse criterion: $\mathscr{G}$ is a gauge group *only if* $\mathscr{G}$ is a variational symmetry. I.e., he needs a criterion that tells us when *not* to see gauge freedom. Earman freely admits that the apparatus is silent on non-Lagrangian theories (e.g., Earman 2006a, 454) which, we shall see, is significant.

2. Physicists' identification of (local) variational symmetries as gauge symmetries is not simply read off from the Lagrangian formalism. As Earman himself carefully explains, the identification is motivated by a desire to avoid indeterminism. But then there are equally good grounds for regarding a theory for which Diff(M) is a symmetry group in the sense of GC3, but which is not derivable from an action principle for which Diff(M) is a variational symmetry group, as also satisfying GC4. I.e., there are equally good grounds for regarding Diff(M) as a gauge group with respect to such a theory too. The reason there is not a good precedent in the physics literature for such a move is indicative of the fact that such theories are almost never discussed (in this literature); it does not indicate that in such theories diffeomorphisms should not be regarded as gauge.
3. Finally, what of Earman's claim that the Poincaré group is a variational symmetry of 19.1? Although he does not explicitly say that Diff(M) fails to be a variational symmetry group, this would appear to be implicit in his discussion. But why should one think this? It is true that one only applies Hamilton's principle to Φ, in order to derive the equation SR3. One does not also consider variations in η. But this is irrelevant to which transformations count as variational symmetries in sense of Noether's second theorem. That theorem applies in full force to 19.1, independently of which of the dependent variables one regards as background structure and which one regards as dynamical. One still obtains mathematical identities relating the Euler expressions. The only difference with the general relativistic case, where all dependent variables are subject to Hamilton's principle, is that the vanishing of the Euler expression corresponding to η (effectively the stress energy tensor of the Klein–Gordon field) is not one of the field equations.[7]

It is true that, if we consider drag-alongs only of Φ under the action of diffeomorphisms, while leaving η unaltered, then only a subgroup of diffeomorphisms isomorphic to the Poincaré group will be symmetries. Why should we consider such transformations? In the next section we will see that there is a reason, and that it connects to whether we regard our specially relativistic theory as derived from an action principle.

19.5 An Alternative Distinction Between Theories

The distinctions between formulations of a theory that we have so far considered have focused on the equations that express the theory, even though our characterization of general covariance has taken a model-theoretic turn. Continuing to think in terms of models is key to making some further, crucial distinctions.

[7] For an illuminating discussion of various connections between Noether's theorems and general covariance, see Brown and Brading (2002).

Let us suppose that the models of our theories have the following basic structure.[8] They are functions from a given space, V, into a given space of field values, W. The theories will involve a space, $\mathscr{K}$, of *kinematically possible models* (i.e., the space of all suitably well-behaved but otherwise arbitrary such functions), and a proper subspace $\mathscr{S} \subset \mathscr{K}$ of *dynamically possible models*, normally picked out via a set of equations.

In these terms, a "theory" corresponding to equations GR2 will involve a differentiable manifold M as the space V. A kinematically possible model will assign a (pseudo)metric tensor and real number to each point of M in a suitably smooth way, and consistently with any necessary boundary conditions. The subspace $\mathscr{S}$ of dynamically possible models is picked out by the equations GR2. Call this theory T_{GR}.

When it comes to the specially relativistic theory, however, we have a choice as to how to proceed. In the first version of such a theory, V is taken to be M *equipped with* a particular Minkowski metric. Each model of the theory then simply maps each point of this space into the real numbers. $\mathscr{K}'$ is the space of all suitably well-behaved such functions. The subspace of physically possibly models, $\mathscr{S}'$, is picked out by a suitable equation, constraining how Φ is adapted to the fixed metric structure of V. Call this theory T_{SR1}.

In the second version of the specially relativistic theory, V is taken, as in T_{GR}, simply to be the differentiable manifold M. We may suppose that the space of kinematically possible models is also the same as that of T_{GR}: each point of M is to be mapped to a metric tensor and real number in a manner consistent with boundary conditions and smoothness requirements. The theory will differ from T_{GR} in terms of its subspace, $\mathscr{S}''$, of dynamically possible models. This will be picked out (obviously) by a different set of equations to those that pick out $\mathscr{S}$. In addition to the Klein–Gordon equation, there will be an equation requiring the vanishing of the Riemann curvature tensor: $R_{abcd} = 0$. Call this theory T_{SR2}.

Which of the equations SR1, SR2 or SR3 is suitable to T_{SR1} and T_{SR2}? It is clear that any one of these equations can be understood as picking out the space $\mathscr{S}'$ of T_{SR1}. Provided coordinate charts on V that are adapted to its metric structure are chosen, SR1 will be satisfied by all and only those models in $\mathscr{S}'$. If we allow arbitrary coordinates, and interpret $\eta_{\mu\nu}$ as the coordinate components of the metric structure of V, then SR2 will be satisfied by all and only those models in $\mathscr{S}'$. If we interpret η_{ab} as referring directly to the fixed metric structure of V, then SR3 is satisfied by all and only those models in $\mathscr{S}'$.

This is not quite true for T_{SR2}. SR2 or SR3 are the natural equations to combine with the vanishing of the Riemann tensor. Although, for every model in $\mathscr{S}''$, there is a coordinate chart such that SR1 holds, for two arbitrary models in $\mathscr{S}''$ different coordinatizations will be needed. Nonetheless, I think it is clear that the different covariance properties of SR1, SR2 or SR3 do not track in any perspicuous way the difference between T_{SR1} and T_{SR2}. To repeat, all three equations are equally legitimate ways of specifying T_{SR1}. The difference between the theories (or formulations of the theory) *can* be made out in terms of equations (T_{SR2}, but not

[8] The following is, very loosely, based on the much more sophisticated material in Belot (2007, §4).

T_{SR1}, involves an equation constraining the geometry), but this does not seem like the most perspicuous way to do so.

19.6 In Search of Substantive General Covariance Again

It is time to assess the general covariance of our new formulations of special and generally relativistic Klein–Gordon theory. Before doing so, I introduce yet another notion of (substantive?) general covariance, advocated by Carlo Rovelli. According to Rovelli:

> A field theory is formulated in manner invariant under passive diffs (or change of co-ordinates), if we can change the co-ordinates of the manifold, re-express all the geometric quantities (dynamical *and non-dynamical*) in the new coordinates, and the form of the equations of motion does not change. A theory is invariant under active diffs, when a smooth displacement of the dynamical fields (*the dynamical fields alone*) over the manifold, sends solutions of the equations of motion into solutions of the equations of motion. (Rovelli 2001, 122, original emphasis)

Rovelli's terminology of "active" versus "passive" *diffeomorphisms* (as opposed to coordinate transformations) is somewhat novel. Let me make a few, hopefully clarifying, remarks. One should not think of diffeomorphisms (as is sometimes unfortunately suggested) as "moving points around". The map $d : M \to M$ simply *associates* each point of M in its domain with another. This map induces maps on fields defined on M, e.g., $d^* : g \mapsto d^*g$. One can think, perhaps, of *these* maps as 'moving g around' (although even this is a bit picturesque; really we use the map to define a new field in terms of an old one). One set of fields on M are the coordinate charts. This suggests the following way of distinguishing 'active' from 'passive' diffeomorphisms:

1. When d is thought of as inducing a change of coordinate chart, but the *physical fields* are left unchanged, d is a 'passive diffeomorphism'.
2. When d is thought of as inducing changes to all the physical fields, d is an 'active diffeomorphism'.[9]

With the notions of active and passive diffeomorphisms so defined, a theory T satisfies GC1 and GC2 if it is invariant under passive diffeomorphisms. It satisfies GC3 if it is invariant under active diffeomorphisms. But the requirement Rovelli in fact labels "active diffeomorphism invariance" in the quotation above is stronger than this. It relies crucially on a distinction between a theory's *dynamical* and *non-dynamical* fields. Let models of T be of the form (M, A_i, D_i), where the A_i are the non-dynamical fields and the D_i are the dynamical fields. A theory T is then generally covariant according Rovelli's version of substantive general covariance iff:

[9] I believe this fits with a more recent characterisation that Rovelli has given (2004, 62–5).

GC5 If $(M, A_1, \ldots, D_1, \ldots)$ is a model of T, then so is $(M, A_1, \ldots, d^*D_1, \ldots)$ for any $d \in \mathrm{Diff}(M)$.[10]

How do our theories measure up against this (and the previous) notions of general covariance? Consider first T_{SR1}, whose space of kinematically possible models $\mathscr{K}'$ was constituted by maps from a manifold equipped with metric structure to the reals. We have already seen that its defining equation can be so formulated that it satisfies both GC1 and GC2. Suppose (M, g, Φ) is a dynamically possible model, i.e., $(M, g, \Phi) \in S'$. In general, $(M, d^*g, d^*\Phi)$ will not be *kinematically* possible, let alone dynamically possible. Hence this theory does not satisfy GC3 (and thus also fails to satisfy GC4). If we consider just dragging-along the sole dynamical field, Φ, we obtain a model $(M, g, d^*\Phi)$ that is in $\mathscr{K}'$. (So restricted, $\mathrm{Diff}(M)$ does have a well defined action on $\mathscr{K}'$.) However, $(M, g, d^*\Phi) \notin S'$, hence T_{SR1} also fails to satisfy GC5.

Consider next T_{GR} and suppose that $(M, g, \Phi) \in S$. $(M, d^*g, d^*\Phi) \in \mathscr{S}$ and hence T_{GR} satisfies GC3. (Note that GC3 is just the requirement that the action of $\mathrm{Diff}(M)$ on the space of kinematically possible models fixes the solution subspace.) The hole argument, therefore, suggests that it should also be classified as satisfying GC4. Finally, what of Rovelli's GC5? Since there are no non-dynamical fields, the requirement is again that $(M, d^*g, d^*\Phi) \in \mathscr{S}$ and GC5 is satisfied.

Finally, we consider T_{SR2} and suppose that (M, g, Φ) is an arbitrary model in S''. Since $(M, d^*g, d^*\Phi) \in S''$ for arbitrary d it follows that T_{SR2}, unlike T_{SR1}, satisfies GC3, and (*pace* Earman) GC4 (recall the hole argument). Does it satisfy GC5?

That depends on whether g counts as a dynamical field. The passage quoted from Rovelli above continues:

> Distinguishing a truly dynamical field, namely a field with independent degrees of freedom, from a nondynamical field disguised as dynamical (such as a metric field g with the equations of motion $\mathrm{Riemann}[g] = 0$) might require a detailed analysis (of, for instance, the Hamiltonian) of the theory (Rovelli 2001, 122).

It is certainly the case that the Anderson–Friedman notion of an absolute object can be invoked to classify g as a non-dynamical field. T_{SR2} then fails to be generally covariant in the sense of GC5 (cf. Giulini 2007). But how much illumination does this piece of classification achieve? In some intuitive sense, the metric structure of a specially relativistic theory plays the role of a fixed background against which the real dynamics is defined and unfolds. There is no such background in GR. Not only is the metric a genuine dynamical player; its dynamical evolution is affected by the material content of spacetime.[11] The action–reaction principle is satisfied. I doubt that the right way to make these ideas more precise is to discover a criterion that, e.g., T_{GR} meets but T_{SR2} fails to meet.

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[10] Compare Earman's definition of a *dynamical symmetry* (Earman 1989, 45).

[11] The former need not entail the latter. For example, consider the, admittedly somewhat contrived, theory whose field equations are $g^{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0$ and $R_{ab}(g) = 0$. The metric in this theory has a non-trivial dynamics and constrains, but is unaffected by, the evolution of Φ.

19.7 Conclusion

Recall Earman's claim that, if we restrict attention to the class of Lagrangian field theories, generally relativistic theories, but not specially relativistic theories, satisfy GC4. In light of the previous section we can partially endorse this claim, for of the two specially relativistic theories, only T_{SR1} is a Lagrangian theory. *All* of its equations can be derived from the action described in Section 19.4. T_{SR2}, on the other hand, does not appear to be a Lagrangian theory.[12] The obvious ways to derive $R_{abcd} = 0$ as an Euler–Lagrange equation (in addition to the Klein–Gordon equation) requires an additional field and thus alters the space of kinematically possible models (Sorkin 2002; Earman 2006a, 455–6). However, it should be stressed that the reason diffeomorphisms do not count as *gauge* symmetries of the relevant formulation of the specially relativistic theory is because *they are not symmetries*; the theory does not satisfy GC3. And when we do consider a formulation of the specially relativistic theory for which diffeomorphisms *are* symmetries, viz. T_{SR2}, Earman's Lagrangian apparatus is simply silent. The points of comparison between T_{SR2} and T_{GR} strongly suggest that – re ontology, the nature of what is observable and the gauge status of diffeomorphisms – what goes for one should go for the other.

There are (at least) two ways of conceiving of pre-relativistic theories: as theories that fail both GC3 and GC5 (such as T_{SR1}) and as theories satisfying both GC3 and GC4 (such as T_{SR2}). Generally relativistic theories appear distinguished in that only the second kind of conception is available. If the second kind of conception of pre-relativistic theories is adopted, and they are then compared to GR, it seems doubtful that the interesting differences between GR and such theories is to be made out in terms of a variety of general covariance, or a difference in the status of the diffeomorphism group.

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[12] David Wallace has pointed out to me that, in the vacuum case, there is no difficulty in principle in obtaining the flatness of the metric via the extremization of an action whose only dependent variables are the components of the metric. What needs to be investigated is whether this observation can be extended to theories involving matter fields. (In vacuum GR the extremization of the gravitational action imposes Ricci flatness but spacetime is certainly not Ricci flat when the theory includes a non-trivial matter Lagrangian.)

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