Toward an Understanding of Parochial Observables

Benjamin Feintzeig

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Abstract

Ruetsche (2011) claims that an abstract C*-algebra of observables will not contain all of the physically significant observables for a quantum system with infinitely many degrees of freedom. This would signal that in addition to the abstract algebra, one must use Hilbert space representations for some purposes. I argue to the contrary that there is a way to recover all of the physically significant observables by purely algebraic methods.

1 Introduction

It is by now well known that in quantum theories with infinitely many degrees of freedom like quantum field theory and quantum statistical mechanics, the presence of unitarily inequivalent representations stymies the extension of our interpretive practices from the ordinary quantum mechanics of finite systems. Ruetsche (2011) lays out the interpretive options in the wake of this problem of unitarily inequivalent representations. One can be a *Hilbert Space Conservative* and maintain that possible worlds correspond to density operators on a particular privileged Hilbert space containing a concrete irreducible representation of the algebra of observables. Or one can be an *Algebraic Imperialist* and hold that possible worlds are represented by the states on the abstract C*-algebra of observables, which captures the structure all representations have in common. Finally, one can be a *Universalist*, aligning with the Conservative in viewing states as density operators on a Hilbert space, but picking the (reducible) universal representation of the abstract

C*-algebra as the collection of physically measurable quantities.[1] Ruetsche argues that none of the interpretations just mentioned is adequate.

This paper will focus on Ruetsche's argument against Imperialism and Universalism,[2] which I will call *the problem of parochial observables*. Ruetsche argues that there are certain observables, such as particle number, temperature, and net magnetization, that the Hilbert Space Conservative acquires and employs in physically significant explanations, and which the Imperialist and Universalist do not have access to. Ruetsche calls these the *parochial observables*. Since the Imperialist and Universalist do not have access to these observables, they cannot recover physically significant explanations of (for example) particle content, phase transitions, and symmetry breaking. Since an interpretation is adequate only insofar as it can recover physically significant explanations, Imperialism and Universalism are unacceptable interpretations of quantum theories with infinitely many degrees of freedom, according to Ruetsche.

I will argue that the problem of parochial observables is only an apparent problem, which disappears once one recognizes that the Imperialist and Universalist have additional resources for representing parochial observables. Parochial observables arise for Ruetsche as approximations to or idealizations from observables in a particular Hilbert space representation (once we have chosen a physically relevant notion of idealization). I will show that the Imperialist and the Universalist can use the same sorts of tools in their respective settings to acquire the parochial observables. Namely, the Imperialist can define a physically relevant notion of approximation or idealization on the abstract algebra and the Universalist can similarly define a physically relevant notion of approximation or idealization on the bounded operators on the universal Hilbert space. According to both of these notions, parochial observables arise as idealizations from the observables we began with in the abstract algebra or the universal representation, respectively. When the Imperialist and Universalist are allowed the same mathematical tools that the Hilbert Space Conservative uses for representing approximations and idealizations, the problem of parochial observables does not arise. The Imperialist and Universalist each have access to all of the observables they need.

[1] Of course, these are not the *only* possible interpretations of algebraic quantum theories (See Ruetsche 2011, Ch. 6), but we will restrict our attention to these positions for the purpose of this paper.

[2] In what follows, Hilbert Space Conservatism will mainly play the role of a foil to help us understand Imperialism and Universalism. We will mention Ruetsche's argument against Conservatism only briefly.

Furthermore, when we allow the Imperialist and the Universalist access to these idealized observables, one can show a precise sense in which they are *equivalent interpretations*. There is a sense in which the Imperialist and the Universalist subscribe to the same physical possibilities; they simply use different mathematical tools to describe these possibilities. Thus, Imperialism and Universalism amount to the same interpretation, and it is one which avoids Ruetsche's problem of parochial observables.

2 Preliminaries

In algebraic quantum theories[3] the measurable quantities for a system are represented by the self-adjoint elements of an abstract C*-algebra $\mathfrak{A}$. A *state* on $\mathfrak{A}$ is given by a positive, normalized, linear functional $\omega : \mathfrak{A} \to \mathbb{C}$. A state ω has the following initial interpretation: for each self-adjoint $A \in \mathfrak{A}$, $\omega(A)$ corresponds to the expectation value of A in the state ω.

Importantly, this algebraic formalism translates back into the familiar Hilbert space theory once we are given a state. A *representation* of a C*-algebra $\mathfrak{A}$ is a pair $(\pi, \mathcal{H})$, where $\pi : \mathfrak{A} \to \mathcal{B}(\mathcal{H})$ is a *-homomorphism into the bounded linear operators on some Hilbert space $\mathcal{H}$. One of the most fundamental results in the theory of C*-algebras, known as the GNS theorem (Kadison & Ringrose 1997, p. 278, Thm. 4.5.2), asserts that for each state ω on $\mathfrak{A}$, there exists a representation $(\pi_\omega, \mathcal{H}_\omega)$ of $\mathfrak{A}$, known as the *GNS representation for* ω, and a (cyclic) vector $\Omega_\omega \in \mathcal{H}_\omega$ such that for all $A \in \mathfrak{A}$,

$$\omega(A) = \langle \Omega_\omega, \pi_\omega(A)\Omega_\omega \rangle$$

One may find representations of $\mathfrak{A}$ on different Hilbert spaces, and in this case one wants to know when these can be understood as "the same representation". This notion of "sameness" is given by the concept of unitary equivalence:[4] two representations $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ are *unitarily*

[3]For more on the theory of C*-algebras and their representations, see Kadison & Ringrose (1997). For more on algebraic quantum field theory, see Ruetsche (2011) and Halvorson (2006).

[4]See Ruetsche (2011, Ch. 2.2) and Clifton & Halvorson (2001, Sec. 2.2-2.3) for more on unitary equivalence as a notion of "sameness of representations." There is another notion of "sameness of representations" given by *quasi-equivalence* (Kadison & Ringrose 1997, p. 735). Quasi-equivalence and unitary equivalence coincide for irreducible representations, but quasi-equivalence may be a more natural notion (for some purposes) when dealing with reducible representations.

equivalent if there is a unitary mapping $U : \mathcal{H}_1 \to \mathcal{H}_2$ which intertwines the representations, i.e. for each $A \in \mathfrak{A}$,

$$U\pi_1(A) = \pi_2(A)U$$

The specified unitary mapping U sets up a way of translating between density operator states on $\mathcal{H}_1$ and density operator states on $\mathcal{H}_2$, and between observables in $\mathcal{B}(\mathcal{H}_1)$ and observables in $\mathcal{B}(\mathcal{H}_2)$. The GNS representation for a state ω is unique in the sense that any other representation $(\pi, \mathcal{H})$ of $\mathfrak{A}$ containing a cyclic vector corresponding to ω is unitarily equivalent to $(\pi_\omega, \mathcal{H}_\omega)$ (Kadison & Ringrose 1997, p. 279, Prop. 4.5.3).

Finally, I would like to note a distinction between kinds of representations that will play an important role in distinguishing between Hilbert Space Conservatism and Universalism. A representation $(\pi, \mathcal{H})$ of $\mathfrak{A}$ is *irreducible* if the only subspaces of $\mathcal{H}$ invariant under $\pi(\mathfrak{A})$ are $\mathcal{H}$ and $\{0\}$. Otherwise, a representation is called *reducible*. A reducible representation contains subrepresentations, i.e. representations on restricted nonzero subspaces of $\mathcal{H}$ invariant under the action of $\pi(\mathfrak{A})$.

3 Three Extremist Interpretations

Quantizing a classical theory involves two steps: first, one isolates from the observables of the classical theory certain algebraic relations (usually commutation or anti-commutation relations) to build the abstract algebra $\mathfrak{A}$. And second, one finds a representation of the resulting algebra in the bounded operators on some Hilbert space. In the case where the original classical theory has finitely many degrees of freedom,[5] the Stone-von Neumann theorem (Ruetsche 2011, p. 41; Clifton & Halvorson 2001, p. 427) shows that all of the Hilbert space representations we end up with are unitarily equivalent and so the resulting quantum theory is unique. However, it is well known that theories with infinitely many degrees of freedom, including field theories and statistical theories in the thermodynamic limit, violate the assumptions of the Stone-von Neumann

[5] The Stone-von Neumann theorem carries other substantive assumptions as well. It assumes that the representation is continuous in an appropriate sense and that the configuration space of the classical theory is the manifold $\mathbb{R}^n$ (See Ruetsche 2011, Ch. 2-3).

theorem. Even though the first step of the quantization procedure yields a unique algebra for the quantum theory, this algebra admits many unitarily inequivalent representations when we attempt to perform the second step (Ruetsche 2011, Ch. 3.3). Because of this, it appears that many theories of interest for physics do not have a unique quantization. Three possible interpretations of quantum theories with infinitely many degrees of freedom arise from consideration of unitarily inequivalent representations.[6]

3.1 Algebraic Imperialism

First, one can be an *Algebraic Imperialist* by asserting that a quantum theory is given in full by the abstract algebra of observables and the states on that algebra rather than its Hilbert space representations. The abstract algebra captures a structure that all Hilbert space representations have in common, so the Algebraic Imperialist chooses to focus only on this structure. To do so is to proclaim that all the work that has been done on interpreting the Hilbert space formalism for ordinary quantum mechanics with finitely many degrees of freedom cannot yield a complete and adequate interpretation for the case of infinitely many degrees of freedom. According to the Algebraic Imperialist, "the extra structure one obtains along with a concrete representation of [$\mathfrak{A}$] is extraneous." (Ruetsche 2011, 132). All that matters is the abstract algebraic structure. The physically measurable quantities are given by the observables (self-adjoint elements) in $\mathfrak{A}$, and the physically possible states are given by the states on $\mathfrak{A}$.

3.2 Hilbert Space Conservatism

On the other hand, if one wants to be a *Hilbert Space Conservative* and maintain an interpretation via the Hilbert space formalism like those usually discussed for ordinary quantum mechanics, then one must pick a particular Hilbert space representation to interpret. When one is working in the context of a particular Hilbert space $\mathcal{H}$, one can define the weak operator topology on $\mathcal{B}(\mathcal{H})$ by the following criterion for convergence (Reed & Simon 1980, p. 183): a net $\{A_i\}_{i\in\mathcal{X}}$ converges

[6] For more on these positions and their advantages and disadvantages, see Arageorgis (1995) and Ruetsche (2002, 2003, 2006, 2011 Ch. 6). Of course, as Ruetsche describes, there are many more subtle interpretive options, but we deal here only with three of the simplest cases.

weakly to A (written $A_i \to A$) just in case for all $\phi, \psi \in \mathcal{H}$,

$$\langle \phi, A_i \psi \rangle \to \langle \phi, A\psi \rangle$$

in $\mathbb{C}$. Recall that the GNS theorem allows us to take any C*-algebra of observables $\mathfrak{A}$ and, having chosen some state ω, represent it via the representation π_ω as a subalgebra $\pi_\omega(\mathfrak{A}) \subseteq \mathcal{B}(\mathcal{H}_\omega)$ for some Hilbert space $\mathcal{H}_\omega$. Using the weak operator topology on $\mathcal{B}(\mathcal{H}_\omega)$ as a physically relevant notion of approximation,[7] one can include in any algebra of observables the operators that are physically indistinguishable from or well approximated by the observables already picked out in our algebra. To do so, we take the weak operator closure (written $\overline{\pi_\omega(\mathfrak{A})}$) of our original algebra of observables $\pi_\omega(\mathfrak{A})$—this adds to our original algebra all limit points of weak operator converging nets. Any weak operator closed subalgebra of a Hilbert space is called a von Neumann algebra, so $\overline{\pi_\omega(\mathfrak{A})}$ will be called the von Neumann algebra affiliated with the representation π_ω. If the state ω that we used to take our GNS representation is pure,[8] then, because the representation π_ω is irreducible (Kadison & Ringrose 1997, p. 728, Thm. 10.2.3), it follows that $\overline{\pi_\omega(\mathfrak{A})} = \mathcal{B}(\mathcal{H}_\omega)$ (see Sakai 1971, Prop. 1.21.9, p. 52). So taking the GNS representation for a pure state and closing in the weak operator topology brings us back to the familiar situation where our observables are *all* of the bounded self-adjoint operators on a Hilbert space.

For the Hilbert Space Conservative, one such particular irreducible Hilbert space representation of the abstract algebra specifies the physical possibilities. Having chosen a pure state ω on $\mathfrak{A}$ and obtained its GNS representation $(\pi_\omega, \mathcal{H}_\omega)$, the Hilbert Space Conservative follows the standard practice in ordinary quantum mechanics (for finitely many degrees of freedom). For the Hilbert Space Conservative, the physically measurable quantities are the self-adjoint elements of $\overline{\pi_\omega(\mathfrak{A})} = \mathcal{B}(\mathcal{H}_\omega)$, and the physically possible states are density operators on $\mathcal{H}_\omega$.

Ruetsche argues that Hilbert Space Conservatism is inadequate because it does not give us access

[7] The motivation for this standard practice is that in the weak operator topology, a net of observables well approximates (i.e. converges to) another observable just in case it approximates it with respect to all possible expectation values and transition probabilities, and hence with respect to the empirical predictions of the theory. We will discuss the significance of this in the next section.

[8] A state ω is *pure* if whenever $\omega = a_1\omega_1 + a_2\omega_2$ for states ω_1, ω_2, it follows that $\omega_1 = \omega_2 = \omega$.

to enough states. In theories with infinitely many degrees of freedom, the existence of unitarily inequivalent representations entails that for any privileged irreducible representation $(\pi, \mathcal{H})$, there is some algebraic state that cannot be implemented as a density operator on $\mathcal{H}$. This would be fine if we only ever needed the density operator states on a single Hilbert space to accomplish the goals of physics, but there are instances in which we need to appeal to two states which cannot be represented as density operators on the same irreducible Hilbert space representation in the course of giving a single physically significant explanation (Ruetsche 2003, 2006). For example, states of different pure thermodynamic phases cannot be represented as density operators on the same irreducible Hilbert space representation. But certainly we need to be able to account for states of different pure thermodynamic phases as simultaneously physically possible in order to explain phase transitions in quantum statistical mechanics. So the Hilbert Space Conservative lacks the resources to recover such physically significant explanations.[9]

This may push the Hilbert Space Conservative to try to find a Hilbert space on which *all* states can be represented as density operators. The Universalist seeks to do precisely this.

3.3 Universalism

The Universalist agrees with the Hilbert Space Conservative that we need a Hilbert space representation of the abstract algebra $\mathfrak{A}$ to interpret our quantum theory but disagrees that we need an irreducible representation. The Universalist holds that the *universal representation* is the privileged representation of the abstract algebra. Letting $\mathcal{S}_{\mathfrak{A}}$ be the set of states on $\mathfrak{A}$ and the GNS representation of any $\omega \in \mathcal{S}_{\mathfrak{A}}$ be denoted by $(\pi_\omega, \mathcal{H}_\omega)$, the universal representation is given by $(\pi_U, \mathcal{H}_U)$, where

$$\mathcal{H}_U = \bigoplus_{\omega \in \mathcal{S}_{\mathfrak{A}}} \mathcal{H}_\omega$$

is the universal Hilbert space and for each $A \in \mathfrak{A}$

$$\pi_U(A) = \bigoplus_{\omega \in \mathcal{S}_{\mathfrak{A}}} \pi_\omega(A)$$

[9]This argument is elaborated in Ruetsche (2002, 2003, 2006, 2011) and discussed in Feintzeig (2015).

The universal representation is guaranteed to be faithful (Kadison & Ringrose 1997, p. 281, Remark 4.5.8) so every element $A \in \mathfrak{A}$ has a unique counterpart in $\pi_U(\mathfrak{A})$. The Universalist, like the Hilbert Space Conservative, has access to more observables than the Algebraic Imperialist. The universal Hilbert space carries its own weak operator topology defined precisely as above, which allows us to take the weak operator closure $\overline{\pi_U(\mathfrak{A})}$ of our original algebra, thereby obtaining *the universal enveloping von Neumann algebra of* $\mathfrak{A}$. Since π_U is reducible, it follows that

$$\pi_U(\mathfrak{A}) \subseteq \overline{\pi_U(\mathfrak{A})} \subsetneq \mathcal{B}(\mathcal{H}_U)$$

For the Universalist, the physically measurable quantities are the self-adjoint elements of $\overline{\pi_U(\mathfrak{A})}$, and the physically possible states are density operators on $\mathcal{H}_U$.

Furthermore, every state on the abstract algebra $\mathfrak{A}$ can be represented by a density operator (in fact, a vector and hence a *finite rank density operator*; see Kadison & Ringrose 1997, p. 281, Remark 4.5.8) on the universal Hilbert space $\mathcal{H}_U$ so the Universalist has access to as many states as there are on the abstract algebra $\mathfrak{A}$ and avoids one of the pitfalls of the Hilbert Space Conservative.

It is worth noting that although Universalism uses a Hilbert space representation to think about states and observables, it differs crucially from the ways we are accustomed to thinking about these mathematical tools in ordinary quantum mechanics, and thus it may not give the Hilbert Space Conservative everything she desired. First, in ordinary quantum mechanics we are accustomed to working in an irreducible representation of the algebra of observables. We have seen that the universal representation is manifestly reducible. Second, in ordinary quantum mechanics we are accustomed to thinking of vector states as pure states (because we work in an irreducible representation). Vector states on the universal representation, or universal enveloping von Neumann algebra, are not necessarily pure states. In fact, every mixed state on $\mathfrak{A}$ can be thought of as a vector state as well. Third, in ordinary quantum mechanics we are accustomed to working in a separable Hilbert space. Because the universal Hilbert space is the direct sum of an uncountable number of nontrivial Hilbert spaces, it follows that the universal Hilbert space is nonseparable. These are certainly important differences from the mathematical tools that we use in ordinary quantum mechanics, and I save further discussion of their significance for future work.

4 Parochial Observables

Ruetsche's *problem of parochial observables* begins as an argument against Algebraic Imperialism and at times is extended to an argument against Universalism. The basic claim of the argument is that the Algebraic Imperialist does not have the resources to represent all of the physically possible observables (Ruetsche 2002, p. 367; Ruetsche 2003, p. 1330). We saw that the Hilbert Space Conservative, having privileged some pure state ω and its GNS representation $(\pi_\omega, \mathcal{H}_\omega)$, acquires all of the observables in $\overline{\pi_\omega(\mathfrak{A})} = \mathcal{B}(\mathcal{H}_\omega)$. The new observables are the ones that Ruetsche calls *parochial observables*; these are limit points in the weak operator topology of nets of observables from the original algebra and so they can be thought of as approximations to or idealizations from the observables we already recognized.[10] Many of these observables have real physical import (e.g., the temperature observable) but have no analogue in the abstract algebra. So the Hilbert Space Conservative gains access to more observables than the Algebraic Imperialist. These observables are physically significant, e.g. for giving explanations of thermodynamic phase transitions. According to Ruetsche, the Algebraic Imperialist runs into a problem because she cannot recognize these operators as physically possible observables, and so cannot vindicate such explanations.

I will argue that there is a way for both the Imperialist and the Universalist to account for parochial observables. The problem of parochial observables only appears because we have given the Conservative more tools than the Imperialist—specifically, tools for representing idealizations and approximations. Once we give the Imperialist the analogous tools on the abstract algebra, she has no trouble representing the parochial observables.

[10]By saying this, I do not mean to take a stance on whether the limit points are bona fide observables or "merely" idealizations, and I also do not mean to take a stance on whether the idealizations here are indispensable (See, eg., Callender 2001; Batterman 2005, 2009). I hope only to invoke the notion that a topology captures a notion of similarity or resemblance and that limit points can be thought of as relevantly resembling elements of the original algebra to arbitrarily high accuracy so that we are licensed to use them for some scientific purposes. For more on limiting relations capturing a notion of similarity, resemblance, and approximation, see Butterfield (2011a, 2011b) and Fletcher (2014).

4.1 Parochial Observables for the Imperialist

When one thinks of an abstract C*-algebra, one usually thinks of it as coming equipped with the topology induced by its norm. This of course corresponds to the uniform topology of a concrete C*-algebra of operators acting on a Hilbert space. But just as one can consider alternative topologies on concrete algebras of operators, one can consider alternative topologies on the abstract algebra *prior to taking a representation.* One of the alternative topologies on the abstract algebra corresponds in a certain sense to the algebraic translation of the weak operator topology. To motivate this, we must first think about the significance of the weak operator topology. I stated in the previous section that the weak operator topology gives us a criterion of convergence based on expectation values and transition probabilities, and so gives us a notion of approximation relevant to the empirical content of the theory. But this can be made more precise. The following proposition[11] shows that the weak operator topology of a representation is the topology for convergence of expectation values *with respect to a privileged collection of states*—namely the finite rank density operators on a Hilbert space representation.

Proposition 1. *Let* $(\pi, \mathcal{H})$ *be a representation of a C*-algebra* $\mathfrak{A}$*. Let* $\{A_i\} \subseteq \pi(\mathfrak{A})$ *be a net of operators. Then the following are equivalent:*

(1) The net $\{A_i\}$ *converges to* $A \in \mathcal{B}(\mathcal{H})$ *in the weak operator topology on* $\mathcal{B}(\mathcal{H})$*.*

(2) For all states ϕ *on* $\mathfrak{A}$ *implementable by a finite rank density operator* Φ *on* $\mathcal{H}$*,* $\phi(A_i) = Tr(\Phi A_i)$ *converges to* $Tr(\Phi A)$ *in* $\mathbb{C}$*.*

One remark before we proceed: in clause (2) the expectation values converge as complex numbers even though there is in general no element of the abstract algebra that the net of observables converges to. This is because the parochial observable (the limit point of the net) in general will not have an analog in the abstract algebra.

Prop. 1 shows that the weak operator topology on a representation of $\mathfrak{A}$ gives us a notion of approximation relevant to only those states that can be implemented as finite rank density operators in this representation. A Hilbert Space Conservative might have reason to restrict attention to

[11] See the appendix for proofs of propositions 1, 2, 3, 4, and 6. The proofs of propositions 5 and 7 are included in the references and are not reproduced here.

these states, but an Algebraic Imperialist does not. The Algebraic Imperialist sees all states on the abstract algebra as on a par and believes that the empirical content of the theory comes from *all* expectation values.

However, the Algebraic Imperialist has access to a topology that defines a notion of convergence using the expectation values of all of the states she deems physically possible. The *weak ($\sigma(\mathfrak{A}, \mathfrak{A}^*)$) topology*[12] *on an abstract C*-algebra* $\mathfrak{A}$ is defined as follows. Let $\mathfrak{A}^*$ refer to the dual space of continuous linear functionals on the Banach space $\mathfrak{A}$. A net $\{A_i\} \in \mathfrak{A}$ converges in the weak ($\sigma(\mathfrak{A}, \mathfrak{A}^*)$) topology to $A \in \mathfrak{A}$ just in case for all $\phi \in \mathfrak{A}^*$

$$\phi(A_i) \to \phi(A)$$

in $\mathbb{C}$. The notation $\sigma(\mathfrak{A}, \mathfrak{A}^*)$ signifies that the weak topology is the weakest topology on $\mathfrak{A}$ that makes all of the bounded linear functionals in $\mathfrak{A}^*$ continuous. Now we see from Prop. 1 that the notion of convergence given by the weak operator topology on a representation is simply the notion of convergence that we get by using the weak ($\sigma(\mathfrak{A}, \mathfrak{A}^*)$) topology with attention restricted to a particular set of states—those that can be represented by finite rank density operators. Likewise, the weak ($\sigma(\mathfrak{A}, \mathfrak{A}^*)$) topology is the analogue of the weak operator topology for the abstract algebra in the sense that it is an appropriate generalization to a notion of convergence with respect to the expectation values of all states on the abstract algebra.

Insofar as the Hilbert Space Conservative is justified in using the weak operator topology as a physically relevant standard of approximation or idealization, the Algebraic Imperialist is justified in using the weak ($\sigma(\mathfrak{A}, \mathfrak{A}^*)$) topology as a physically relevant standard of approximation or idealization too. The Imperialist is concerned with approximation and idealization with respect to the expectation values of *all states on the abstract algebra*.[13]

[12] The nomenclature here is a bit unfortunate. The weak topology (sometimes called the weak Banach space topology) on an abstract C*-algebra is to be distinguished from the weak operator topology on its particular representations. We will see the difference in what follows.

[13] I do not claim that the weak topology is the *right* one for the Algebraic Imperialist or that there even is a *right* topology to use (See Sec. 5). Just as the Hilbert Space Conservative may have access to multiple topologies on $\mathcal{B}(\mathcal{H})$, the Algebraic Imperialist may have access to multiple topologies on $\mathfrak{A}$. My claim is simply that the weak topology on $\mathfrak{A}$ is analogous to the weak operator topology on $\mathcal{B}(\mathcal{H})$ in the sense that they derive from the same physical motivations for approximation and idealization, and they have analogous conditions for convergence.

There is a sense in which the abstract algebra $\mathfrak{A}$ and its representations are not complete with respect to the weak and weak operator topologies, respectively (even though they are both complete with respect to the norm topology). There are nets of observables whose relevant expectation values converge in $\mathbb{C}$ but which have no limit point in the abstract algebra. To find those limit points, we must think of the algebra of observables as living in some kind of ambient space of observables; for the Hilbert Space Conservative, this is just the collection $\mathcal{B}(\mathcal{H})$ of operators on some Hilbert space that the Imperialist eschews. But the Imperialist also has access to an ambient space of observables of her own, and we will see that this ambient space allows us to find the limit points of nets of observables in the weak ($\sigma(\mathfrak{A}, \mathfrak{A}^*)$) topology just as $\mathcal{B}(\mathcal{H})$ allows the Hilbert Space Conservative to find the limit points of nets of observables in the weak operator topology.

Recall that the dual of any Banach space X, written X^*, is the set of continuous linear functionals on X (in the norm topology), and the *bidual* of X, written X^{**} is the set of continuous linear functionals on X^*. The original space X can be embedded in X^{**} via the canonical evaluation map $J : X \to X^{**}$ given by $x \in X \mapsto \hat{x} \in X^{**}$, where we define $\hat{x}$ by

$$\hat{x}(l) = l(x)$$

for all $l \in X^*$. When X is finite-dimensional, J always provides an isomorphism, but in general when X is infinite-dimensional J may not be an isomorphism because it may not be onto.

The bidual $\mathfrak{A}^{**}$ of the abstract algebra $\mathfrak{A}$ provides the ambient space of observables in which to look for limit points or idealizations of observables from the original algebra. The elements of $\mathfrak{A}^{**}$ can be thought of as observables because for each state ω on $\mathfrak{A}$, we can think of the expectation value of any $A \in \mathfrak{A}^{**}$ as the value $A(\omega)$. Notice that this makes sense because for any element of our original algebra $A \in \mathfrak{A}$, letting $\hat{A} = J(A) \in \mathfrak{A}^{**}$, we see that $\hat{A}(\omega) = \omega(A)$ is the expectation value of A in the state ω. Thinking of observables and expectation values in the bidual like this suggests a way of extending the weak topology of $\mathfrak{A}$ to the bidual $\mathfrak{A}^{**}$: focus on convergence of expectation values with respect to the same collection of states now considered as states on an enlarged algebra of observables. This brings us to the *weak* ($\sigma(\mathfrak{A}^{**}, \mathfrak{A}^*)$) topology on the bidual*

$\mathfrak{A}^{**}$. A net $\{A_i\} \in \mathfrak{A}^{**}$ converges in the weak* ($\sigma(\mathfrak{A}^{**}, \mathfrak{A}^*)$) topology to $A \in \mathfrak{A}^{**}$ just in case for all $\phi \in \mathfrak{A}^*$

$$A_i(\phi) \to A(\phi)$$

in $\mathbb{C}$. The notation $\sigma(\mathfrak{A}^{**}, \mathfrak{A}^*)$ signifies that the weak* topology on the bidual is the weakest topology that makes all of the bounded linear functionals in $\mathfrak{A}^*$ continuous when considered as linear functionals on $\mathfrak{A}^{**}$ (i.e. when the elements of $\mathfrak{A}^*$ are considered as elements of $\mathfrak{A}^{***}$ by the canonical evaluation map). The weak* topology on $\mathfrak{A}^{**}$ is the natural extension of the weak topology on $\mathfrak{A}$ because it makes precisely the same linear functionals, and more specifically states, continuous—namely, the linear functionals and states in $\mathfrak{A}^*$. Furthermore, each state on $\mathfrak{A}$ has a unique continuous extension to a state on $\mathfrak{A}^{**}$ in the weak* topology. The following proposition shows that every element of the bidual $\mathfrak{A}^{**}$ can be understood as a limit point of a net of observables in the abstract algebra $\mathfrak{A}$ in the weak* ($\sigma(\mathfrak{A}^{**}, \mathfrak{A}^*)$) topology on the bidual $\mathfrak{A}^{**}$. This shows that elements of the bidual can be thought of as approximations to or idealizations from our original observables in a topology that the Algebraic Imperialist deems physically relevant.

Proposition 2. *Let $\mathfrak{A}$ be a C*-algebra, $\mathfrak{A}^{**}$ be its bidual, and $J : \mathfrak{A} \to \mathfrak{A}^{**}$ be the canonical evaluation map. Then $J(\mathfrak{A})$ is dense in $\mathfrak{A}^{**}$ in the weak* ($\sigma(\mathfrak{A}^{**}, \mathfrak{A}^*)$) topology.*

Just as the Hilbert Space Conservative, upon adding limit points in the weak operator topology of some irreducible representation, arrives at the algebra of operators $\mathcal{B}(\mathcal{H})$ of bounded operators on the Hilbert space $\mathcal{H}$, the Algebraic Imperialist, upon adding limit points in the weak (or really weak*) topology arrives at the collection of observables $\mathfrak{A}^{**}$. When allowed the same methods for constructing idealizations, the Algebraic Imperialist gains access to more observables than reside in the abstract algebra $\mathfrak{A}$.[14]

But does the Algebraic Imperialist gain access to the observables that Ruetsche argued are physically significant for giving explanations? Recall that the reason the problem of parochial

[14]At this point, one might want to distinguish between two different kinds of Algebraic Imperialists—one who believes the physically significant observables reside in $\mathfrak{A}$ and the norm topology gives the right notion of convergence and another who believes the physically significant observables reside in $\mathfrak{A}^{**}$ and the weak (or weak*) topology gives the right notion of convergence. All I claim is that there is a systematic relationship between these positions and if one is not wedded to there being a *right* topology (see footnote 13), then one need not make a choice between the two collections of observables.

observables was supposed to be a *problem* is that the Algebraic Imperialist appears to not allow us to reconstruct the physics of, say, phase transitions and symmetry breaking. The idealizations the Hilbert Space Conservative constructs are in a certain sense the "right" ones because they allow us to give these physically significant explanations. We need to check that the observables the Algebraic Imperialist constructs by moving to the bidual have enough structure to be able to recover those physically significant explanations, too. First, notice that although $\mathfrak{A}^{**}$ is initially only a Banach space, it can be made into a C*-algebra by defining multiplication and involution operations. We get these operations by Prop. 2 as the unique extensions of the multiplication and involution operations on the algebra $J(\mathfrak{A})$ (with algebraic structure inherited from $\mathfrak{A}$) such that multiplication is separately continuous in its individual arguments in the weak* topology[15] and involution is continuous in its only argument in the weak* topology. When we refer to C*-algebraic structure on $\mathfrak{A}^{**}$ in what follows, these are the operations we will have in mind. The following proposition shows that the Algebraic Imperialist, using this structure on the bidual, has access to every idealized parochial observable that every possible Hilbert Space Conservative (for each possible distinct privileged representation) has access to.

Proposition 3. *If π is a representation of a C*-algebra $\mathfrak{A}$, then there is a central*[16] *projection $P \in \mathfrak{A}^{**}$ and a *-isomorphism α from $\mathfrak{A}^{**}P$ onto $\overline{\pi(\mathfrak{A})}$ such that $\pi(A) = \alpha(J(A)P)$ for all $A \in \mathfrak{A}$.*

So the von Neumann algebra affiliated with any representation of $\mathfrak{A}$ is canonically isomorphic to a subalgebra of $\mathfrak{A}^{**}$. This shows that every parochial observable can be thought of as an element of the bidual $\mathfrak{A}^{**}$ and so can be thought of as an idealization from observables in the abstract algebra that we arrive at by a notion of approximation or idealization that is physically relevant by the Imperialist's lights. The Algebraic Imperialist, when allowed the same tools for constructing idealizations that we allowed the Hilbert Space Conservative, has access to all of the parochial observables, like temperature, particle number, and net magnetization, that we need to give the

[15] In general, multiplication is not jointly continuous in the weak* topology.

[16] The center of a C*-algebra $\mathfrak{A}$ consists in those elements $A \in \mathfrak{A}$ such that for all $B \in \mathfrak{A}$, $AB = BA$.

kinds of physically significant explanations Ruetsche is worried about.[17,18]

4.2 Parochial Observables for the Universalist

When we think about Universalism, we do not need to go through the rigamarole of defining a new kind of topology as we did for Imperialism in the previous section. The universal representation already comes with a topology that is precisely analogous to the weak operator topology used by the Hilbert Space Conservative: namely the weak operator topology on the collection $\mathcal{B}(\mathcal{H}_U)$ of bounded operators on the universal Hilbert space $\mathcal{H}_U$. But how does the weak operator topology of the Universalist compare to the weak operator topology of the Hilbert Space Conservative? While the Hilbert Space Conservative restricts attention to a privileged collection of states from the larger collection of states on the abstract algebra, the Universalist considers all states on the abstract algebra to be physically possible because they can all be implemented as finite rank density operators on the universal Hilbert space. In this sense, the Universalist is just like the Imperialist, so we expect their notions of approximation with respect to the empirical content of the theory to match up. As the following proposition shows, the notion of convergence of the weak operator topology on the universal representation corresponds exactly to the notion of convergence of the weak (or weak*) topology on the abstract algebra.

Proposition 4. *Let $(\pi_U, \mathcal{H}_U)$ be the universal representation of $\mathfrak{A}$. Let $\{A_i\} \subseteq \pi_U(\mathfrak{A})$ be a net of operators. Then the following are equivalent:*

(1) The net $\{A_i\}$ converges to $A \in \mathcal{B}(\mathcal{H}_U)$ in the weak operator topology on $\mathcal{B}(\mathcal{H}_U)$

(2) For all states ϕ on $\mathfrak{A}$, $\phi(A_i) = Tr(\Phi A_i)$ converges to $Tr(\Phi A)$ in $\mathbb{C}$, where Φ is any finite rank density operator (and there is always at least one) implementing the state ϕ on $\pi_U(\mathfrak{A})$.

Just as for Prop. 1, in clause (2) the expectation values converge as complex numbers even though there is no element of the abstract algebra that the net is converging to. The collection

[17] See Kronz and Lupher (2005) and Lupher (2008), who also assert that the bidual contains all of the parochial observables. Here, I have added a precise characterization of how the parochial observables arise from the original algebra via limiting relations.

[18] I do not claim that the observables the Algebraic Imperialist gets in Prop. 3 (or the analogous observables for the Universalist in the next section) are somehow the *real* or *fundamental* temperature, particle number, and net magnetization. All I claim is that these observables suffice to give the kinds of explanations that Ruetsche worries about. (See Sec. 5 for a view on which we do not need to worry about which observables are real or fundamental.)

of bounded operators on the universal Hilbert space provides the ambient space of observables in which to think about these limit points or idealizations. This allows us to construct the universal enveloping von Neumann algebra, which contains the idealizations of our original observables with respect to a notion of idealization that is physically relevant by the lights of the Universalist.

We also note that Prop. 4 shows a sense in which the universal representation is privileged if one wants to think algebraically—its weak operator topology reproduces the precise condition of convergence of the weak (or weak*) topology on the abstract algebra $\mathfrak{A}$. Restricting attention to finite rank density operator states on the universal representation amounts to no restriction at all because *every state is implementable as a finite rank density operator on the universal representation.*

We already knew that the Universalist could acquire more observables by using the universal enveloping von Neumann algebra, but as in the previous section, we still need to ask whether the Universalist acquires the "right" ones. Does the Universalist have access to parochial observables like temperature, particle number, and net magnetization? The following proposition shows that the Universalist has access to every parochial observable that every possible (for each distinct privileged representation) Hilbert Space Conservative has access to (compare with Prop. 3).

Proposition 5. *If π is a representation of a C*-algebra $\mathfrak{A}$ and π_U is the universal representation of $\mathfrak{A}$, then there is a central projection P in $\overline{\pi_U(\mathfrak{A})}$ and a *-isomorphism α from the von Neumann algebra $\overline{\pi_U(\mathfrak{A})P}$ onto $\overline{\pi(\mathfrak{A})}$ such that $\pi(A) = \alpha(\pi_U(A)P)$ for all $A \in \mathfrak{A}$.*[19,20]

The von Neumann algebra affiliated with any representation is canonicaly isomorphic to a subalgebra of the universal enveloping von Neumann algebra. This shows that every parochial observable can be thought of as an element of the universal enveloping von Neumann algebra and so can be thought of as an idealization from observables in the universal representation with respect to a notion of idealization that is physically relevant by the lights of the Universalist. Just as in the previous section, the Universalist gains access to all of the observables including temperature, particle number, and net magnetization, that we need to recover the physically significant explanations that Ruetsche is worried about.

[19] See Kadison & Ringrose (1997, p. 719, Thm. 10.1.12) for a proof.

[20] For those interested in the notion of *quasi-equivalence* (footnote 4), this shows that every representation of $\mathfrak{A}$ is quasi-equivalent to a subrepresentation of the universal representation (See Kadison & Ringrose 1997, p. 735).

Ruetsche, however, presents a number of objections to the claim that the universal representation contains all parochial observables. I will consider what I take to be two of the most prominent objections here and argue that they fail. Seeing why they fail illustrates how the universal representation gives us access to all parochial observables.

First, Ruetsche asserts (2011, p. 145, fn. 10) that in the universal representation we would expect an observable $\pi_\omega(A)$ from the GNS representation for ω to be implemented in the universal representation by a "portmanteau" operator of the form $\pi_\omega(A) \oplus_{\phi\neq\omega} 0$, and similarly for the parochial observables from the representation π_ω which will be weak operator limits of these "portmanteau" operators. But, Ruetsche correctly argues that these operators may not be contained in the universal enveloping von Neumann algebra because it follows from Kadison & Ringrose Thm. 10.3.5 (1997, p. 738) that $\overline{\pi_U(\mathfrak{A})} \neq \bigoplus_{\omega\in\mathcal{S}_\mathfrak{A}} \overline{\pi_\omega(\mathfrak{A})}$.

This objection, however, fades in light of Prop. 5. The "portmanteau" operators are obviously intended to capture the observables from the GNS representation for ω in the universal representation. Prop. 5 explicitly asserts that we can think of any observable $\pi_\omega(A)$ in the GNS representation for ω as the observable $\pi_U(A)P$ in the universal representation (and similarly for parochial observables), where P is a central projection in the universal enveloping von Neumann algebra. Because of the presence of the projection P, such an observable acts nontrivially only on some subspace of the universal Hilbert space $\mathcal{H}_U$ and acts like the zero operator everywhere else. This means that the operator $\pi_U(A)P$ has some properties analogous to the operator $\pi_\omega(A) \oplus_{\phi\neq\omega} 0$. But since operators of the latter form are not required to belong to the universal enveloping von Neumann algebra, it follows that P may not be the projection onto the GNS representation for ω. I know of no simple characterization of the range of P in terms of the direct summands of the universal representation;[21] nevertheless, I will show explicitly that P is not the projection onto the GNS representation for ω and that we should not be surprised by this fact.

Intuitively, the observable $\pi_U(A)P$ ought to act like $\pi_\omega(A)$ on all of the vector states that $\pi_\omega(A)$ acts on. But in general there will be many vectors in the universal Hilbert space $\mathcal{H}_U$ that implement any vector state ψ corresponding to the vector $\Psi \in \mathcal{H}_\omega$, which is the sort of vector

[21] See Kadison & Ringrose Thm. 6.8.8 (1997, p. 443) and Thm. 10.1.12 (1997, p. 719) for the general construction of P.

that $\pi_\omega(A)$ acts on. For example, the vector $\Psi \oplus_{\phi \neq \omega} 0$ will implement the state ψ in the universal representation. But so will the vector $\Omega_\psi \oplus_{\phi \neq \psi} 0$, where Ω_ψ is the cyclic vector implementing ψ in the GNS representation for ψ. In fact, there will be a vector implementing ψ in each direct summand of the universal representation that corresponds to a GNS representation unitarily equivalent to π_ω. And furthermore, if ϕ is any mixture having nonzero component on the state ω, then the GNS representation of ϕ will be a direct sum containing the GNS representation of ω as one of its summands. It follows by similar considerations that there will be some vector in the GNS representation of ϕ (and hence in another summand of the universal representation) that implements the state ψ. The following proposition shows that the range of the projection P must contain *all* of these vectors implementing the state ψ.

Proposition 6. *Let $(\pi, \mathcal{H})$ be a representation of a C*-algebra $\mathfrak{A}$, let $(\pi_U, \mathcal{H}_U)$ be the universal representation of $\mathfrak{A}$, and let P be the corresponding central projection in Prop. 5. Choose an arbitrary unit vector $\Psi \in \mathcal{H}$. Then for any vector $\Phi \in \mathcal{H}_U$ implementing the vector state Ψ in the sense that*

$$\langle \Phi, \pi_U(A)\Phi \rangle = \langle \Psi, \pi(A)\Psi \rangle$$

for all $A \in \mathfrak{A}$, it follows that $P\Phi = \Phi$.

This shows that the operators in the universal representation that correspond to operators from some particular GNS representation must take a more complicated form than the “portmanteau” operators Ruetsche suggests. In the universal representation, the correct operator must act on a whole host of vectors implementing the states from the GNS representation that we started with, and many of these vectors will lie elsewhere in the universal representation, outside of the GNS representation we began with. However, it is most important to recognize that even though the elements of the universal representation do not take the simple form we might have expected, Prop. 5 guarantees us that there is always some operator in the universal enveloping von Neumann algebra corresponding to the observable we are interested in.

Ruetsche’s second objection (2011, p. 283) claims that parochial observables (in particular, phase observables like the net magnetization of a ferromagnet (Ruetsche 2006, p. 478)) may have a domain that is a proper subset of $\mathcal{S}_{\mathfrak{A}}$ and that no observable qua bounded operator on the entire universal Hilbert space can have this property. But we have already seen that parochial observables, by virtue of Prop. 5, can be thought of as acting on the subspace of $\mathcal{H}_U$ that is the range of the projection P given in that proposition. Parochial observables act like the zero operator everywhere else in $\mathcal{H}_U$ and so we can think of the orthogonal complement of the range of P as the subspace generated by the collection of states that are outside the domain of that parochial observable. Hence, we have a way of making sense in the universal representation of Ruetsche's claim that the domain of parochial observables may be smaller than $\mathcal{S}_{\mathfrak{A}}$.

It may be helpful to consider a particular example. Ruetsche (2011, p. 226) considers and rejects the universal representation as a way of accounting for inequivalent particle notions in quantum field theory.[22,23] The universal representation gives rise to a total number operator representing the number of particles of any variety as an observable in the universal enveloping von Neumann algebra. Ruetsche argues that this total number operator does not suffice for doing physics because it identifies states with the same total number of quanta spread among the different varieties, and of course a state with n quanta of one variety is different from a state with n quanta of a different variety. For example, the total number operator cannot distinguish the state with one Minkowski quantum from the state with one Rindler quantum because it tells us that both states simply have one total quantum.[24] But Prop. 5 tells us that the universal representation will contain (in addition to the total number operator for all varieties) the Minkowski quanta total number operator and the Rindler quanta total number operator, which are parochial observables to particular GNS representations of the abstract algebra. The Minkowski quanta total number operator and the Rindler quanta total number operator will have different expectation values in the state with one Minkwoski quantum and the state with one Rindler quantum. Hence, they will distinguish between

[22] I am taking Ruetsche's claims out of context here. Really, she rejects the universalized particle notion as a "fundamental" particle notion (See Ruetsche 2011, Ch. 9 for more detail). However, considering her remarks as an objection to the view outlined here is illustrative.

[23] See also Clifton & Halvorson (2001) for more on inequivalent particle notions.

[24] Really, Ruetsche makes this claim in the context of the reduced atomic representation (See Kadison & Ringrose 1997, p. 740-1). But her objection may be carried over to the universal representation and my response may be carried back to the reduced atomic representation as well because Kadison & Ringrose Thm. 10.3.10 (1997, p. 741) shows that the von Neumann algebra affiliated with the reduced atomic representation contains all of the parochial observables for all irreducible representations just as the universal enveloping von Neumann algebra contains the parochial observables associated with all (not just irreducible) representations.

these two distinct states. The universal representation, by virtue of containing all of the parochial observables, gives us the ability to make as many distinctions between states as we might like.

I hope that these remarks concerning Ruetsche's objections suffice to show that all parochial observables really are contained in the universal representation, or more specifically the universal enveloping von Neumann algebra. Now I want to remark upon the fact that the technical results I have presented for the Imperialist's and Universalist's solution to the problem of parochial observables appear so similar—this is no coincidence. We have already mentioned that the universal representation (or really universal enveloping von Neumann algebra) and the abstract algebra (or really its bidual) share the same topological structure in the sense that the notion of convergence provided by the weak (or weak*) topology on $\mathfrak{A}$ (or $\mathfrak{A}^{**}$) reproduces precisely the notion of convergence provided by the weak operator topology on $\mathcal{B}(\mathcal{H}_U)$. But these objects share much more structure than that. The following proposition shows that the bidual of a C*-algebra in a certain sense carries the same algebraic structure as the universal enveloping von Neumann algebra.

Proposition 7. *There is a *-isomorphism*[25] α *from the bidual* $\mathfrak{A}^{**}$ *of a C*-algebra* $\mathfrak{A}$ *to its universal enveloping von Neumann algebra* $\overline{\pi_U(\mathfrak{A})}$ *such that* $\pi_U(A) = \alpha(J(A))$ *for all* $A \in \mathfrak{A}$.[26]

Since the Imperialist and the Universalist invoke the same algebraic and topological structure to represent quantum systems, they end up believing in the same physically significant observables and the same physically possible states while using the same notion of approximation or idealization. This shows a sense in which Algebraic Imperialism and Universalism amount to the same position. Of course I do not claim that Imperialism and Universalism are equivalent with respect to every purpose we might put quantum theory to, but at least they are equivalent with respect to the interpretive uses just outlined. One may have pragmatic reasons for choosing one or the other—for example, one might want to use the universal representation if one is familiar with interpreting Hilbert spaces from ordinary quantum mechanics. Nevertheless, whatever quantum states and observables the Imperialist can represent, the Universalist can represent too (and vice versa). So the

[25] Prop. 3 shows that this *-isomorphism is a W*-isomorphism in the sense of Sakai (1971, p. 40), i.e. it is also a homeomorphism in the weak* and weak operator topologies, respectively. This is a relevant notion of isomorphism because both $\mathfrak{A}^{**}$ and $\overline{\pi_U(\mathfrak{A})}$ are W*-algebras (the abstract version of von Neumann algebras).

[26] Adapted from Kadison & Ringrose (1997, p. 726, Prop. 10.1.21) and Emch (1972, p. 121-2, Thm. 11). See those sources for a proof and see Lupher (2008, p. 95) for more discussion.

Imperialist and the Universalist, when allowed the same tools as the Hilbert Space Conservative for representing idealizations, come up with the *same* solution to the problem of parochial observables.

5 Conclusion: On Pristine and Adulterated Interpretation

I have argued that the Algebraic Imperialist and the Universalist both have solutions to Ruetsche's problem of parochial observables and that these are in a certain sense equivalent interpretations of quantum theories. Some may see this as an argument for Imperialism or for Universalism. Nevertheless, I subscribe to neither view. As Ruetsche describes them, Imperialism and Universalism are both instances of what she calls *pristine interpretation*. Pristine interpretations are first of all characterized as ones that articulate the content of a theory by identifying the physically possible worlds according to the theory. And, in order to be pristine, such an interpretation must specify those physically possible worlds once and for all, using only general metaphysical principles rather than contingent facts about specific applications. Ruetsche's work presents an extended argument against this ideal through the consideration of infinite quantum systems; Ruetsche argues that we should reject pristine interpretation because none of the pristine interpretations of infinite quantum theory are adequate. The problem of parochial observables comes in service to this more general project as a reason for rejecting Imperialism and Universalism.

I have argued that the problem of parochial observables fails to give us reason to reject Imperialism and Universalism, but I do not think that we ought to adopt those interpretations and I do not take this to be a defense of pristine interpretation. In showing that one can use the abstract algebra or its universal representation to adequately recover physically significant explanations I did not rely on the fact that Imperialism and Universalism are pristine interpretations. In particular, none of my arguments depended on thinking of theories as circumscribing a set of physically possible worlds by appealing to general metaphysical principles. To see this, I will outline two alternative adulterated interpretations that the arguments given in this paper support at least as well.[27]

[27] In fact, some might think my arguments support the adulterated interpretations over the pristine ones in light of the comments of footnotes 10, 13, 14, and 18. Those footnotes make caveats to avoid questions about the "right" topology, the "fundamental" observables, and the "reality" of idealizations, which the pristine interpreter might demand an answer to. I am not optimistic that one could provide answers to satisfy her. This, however, does not

The idea lurking behind the following adulterated interpretations is as follows. It is misleading to think that our scientific theories aim at specifying a set of physically possible worlds in the first place. Our scientific theories aim at providing a formalism or collection of tools that we can use for a variety of different purposes: making predictions, constructing explanations, modeling particular systems, generating new theories, examining relationships with past theories, and more.[28] This is not some sort of simple operationalism or instrumentalism, but rather a way of taking the practice of science seriously.

Whereas the Algebraic Imperialist would force algebraic methods upon everyone as the correct ones to describe the physical possibilities, her adulterated counterpart—let us call her the *Algebraic Colonialist*—merely asserts that we can use the abstract algebra to accomplish all our scientific goals. We can, for example, think of any two states—even states describing different thermodynamic phases of a statistical system—as states on the abstract algebra. As Prop. 3 shows, we can also think of any observable—even a parochial observable like temperature or net magnetization—as a limit point of a net of observables in the abstract algebra with respect to the weak (or really weak*) topology. Hence, we can think of any observable as an idealization from a collection of observables in the abstract algebra. If we desired, we could also focus attention on a particular collection of states on the abstract algebra and use the weakest topology that makes those states continuous. That is essentially what we do when we take a Hilbert space representation and use the weak operator topology on that Hilbert space. For some scientific purposes, restricting our attention to certain states and changing our topology may turn out to be fruitful.[29] All the Algebraic Colonialist claims is that this practice can be understood algebraically and that we need not take representations at all to give the desired explanations, e.g. of phase transitions.

Similarly, whereas the Universalist claims that the universal representation once and for all specifies the physical possibilities, her adulterated counterpart—let us call her the *Unitarian Universalist*—is a bit more lax. The Unitarian Universalist simply claims that we can use the universal

undermine the central arguments of the paper because an adulterated interpreter of the sort I describe here simply *does not need to answer those questions* for her purposes.

[28]See Stein (1989) for a statement of a similar view and how it bears on the scientific realism debate. Also compare this view with the alternative adulterated interpretation that Ruetsche (2003, 2011) ends up supporting.

[29]See Fletcher (2014) for an argument that we ought to allow ourselves access to different topologies for different scientific purposes, at least in the case of general relativity.

representation to accomplish all of our scientific goals. We can, for example, think of any two states as density operators (or even vectors!) on the Hilbert space $\mathcal{H}_U$ of the universal representation. And, as Prop. 5 shows, we can think of any observable—again, even a parochial observable—as being contained in the universal enveloping von Neumann algebra. Hence, we can think of any observable as well approximated by or physically indistinguishable from a collection of observables with respect to the weak operator topology of the universal representation. Similarly, if we desired, we could restrict attention to some subrepresentation of the universal representation π_U on a subspace of $\mathcal{H}_U$, thereby restricting our attention to a particular collection of density operator and vector states and defining a new topology by using the weak operator topology on this subspace. Again, this may be useful for many scientific purposes, but the universal representation gives us all of the resources we need to accomplish these tasks.

We have seen that Algebraic Imperialism and Universalism amount to the same position, and for precisely the same reasons Algebraic Colonialism and Unitarian Universalism are equivalent as well. Prop. 4 shows that the weak topology on the abstract algebra yields the same criterion for convergence as the weak operator topology on the universal representation. And Prop. 7 shows that the bidual of the abstract algebra, which is just the original algebra with the addition of its limit points in the weak topology, can be thought of as the same collection of observables as the universal enveloping von Neumann algebra, which of course is a faithful representation of the original algebra with the addition of its limit points in the weak operator topology of that representation. So the abstract algebra and the universal representation allow us to countenance the same physically significant states and observables.

Even though I have targeted and attacked a crucial piece of Ruetsche's argument against pristine interpretation, none of what I have said here provides support for that ideal. All I have done is show that the use of algebraic methods and the universal representation are equivalent and adequate for interpreting infinite quantum theories. Both of these sets of tools—the abstract algebra and its universal representation—allow us to accomplish the same tasks, and in particular to begin making sense of scientific practice in quantum field theory and quantum statistical mechanics.

Appendix A: Proofs of Propositions

This appendix contains the proofs of propositions from the body of the paper. Props. 1 and 4 follow immediately from the following lemma, which is a restatement of facts described in Reed & Simon (1980, p. 213):

Lemma 1. *Let $\mathcal{H}$ be a Hilbert space and let $\{A_i\} \subseteq \mathcal{B}(\mathcal{H})$ be a net of operators. Then the following are equivalent:*

(1) The net $\{A_i\}$ converges to $A \in \mathcal{B}(\mathcal{H})$ in the weak operator topology on $\mathcal{B}(\mathcal{H})$.

(2) For all finite rank density operators ρ on $\mathcal{H}$, $Tr(\rho A_i)$ converges to $Tr(\rho A)$ in $\mathbb{C}$.

Proposition 1. *Let $(\pi, \mathcal{H})$ be a representation of a C*-algebra $\mathfrak{A}$. Let $\{A_i\} \subseteq \pi(\mathfrak{A})$ be a net of operators. Then the following are equivalent:*

(1) The net $\{A_i\}$ converges to $A \in \mathcal{B}(\mathcal{H})$ in the weak operator topology on $\mathcal{B}(\mathcal{H})$.

(2) For all states ϕ on $\mathfrak{A}$ implementable by a finite rank density operator Φ on $\mathcal{H}$, $\phi(A_i) = Tr(\Phi A_i)$ converges to $Tr(\Phi A)$ in $\mathbb{C}$.

Proof. $(1 \Rightarrow 2)$ This follows immediately by Lemma 1.

$(2 \Rightarrow 1)$ Suppose that for all states ϕ on $\mathfrak{A}$ implementable by a finite rank density operator Φ on $\mathcal{H}$, $\phi(A_i) = Tr(\Phi A_i)$ converges to $Tr(\Phi A)$ in $\mathbb{C}$. Every finite rank density operator ρ on $\mathcal{H}$ defines a state on $\mathfrak{A}$ implementable by a finite rank density operator. So it follows from the assumption that $Tr(\rho A_i)$ converges to $Tr(\rho A)$ for all finite rank density operators and hence A_i converges to A by Lemma 1. □

Proposition 4. *Let $(\pi_U, \mathcal{H}_U)$ be the universal representation of $\mathfrak{A}$. Let $\{A_i\} \subseteq \pi_U(\mathfrak{A})$ be a net of operators. Then the following are equivalent:*

(1) The net $\{A_i\}$ converges to $A \in \mathcal{B}(\mathcal{H}_U)$ in the weak operator topology on $\mathcal{B}(\mathcal{H}_U)$

(2) For all states ϕ on $\mathfrak{A}$, $\phi(A_i) = Tr(\Phi A_i)$ converges to $Tr(\Phi A)$ in $\mathbb{C}$, where Φ is any finite rank density operator (and there is always at least one) implementing the state ϕ on $\pi_U(\mathfrak{A})$.

Proof. $(1 \Rightarrow 2)$ Suppose $A_i \to A$ in the weak operator topology on $\mathcal{B}(\mathcal{H}_U)$. Let Φ be any finite rank density operator (there is always at least one) implementing a state ϕ on $\pi_U(\mathfrak{A})$. Then by Prop. 1,

$\phi(A_i) = Tr(\Phi A_i)$ converges to $Tr(\Phi A)$ in $\mathbb{C}$.

($2 \Rightarrow 1$) Suppose that for all states ϕ on $\mathfrak{A}$, $Tr(\Phi A_i)$ converges to $Tr(\Phi A)$ in $\mathbb{C}$, where Φ is a finite rank density operator implementing ϕ. Let Φ be any finite rank density operator on $\mathcal{H}$. Then $X \mapsto Tr(\Phi X)$ for all $X \in \pi_U(\mathfrak{A})$ defines a state on $\pi_U(\mathfrak{A})$ and hence $Tr(\Phi A_i)$ converges to $Tr(\Phi A)$ in $\mathbb{C}$, which shows by Prop. 1 that A_i converges in the weak operator topology to A. □

Prop. 2 is an immediate corollary of the following elementary lemma about Banach spaces.

Lemma 2. *Let X be a Banach space, X^{**} its bidual, and $J : X \to X^{**}$ the canonical evaluation map. Then $J(X)$ is dense in X^{**} in the weak* ($\sigma(X^{**}, X^*)$) topology.*

Proof. Let $\tilde{x} \in X^{**}$ and let U be an open neighborhood of $\tilde{x}$. We will show that U contains some element $y = J(x) \in J(X)$.

By the definition of the $\sigma(X^{**}, X^*)$-topology, there are linear functionals $l_1, ..., l_n \in X^*$ and $\epsilon_1, ..., \epsilon_n > 0$ such that

$$\bigcap_{i=1}^{n} (\tilde{x} + N(l_i, \epsilon_i)) \subseteq U$$

where $\tilde{x} + N(l_i, \epsilon_i) = \{\tilde{y} \in X^{**} : |\tilde{y}(l_i) - \tilde{x}(l_i)| < \epsilon_i\}$.

Now let $N'(l_i, \epsilon_i) = \{x \in X : |l_i(x) - \tilde{x}(l_i)| < \epsilon_i\}$. We will show that

$$\bigcap_{i=1}^{n} N'(l_i, \epsilon_i) \subseteq \bigcap_{i=1}^{n} (\tilde{x} + N(l_i, \epsilon_i)) \cap J(X)$$

is non-empty, or in other words we will show that there is a $y = J(x) \in J(X)$ such that for all $1 \leq i \leq n$,

$$|y(l_i) - \tilde{x}(l_i)| = |l_i(x) - \tilde{x}(l_i)| < \epsilon_i \tag{1}$$

It suffices to consider only a linearly independent subset of the linear functionals $l_1, ..., l_n \in X^*$ forming a basis for the subspace of X^* spanned by these functionals: if $x \in X$ satisfies the above inequality for this linearly independent subset of $l_1, ..., l_n \in X^*$, then it must also satisfy the inequalities for the rest of the linear functionals, or else the inequalities would contradict each other and then $\bigcap_{i=1}^{n} (\tilde{x} + N(l_i, \epsilon_i))$ would have to be empty, which it is not because it contains $\tilde{x}$.

Choose this linearly independent set of functionals $l_{k_1}, ..., l_{k_m}$. We know (Schechter 2001, p. 93, Lemma 4.14) that there exists a dual basis $e_1, ..., e_m \in X$ for a subspace of X such that $l_{k_i}(e_j) = \delta_{ij}$ for all $1 \leq i, j \leq m$. Consider the vector $x = \sum_{i=1}^{m} \tilde{x}(l_i)e_i$. We have for all $1 \leq j \leq m$,

$$l_j(x) = \sum_{i=1}^{m} \tilde{x}(l_i)l_j(e_i) = \tilde{x}(l_j)$$

Hence, x satisfies the above inequalities in (1) and it follows that

$$y = J(x) \in \bigcap_{i=1}^{n} N'(l_i, \epsilon_i) \subseteq \bigcap_{i=1}^{n} (\tilde{x} + N(l_i, \epsilon_i)) \subseteq U$$

Therefore, since $U \cap J(X) \neq \varnothing$, $J(X)$ is dense in X^{**}. □

Proposition 2. *Let $\mathfrak{A}$ be a C*-algebra, $\mathfrak{A}^{**}$ be its bidual, and $J : \mathfrak{A} \to \mathfrak{A}^{**}$ be the canonical evaluation map. Then $J(\mathfrak{A})$ is dense in $\mathfrak{A}^{**}$ in the weak* ($\sigma(\mathfrak{A}^{**}, \mathfrak{A}^{*})$) topology.*

Proof. Since every C*-algebra $\mathfrak{A}$ is a Banach space, this follows immediately from Lemma 2. □

Prop. 3 is an immediate corollary of Prop. 5 and Prop. 7, whose proofs are contained in Kadison & Ringrose (1997, p. 719, Thm. 10.1.12 and p. 726, Prop. 10.1.21, respectively).

Proposition 3. *If π is a representation of a C*-algebra $\mathfrak{A}$, then there is a central projection $P \in \mathfrak{A}^{**}$ and a *-isomorphism α from $\mathfrak{A}^{**}P$ onto $\overline{\pi(\mathfrak{A})}$ such that $\pi(A) = \alpha(J(A)P)$ for all $A \in \mathfrak{A}$.*

Proof. By Prop. 5, there is a projection $\tilde{P}$ in $\overline{\pi_U(\mathfrak{A})}$ in the center of $\overline{\pi_U(\mathfrak{A})}$ and a *-isomorphism α_1 from $\overline{\pi_U(\mathfrak{A})}\tilde{P}$ to $\pi(\mathfrak{A})$ such that $\pi(A) = \alpha_1(\pi_U(A)\tilde{P})$ for all $A \in \mathfrak{A}$. By Prop. 6, there is a *-isomorphism α_2 from the bidual $\mathfrak{A}^{**}$ to $\overline{\pi_U(\mathfrak{A})}$ such that $\pi_U(A) = \alpha_2(J(A))$ for all $A \in \mathfrak{A}$. The projection $P = \alpha_2^{-1}(\tilde{P})$ and the *-isomorphism $\alpha = \alpha_1 \circ (\alpha_2)_{|\mathfrak{A}^{**}P}$ serve as a witness to the current theorem, because for any $A \in \mathfrak{A}$,

$$\alpha(J(A)P) = \alpha_1 \circ \alpha_2(J(A)P) = \alpha_1[\alpha_2(J(A)) \cdot \alpha_2(P)] = \alpha_1(\pi_U(A)\tilde{P}) = \pi(A)$$

□

Proposition 6. *Let* $(\pi, \mathcal{H})$ *be a representation of a* C^**-algebra* $\mathfrak{A}$ *and let* $(\pi_U, \mathcal{H}_U)$ *be the universal representation of* $\mathfrak{A}$*. Let* P *be the central projection in Prop. 5, and choose some unit vector* $\Psi \in \mathcal{H}$*. Then for any vector* $\Phi \in \mathcal{H}_U$ *implementing the vector state* Ψ *in the sense that*

$$\langle \Phi, \pi_U(A)\Phi \rangle = \langle \Psi, \pi(A)\Psi \rangle$$

for all $A \in \mathfrak{A}$*, it follows that* $P\Phi = \Phi$*.*

Proof. By construction (See Kadison & Ringrose Thm. 6.8.8 (1997, p. 443) and Thm. 10.1.12 (1997, p. 719)) , the projection P takes the form

$$P = I - E$$
$$E = \bigvee_{T \in Ker(\overline{\beta})} R(T^*)$$

where $\overline{\beta}$ is the ultraweakly continuous extension of the map $\beta = \pi \circ \pi_U^{-1}$ and $R(T^*)$ is the projection onto the range of T^*. With Φ and Ψ as above,

$$P\Phi = \Phi - E\Phi$$

It suffices to show that $\langle \Phi, E\Phi \rangle = 0$, which shows that the second term above is zero.

We know that $E = \bigvee_{T \in Ker(\overline{\beta})} R(T^*) \in Ker(\overline{\beta})$, so it follows that

$$\langle \Psi, \overline{\beta}(E)\Psi \rangle = 0$$

Choose a net $A_i \in \mathfrak{A}$ such that $\pi_U(A_i)$ converges in the weak operator topology on $\mathcal{B}(\mathcal{H}_U)$ to E. It follows immediately that $\pi(A_i) = \overline{\beta}(\pi_U(A_i))$ converges in the weak operator topology on $\mathcal{B}(\mathcal{H})$ to

$\overline{\beta}(E)$, so

$$\begin{aligned}\langle \Phi, E\Phi\rangle &= \langle \Phi, w\text{-}\lim(\pi_U(A_i))\Phi\rangle \\ &= \lim\langle \Phi, \pi_U(A_i)\Phi\rangle \\ &= \lim\langle \Psi, \pi(A_i)\Psi\rangle \\ &= \langle \Psi, w\text{-}\lim(\pi(A_i))\Psi\rangle \\ &= \langle \Psi, \overline{\beta}(E)\Psi\rangle = 0\end{aligned}$$

where $w\text{-}\lim$ denotes the weak operator limit in the relevant Hilbert space. □

Appendix B: An Illustration in Classical Systems

Suppose that the system under consideration is classical so that $\mathfrak{A}$ is abelian. Examining the weak topology and weak operator topologies of representations of this algebra illustrates the concepts of section 4 in a somewhat more familiar and concrete setting (although admittedly not the most familiar or concrete!).[30] Recall that when $\mathfrak{A}$ is abelian, it is *-isomorphic to $C(\mathcal{P}(\mathfrak{A}))$, the continuous functions on the compact Hausdorff space $\mathcal{P}(\mathfrak{A})$ of pure states of $\mathfrak{A}$ with the weak* $(\sigma(\mathcal{P}(\mathfrak{A}), \mathfrak{A}))$ topology (Kadison & Ringrose 1997, p. 270, Thm. 4.4.3). As such, each observable $A \in \mathfrak{A}$ corresponds to a function $\hat{A} \in C(\mathcal{P}(\mathfrak{A}))$ defined by

$$\hat{A}(\omega) = \omega(A)$$

for each pure state $\omega \in \mathcal{P}(\mathfrak{A})$.

Taking a representation of $\mathfrak{A}$ amounts to choosing a measure on the space $\mathcal{P}(\mathfrak{A})$ (See Kadison & Ringrose 1997, p. 744; Landsman 1998, p. 55), which defines an (L^2) inner product, hence constructing a Hilbert space as follows. By the Riesz-Markov theorem (Reed & Simon 1980, p. 107, Thm. IV.14), each state ω on $\mathfrak{A}$ corresponds to a unique regular Borel measure μ_ω on $\mathcal{P}(\mathfrak{A})$

[30] Feintzeig (2015) similarly uses the classical case to gain insight about interpreting the algebraic formalism. This section can be understood as adding to that project.

such that for all $A \in \mathfrak{A}$

$$\omega(A) = \int_{\mathcal{P}(\mathfrak{A})} \hat{A} d\mu_\omega$$

The GNS representation of $\mathfrak{A}$ for the state ω is unitarily equivalent to the representation[31] $(\pi_\omega, \mathcal{H}_\omega)$ on the Hilbert space $\mathcal{H}_\omega = L^2(\mathcal{P}(\mathfrak{A}), d\mu_\omega)$, with π_ω defined by

$$\pi_\omega : A \mapsto M_{\hat{A}}$$

where the operator $M_{\hat{A}}$ is defined as multiplication by the function $\hat{A}$, i.e. for any $\psi \in \mathcal{H}_\omega$,

$$M_{\hat{A}}\psi = \hat{A} \cdot \psi$$

Now, we can pull the discussion of topologies on $\mathfrak{A}$ back to the more familiar topologies on ordinary functions on $\mathcal{P}(\mathfrak{A})$. We recall these topologies now before proceeding. Let f_n be a net of functions on $\mathcal{P}(\mathfrak{A})$. We say that f_n converges to the function f *uniformly* if $\sup_{\psi \in \mathcal{P}(\mathfrak{A})} |f_n(\psi) - f(\psi)|$ converges to zero in $\mathbb{C}$. We say that f_n converges to the function f *pointwise* if $f_n(\psi)$ converges to $f(\psi)$ in $\mathbb{C}$ for each $\psi \in \mathcal{P}(\mathfrak{A})$. And finally, we say that f_n converges to the function f *pointwise almost everywhere* with respect to a measure μ on $\mathcal{P}(\mathfrak{A})$ if $f_n(\psi)$ converges to $f(\psi)$ in $\mathbb{C}$ for all $\psi \in \mathcal{P}(\mathfrak{A})$ except possibly on a set of measure zero with respect to μ. Now it is easy to show that for the GNS representation $(\pi_\omega, \mathcal{H}_\omega)$, the weak operator topology on $\mathcal{B}(\mathcal{H}_\omega)$ is the topology of pointwise convergence almost everywhere with respect to the measure μ_ω. This implies that while $\pi_\omega(\mathfrak{A})$ is the collection of multiplication operators by *continuous functions* (which is uniformly closed), its weak operator closure $\pi_\omega(\mathfrak{A})$ will be the collection of multiplication operators by *essentially bounded measurable functions* with respect to the measure μ_ω (i.e., $L^\infty(\mathcal{P}(\mathfrak{A}), d\mu_\omega)$).

In some cases, taking the weak operator closure (and hence, moving to the essentially bounded measurable functions) does not give rise to any new parochial observables. When ω is pure,

$$\omega(A) = \hat{A}(\omega) = \int_{\mathcal{P}(\mathfrak{A})} \hat{A}\delta(\omega)$$

[31] Here, the relevant cyclic vector Ω_ω is the constant unit function.

for all $A \in \mathfrak{A}$, where $\delta(\omega)$ is the point mass or delta function centered on ω. It follows by the uniqueness clause of the Riesz-Markov theorem that $d\mu_\omega = \delta(\omega)$. So every vector $\psi \in \mathcal{H}_\omega$ will be defined by a single complex number—the value of ψ on $\omega \in \mathcal{P}(\mathfrak{A})$ and $\mathcal{H}_\omega$ will be one-dimensional. Hence, $\mathcal{B}(\mathcal{H}_\omega)$ will be one-dimensional and since $\pi_\omega(\mathfrak{A})$ contains the identity and is closed under scalar multiplication, it follows that $\overline{\pi_\omega(\mathfrak{A})} = \mathcal{B}(\mathcal{H}_\omega) = \pi_\omega(\mathfrak{A})$. This means that there are no parochial observables in this representation. Furthermore, even the GNS representations for many mixed states do not give rise to new parochial observables. Consider an arbitrary state ω on $\mathfrak{A}$ such that μ_ω has support on only a countable subset of $\mathcal{P}(\mathfrak{A})$. In such a special state, since the measure focuses our attention on only a countable subset of $\mathcal{P}(\mathfrak{A})$, the continuous functions coincide with the essentially bounded measurable functions—every discontinuous but essentially bounded measurable function is equivalent to a continuous function when we ignore differences on sets of measure zero. In other words, focusing only on a countable subset of $\mathcal{P}(\mathfrak{A})$ does not allow one to distinguish between continuous and merely bounded functions. So it similarly follows that $\overline{\pi_\omega(\mathfrak{A})} = \pi_\omega(\mathfrak{A})$ and there are no parochial observables in this representation.

However, we can also have an arbitrary mixed state ω on $\mathfrak{A}$ such that μ_ω has support on an uncountable subset of $\mathcal{P}(\mathfrak{A})$. In this case, we may acquire new parochial observables. The weak operator closure will include even discontinuous functions like characteristic functions (projection operators), where the original algebra did not. Since each one of these essentially bounded measurable functions is the weak operator limit point of a collection of continuous functions, we can understand them as idealizations from or approximations to collections of our original observables. So the essentially bounded (but discontinuous) measurable functions with respect to some measure are the parochial observables for an algebra of continuous functions in the representation defined by that measure.

But, comes the obvious retort, in a similar sense *every bounded function* (without considering any measure) can be considered as an idealization from or approximation to a collection of our original observables without appeal to any measure, and hence without appeal to any representation. The sense in which this is true uses the weak topology on $\mathfrak{A}$. In particular, every bounded function is the pointwise limit of a collection of continuous functions. The topology of pointwise convergence for functions is just the weak topology on $\mathfrak{A}$ (really, extended to the weak* topology on $\mathfrak{A}^{**}$), so $\mathfrak{A}^{**}$ is just the collection of bounded functions on $\mathcal{P}(\mathfrak{A})$. Since every parochial observable qua essentially bounded function is equivalent to a bounded function (ignoring differences on sets of measure zero), it follows that every parochial observable can be thought of as the weak (pointwise) limit of observables in the abstract algebra. This provides our algebraic route to all of the parochial observables at once, without reference to any representation.

On the other hand, we can gather all the parochial observables in a single representation as in the previous section by taking the universal representation (See Kadison & Ringrose 1997, p. 746). The universal representation acts on the Hilbert space

$$\mathcal{H}_U = \bigoplus_{\omega \in \mathcal{S}(\mathfrak{A})} L^2(\mathcal{P}(\mathfrak{A}), d\mu_\omega)$$

by the representation

$$\pi_U(A) = \bigoplus_{\omega \in \mathcal{S}(\mathfrak{A})} M_{\hat{A}|supp(d\mu_\omega)}$$

where $M_{\hat{A}|supp(d\mu_\omega)}$ is the multiplication operator by the restriction of $\hat{A}$ to the support of $d\mu_\omega$ because that is equivalent to $M_{\hat{A}}$ on $\mathcal{H}_\omega$. The operator $\pi_U(A)$ in a certain sense amounts to the multiplication operator by $\hat{A}$ *everywhere* because we have taken the direct sum over spaces with all possible regular normalized Borel measures and so for each point $\omega \in \mathcal{P}(\omega)$ there is some summand in which $\{\omega\}$ gets assigned nonzero measure. This is why π_U gives a faithful representation whereas each individual GNS representation is not faithful (because functions that disagree only on a set of measure zero are mapped to the same multiplication operator by that GNS representation). Weak operator convergence in the universal representation is just pointwise convergence everywhere again because each point gets assigned nonzero measure by at least one of the regular normalized Borel measures defining the Hilbert space summands. Thus the weak operator closure in the universal representation gives us back all of the bounded functions as direct sums of essentially bounded measurable functions considered over all possible regular normalized Borel measures.

The upshot is that the abstract algebra and its universal representation give us two routes to acquiring all of the parochial observables, which in this case are the bounded functions. One can stay with the abstract algebra and use the weak topology, which defines the topology of pointwise convergence. Or one can use the universal representation and its weak operator topology, which similarly defines the topology of pointwise convergence. Either way, we start with the continuous functions and construct certain discontinuous idealizations from them. I hope that this special example may take some of the mystery out of the parochial observables and where they come from.

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