Literal vs careful interpretations of scientific theories: the vacuum approach to the problem of motion in general relativity

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Abstract

The problem of motion of general relativity is about how exactly the gravitational field equations, the Einstein equations, are related to the equations of motion of material bodies subject to gravitational fields. This paper compares two approaches to derive the geodesic motion of (test) matter from the field equations: 'the T approach' and 'the vacuum approach'. The latter approach has been dismissed by philosophers of physics because of it apparently representing material bodies by singularities. I shall argue that a careful interpretation of the approach shows that it does not depend on introducing singularities at all, and that it holds at least as much promise as the T approach. I finish with some general lessons about careful vs. literal interpretations of scientific theories.

Contents

1   Introduction

It is a bit of an irony that one of the most embraced definitions of what it means to be a scientific realist is due to arch-anti-realist Bas van Fraassen. His definition of the position starts by saying that "Science aims to give us, in its theories, a literally true story of what the world is like".[1] And indeed, scientific realists often see themselves as committed to 'taking scientific theories at face value': if the best theories of particle physics say that quarks exist, then we should believe that they exist; if general relativity tells us that gravity is really just an aspect of spacetime structure, then we should believe it; if quantum mechanics tells us that the world is at its core non-deterministic, then we should believe that too.

The problem is that scientific theories, or at least the theories of modern physics, are not that straightforward with us. They may seem so at first, but if you listen to the details of their respective story, if you take your time to look under the surface, what exactly we should take them to tell us about the world is far from clear. Murray Gell-Mann, the inventor of the quarks concept, for a long time did not think that quarks should be interpreted as literally existing; neither did Richard Feynman. Albert Einstein passionately resisted the interpretation of general relativity that says that the gravitational force field of Newtonian theory is ontologically reduced to the geometry of spacetime in general relativity. And of course, there is a long-standing battle in foundations of physics about whether quantum mechanics really does tell us that the world is non-deterministic.[2]

In this paper I shall introduce a new case study that provides further evidence to the position that, whether you are a realist or not, the *literal interpretation* of a scientific theory, especially in physics, can be rather misleading. I will argue that what we should aim for is a *careful interpretation*; an interpretation of the theory or model or formalism that engages with its details, both with the details of its mathematical structure and how it is applied to the natural world. Philosophy of science must be willing to look under the hood.

The case study I want to look at is the so-called problem of motion in

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[1]Van Fraassen [1980], p.8.

[2]For a discussion of different interpretations of the quark concept see Pickering [1999], for Einstein's opposition of interpreting general relativity as a geometrization of gravity see Lehmkuhl [2014], and for debate on whether quantum mechanics is really indeterministic see e.g. Saunders et al. [2010].

the general theory of relativity (GR). It asks about the precise relationship between the two sets of equations that are at the very heart of GR. On the one hand there are the Einstein field equations, which give us the dynamics of the gravitational potential (the metric tensor) $g_{\mu\nu}$:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \kappa_E T_{\mu\nu} \quad . \tag{1}$$

On the other hand, we have the geodesic equation that determines which paths through spacetime are geodesics of the connection $\Gamma^\nu{}_{\mu\sigma}$ compatible with the metric $g_{\mu\nu}$:

$$\frac{d^2 x_\tau}{ds^2} + \Gamma^\tau{}_{\mu\nu}\frac{dx_\mu}{ds}\frac{dx_\nu}{ds} = 0. \tag{2}$$

In GR, material bodies subject only to gravitational fields are supposed to move on the geodesics determined by equation (2).[3] The problem of motion in GR is the question of whether the equations of motion of matter subject to gravitational fields (2) can be derived from the gravitational field equations (1).

Einstein himself, in his first publication on the topic, a paper co-written with Jakob Grommer and published in 1927, compares different classes of attempts for giving such a derivation. In particular, Einstein and Grommer distinguish between two classes of attempts at deriving the geodesic motion of matter from the gravitational field equations, which I will term *the T approach* and *the vacuum approach*, respectively. The T approach starts from the realization that the field equations (1) imply the conservation condition, namely that the covariant divergence of the energy-momentum tensor $T_{\mu\nu}$ vanishes:

$$\nabla^\mu T_{\mu\nu} = 0 \quad . \tag{3}$$

From this, together with certain conditions on the energy-momentum tensor $T_{\mu\nu}$, the T approach derives that material particles move on time-like geodesics. It is this kind of approach to the problem of motion that

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[3]It is a big question which systems are actually included into 'material bodies' here. The minimal position is that only test particles are referred to: particles with negligible extension, spin, and self-gravity. However, many actual bodies can be approximated well by test particles in this sense; planets orbiting a star are an example, as we shall see below.

philosophers have engaged with almost exclusively up to now.[4]

Einstein and Grommer end up dismissing the T approach, and suggest an alternative path to deriving geodesic motion instead. It is a particular version of a *vacuum approach to the problem of motion.* Einstein and Grommer start from the vacuum form of the Einstein field equations,

$$R_{\mu\nu} = 0 \quad , \tag{4}$$

and attempt to derive that the equations (4) imply that material particles move on geodesics.

To the extent that philosophers have engaged with this approach at all, they have quickly dismissed it because it seems to model material bodies by singularities in spacetime; while singularities, by definition, are not even part of spacetime. However, in this paper I shall argue that this dismissal was far too fast, and that indeed the vacuum approach deserves at least as much attention by philosophers as the T approach. The vacuum approach, despite first appearances, engages more closely with some of the most major predictions of GR: both the prediction of the perihelion of Mercury and the prediction of light bending by the Sun utilise the vacuum approach to the derivation of motion of material systems. Indeed, even the prediction of gravitational waves resulting from a binary black hole merger that was recently confirmed rests on the vacuum field equations, for black holes are described by vacuum solutions.[5]

My argument in this paper will proceed in three steps. First, I will argue that the vacuum approach to the problem of motion promises certain advantages that the T approach lacks. Second, I will argue that the problems of the vacuum approach for which it has been dismissed are artefacts of a too literal interpretation of the formalism and its application to the problem at hand. Third, I will argue that a careful interpretation makes the problems disappear; I will argue that the approach does not need to interpret singularities as representing material bodies.

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[4]For a comprehensive review of the early history of this approach see Havas [1989] and Kennefick [2005]; for two particularly beautiful exemplars from within this class of proofs see Geroch and Jang [1975] and Ehlers and Geroch [2004], which are investigated by Brown [2007], Malament [2012], and Weatherall [Forthcoming, 2011].

[5]See Abbott et al. [2016] and references therein.

2 A critical comparison of the two research programmes

I said above that the T approach to the problem of motion proceeds via the fact that the Einstein field equations (1) imply the conservation condition (3), which in turn imply the geodesic motion of matter. However, as Malament [2012] pointed out, the conservation condition by itself is not sufficient to prove that the geodesic equation is the equation of motion of material particles. One of the most general proofs from within the T approach, proposed by Geroch and Jang [1975] and further generalised by Ehlers and Geroch [2004], rests not only on the conservation condition (3), but also on the strengthened dominant energy condition, which states:

> Given any timelike covector ξ_μ at any point in M, $T^{\mu\nu}\xi_\mu\xi_\nu \geq 0$
> and either $T^{\mu\nu} = \mathbf{0}$ or $T^{\mu\nu}\xi_\mu$ is timelike.

The first clause is effectively the weak energy condition, which states that the mass-energy-momentum density associated with the body in question is always non-negative. The second clause states that every observer will judge the mass-energy-momentum of the body to propagate along time-like curves only.[6]

It would be rather attractive if we did not have to presume that material particles move on time-like curves to then show that these curves are actually time-like geodesics, and if we did not have to presume that matter cannot have non-negative mass-energy. These are weak assumptions about the nature of matter, but they are assumptions.

The vacuum approach to the problem of motion, on the other hand, aims to make *no* assumptions about the nature of matter and its properties at all, and to still derive that matter moves on geodesics. It starts from the question of whether just knowing the exterior gravitational field of a material body, and how this gravitational field interacts with the gravitational field of its surroundings, is enough to derive that the body will move on a geodesic of the metric surrounding it. Arguably, this programme is far more ambitious that the T approach, for it starts with fewer assumptions. And yet, if successful, it would really fit much better the virtues that philosophers have associated

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[6]For more on the interpretation of the strengthened dominant energy condition seeWeatherall [2011], Weatherall [Forthcoming] and especially Curiel [Forthcoming].

with the geodesic theorem(s) in the first place: derive the intertial motion of matter from knowledge of the dynamics of gravitational fields alone.[7]

Einstein was deeply skeptical of the role of the energy-momentum tensor in GR. Throughout the decades, he emphasised that $T_{\mu\nu}$ provides only a 'phenomenological representation of matter'.[8] In Einstein and Grommer [1927], Einstein elaborates that general relativity with an energy-momentum tensor as a source term on the right-hand side of (1) is just not a complete theory: it does not tell us what kind of matter is present, only that it has a certain mass-energy distribution. This perspective on GR was further strengthened by Tupper [1981, 1982, 1983], who showed that knowing the energy-momentum tensor of a material system does not suffice to tell us what kind of matter is present. For example, one and the same mass-energy-momentum distribution $T_{\mu\nu}$ featuring on the right-hand side of the Einstein equations, and for the same metric solving the Einstein equations, can correspond to an electromagnetic field or a viscous fluid. Knowing the energy-momentum tensor just is not sufficient to know which of these two material systems interacts with the metric field.

Einstein's aim is then to instead start with the vacuum field equations (4), treat material particles as singularities in the metric field,[9] and derive that they move on geodesics of a metric $g_{\mu\nu}$ that solves the vacuum field equations (4) in the region through which the particle moves.

To the extent that philosophers have engaged with this approach at all, they have dismissed it at this point already. The main criticism is that the very idea of the approach is flawed: A singularity is not even part of spacetime. How should it be possible to describe its motion in said spacetime?

Both Torretti and Earman essentially answer that this is not possible and that the whole programme is ill-defined. Earman [1995], p. 12, writes:[10]

> [S]ingularities in the spacetime metric cannot be regarded as taking place at points of the spacetime manifold M. Thus, to speak

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[7]Cf. Brown [2007], p. 141 and 163.

[8]See, for example, Einstein [1922], Einstein to Michele Besso, 11 August 1926 (EA-7-361), and Einstein [1936].

[9]In recent years, the adequate definition of what a singularity is in GR has been a subject of extensive debate, see e.g. Earman [1995] and Curiel [1999]. For Einstein's thoughts on singularities see Earman and Eisenstaedt [1999]; in the context of the Einstein-Grommer paper Einstein clearly thinks of a singularity in the metric field $g_{\mu\nu}$ as a region were the components of the metric tend to infinity.

[10]For similar statements see Torretti [1996], section 5.8.

of singularities in $g_{\mu\nu}$ as geodesics of the spacetime is to speak in oxymorons.

The most detailed discussion of the Einstein-Grommer paper in the philosophical literature is due to Tamir [2012]. After quoting the above statement by Earman, Tamir goes on to write (p.142):

> The proponent of such a "vaccuum-cum-singularity" technique is faced with the rather paradoxical challenge of explaining in what sense we can say that a singular curve (ostensibly constituted by the *missing* points in the manifold) is actually a geodesic of the spacetime from which it is absent. Not only is no metric defined at the singularity, but also technically there are not even spacetime points there: the geodesic does not exist.

Tamir then mentions a key ingredient of the Einstein-Grommer approach, namely the distinction between an 'inner metric' and an 'outer metric'.[11] Einstein and Grommer aim to show that the particle characterized by a singular inner metric moves on geodesics of the non-singular outer metric. Tamir states that the "suggested implication" is that we are to compare a second spacetime whose metric is that of the regular outer metric with the singular first spacetime, and identify the regular geodesic of the second spacetime with the singular curve of the first one. He then argues that the thought that the second singularity-free spacetime can teach us anything about the singular original spacetime is "spurious".

My point in the following will be this. Even if this argument were convincing, its premise (the 'suggested implication' that Einstein and Grommmer intended to deduce something about a singular spacetime by comparing it to a non-singular spacetime) is not. I shall argue that by looking at the details of the Einstein-Grommer approach we come to a different interpretation of the approach, one that sheds a completely different light on the alleged

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[11]There is an interesting relationship between Einstein's and Grommer's distinction between inner and outer metric (discussed further in section 3) on the one hand and the later distinction between interior and exterior black hole solutions on the other. I do believe that bringing together results and concepts developed in the context of black hole solutions (a special case of vacuum solutions) on the one hand and the vacuum approach to the problem of motion on the other hand is very promising indeed. I will have to postpone a detailed discussion to a later paper; it will include the problem of motion of a binary black hole, the black hole equivalent of the Sun-Mercury two-body system discussed below.

presence of singularities. We will see that a careful (rather than literal) interpretation of the vacuum approach (and the Einstein-Grommer paper in particular) does not actually depend on introducing singularities at all.

3 The vacuum approach to the problem of motion

3.1 Two ways of looking at Einstein's model of the Sun-Mercury system

In a way, the story of the vacuum approach to the problem of motion starts in 1915, with Einstein's treatment of the orbit of Mercury around the Sun in the context of GR. It is a two-body problem: a small body (Mercury) with a comparatively small mass orbits around a large body (the Sun). Einstein seems to postulate (more on the 'seems' below) that the Sun be represented by what would soon be recognized as an approximation to the Schwarzschild metric. He definitely postulates (!) that Mercury moves on a geodesic of said metric.[12] In a way, the problem of motion in GR is about the question whether this second postulate is really necessary.

If we now look at Einstein's Mercury paper and recall the kind of criticism that was launched against the vacuum approach to the problem of motion, we may find ourselves puzzled. After all, the Schwarzschild metric is a solution to the vacuum field equations, and it has a singularity at its center.[13] If representing material bodies by singular metrics is so problematic, how does it come that Einstein [1915] successfully predicted the perihelion motion of Mercury? Why is it not problematic to represent the Sun by the singular Schwarzschild metric?

The answer lies in denying the premise of the question. Einstein's treatment of the Sun-Mercury system should *not* be interpreted as involving him representing the Sun by (an approximation of) the Schwarzschild metric. We *know* that the Sun is a material body with non-vanishing mass-energy, and that it does not have a spacetime singularity at its center. What Einstein

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[12]For a careful analysis of Einstein's Mercury paper and how it rests on the Einstein-Besso manuscript see Earman and Janssen [1993], and the Editorial Note on the Einstein-Besso manuscript in Vol. 4 of the Collected Papers of Albert Einstein (CPAE).

[13]For the history and interpretation of the Schwarzschild metric and its analytic extensions see Eisenstaedt [1989] and Bonnor [1992].

really does is to convert the two-body problem Sun-Mercury into a one-body problem, where one body (Mercury) is subject to an external gravitational field. It is the exterior gravitational field of the Sun, *not the Sun itself*, that is represented by the Schwarzschild metric. And that is enough to predict the perihelion of Mercury: we don't need to know what the Sun is made of or what happens in its interior; all that matters is the exterior gravitational field that Mercury is subject to.

Thus, worrying about the singularity at the center of the Schwarzschild metric just misses the point: we do not have to interpret the interior part of the Schwarzschild metric literally, at least not in this application.

In the following I shall argue that we should interpret the appearance of singularities in the Einstein-Grommer vacuum approach to the problem of motion in a similar vein.

3.2 The Einstein-Grommer vacuum approach to the problem of motion

The general scheme of the Einstein-Grommer approach proceeds as follows.[14]

1. Reformulate the vacuum Einstein equations in terms of a surface integral over a three-dimensional hyper-surface such that we can ask whether gravitational energy-momentum represented by the pseudo-tensor t_α^τ passes through the surface.[15]

2. Pick a curve that is supposed to represent the path of a material particle.

3. Impose the linear approximation according to which $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$, i.e. assume that, at least close to the curve, the metric deviates from Minkowski spacetime only slightly.

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[14]The genesis of the Einstein-Grommer approach has been a bit of a mystery up to now, as pointed out by Kennefick [2005]. However, the work on the 15th volume of Einstein's collected papers has revealed the context and correspondence leading up to that paper, and how it fits into Einstein's overall research program. It is a fascinating story; alas, it will have to wait for a separate paper.

[15]There has been a long debate on whether gravitational energy can be adequately represented by a pseudo-tensor; I will not be able to do it justice here. For some details see the introduction to Volume 8 CPAE for the debate between Einstein, Klein, Levi-Civita and Lorentz, for conceptual analysis Hoefer [2000] and especially Trautmann [1962].

4. Realise that not all solutions to the linearized field equations will correspond to solutions of the non-linear field equations that the linearized field equations approximate. Argue that in the case where an 'equilibrium condition' for the energy-pseudo-tensor of the gravitational field holds, the $\gamma_{\mu\nu}$ of the linearized field equations *will* solve the full non-linear equations reformulated as a surface integral.[16]

5. Now split the $\gamma_{\mu\nu}$ in the immediate neighborhood of the particle into the 'inner metric $\bar{\gamma}_{\mu\nu}$ that the particle itself gives rise to and the 'outer metric $\bar{\bar{\gamma}}_{\mu\nu}$ that is due to other sources (or lack thereof). Observe that the 'outer metric' is entirely regular, even if extended to the point at which the material particle is supposed to be located.

6. Integrate the surface integral that is equivalent to the vacuum field equations 'around' the curve that is supposed to represent the path of a material particle. For the case where the integration surface is a sphere, the equilibrium condition for t_α^τ simplifies to $\frac{\partial \bar{\bar{\gamma}}_{44}}{\partial x_\sigma} = 0$.

7. Conclude that the curve that represents the path of a material particle is a geodesic of the outer metric $\bar{\bar{\gamma}}_{\mu\nu}$.[17]

4 Interpreting the Einstein-Grommer approach to the problem of motion

The reader might think that the argument presented in the last section cannot be a faithful representation of the Einstein-Grommer approach; after all, where is the claim that the material particle is represented by a singularity, the reason the approach was dismissed by Earman and Tamir? Indeed, I

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[16]This step is very intricate and it would take me a few pages to do it justice. This point of the Einstein-Grommer paper has not been adressed by the literature at all (neither in physics nor in philosophy); I will argue elsewhere that it sheds new light on Einstein's later doubts of whether the gravitational wave solutions of the linearized equations correspond to gravitational wave solutions in the full non-linear theory.

[17]Einstein and Grommer then go on to generalise this result to the 'non-stationary case', i.e. the case where it is not demanded that the external gravitational field to which the particle is subject to does not change in time. They conclude that in this case, too, the particle will move on a geodesic of the outer metric $\bar{\bar{\gamma}}_{\mu\nu}$ that is a solution to the field equations. For the following this generalisation does not make a difference; I will thus refer only to the stationary scenario described above.

have omitted that after step 5 of the argument Einstein and Grommer *do* say that one *could* assume that the inner metric $\bar{\gamma}_{\mu\nu}$ is given by what is effectively a three-dimensional counterpart of the Schwarzschild metric: it is spherically symmetric and has a singularity at the center. And yet, *Einstein and Grommer never use this assumption in their argument.* They call the material particle 'the singularity' all the time, but their argument does not depend on assuming *any* particular form for the inner metric, let alone one that is necessarily singular. As a matter of fact, they do not even mention a concrete candidate metric for the outer metric $\bar{\bar{\gamma}}_{\mu\nu}$; all they need is that $\gamma_{\mu\nu}$ is split into the inner metric $\bar{\gamma}_{\mu\nu}$ and the outer metric $\bar{\bar{\gamma}}_{\mu\nu}$ in such a way that $\bar{\bar{\gamma}}_{\mu\nu}$ is non-singular everywhere.

Note that this does not mean that we *know* that the inner metric $\bar{\gamma}_{\mu\nu}$ is non-singular. We don't know anything about the inner metric, for the argument is independent of $\bar{\gamma}_{\mu\nu}$ having any particular form, just like the derivation of Mercury's perihelion was independent of whether there is a singularity at the center of the Schwarzschild metric that represented the exterior field of the Sun.

With regard to the Sun-Mercury system I argued that we should not interpret the Schwarzschild metric as representing the Sun, but as representing its exterior gravitational field. The part of the Sun that is within the event horizon, including the singularity at the center, should not be taken as a representation of the *actual* interior of the Sun, but as a *placeholder* or a *blind spot* within the current description of the Sun-Mercury system: a docking station for a theoretical model of the Sun not included in Einstein's Sun-Mercury model.[18]

Likewise, we should interpret the inner metric $\bar{\gamma}_{\mu\nu}$ in the Einstein-Grommer approach as a placeholder for a representation of matter not included in the current theoretical approach. Sure, you *can* set $\bar{\gamma}_{\mu\nu}$ to be a Schwarzschild-like metric with a singularity at the center. But you don't have to do that to make the Einstein-Grommer argument work, and even if you do make that assumption you should still take this particular inner metric with a singularity at its center as a placeholder for a representation or theory of matter not yet delivered.[19]

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[18]Note that there are interior extensions of the Schwarzschild metric that model the interior of the Sun by solutions of the non-vacuum field equations (1), for example by an incompressible perfect fluid. See Bonnor [1992], section 5.

[19]If I had given more historical details, I could have, I believe, shown that Einstein himself saw the occurence of a singularity in the inner metric in exactly this way. This

But now wait a minute. You might have disliked the occurence of singularities as representing particles, but at least the singularity (in lieu of a non-vanishing energy-momentum tensor) gave you an idea of *where* in spacetime the particle was supposed to be. True, Earman and Tamir rightly pointed out that the singularity is not actually part of spacetime, and so it can hardly serve to localize the particle in spacetime. Still, you might think that we're throwing the baby out with the bath water by not choosing any inner metric. After all, is it not the case that then the curve we have been focusing on is just *any* curve, without any reason to think of this curve as the curve of a material particle?[20]

Again, I think we can counter this criticism by comparing the Einstein-Grommer approach to Einstein's treatment of the Sun-Mercury system in Einstein [1915]. What Einstein did there was to assume that Mercury would move on *some* geodesic of the exterior gravitational field produced by the Sun. He calculated an approximation to the external gravitational field of a static, spherically symmetric and asymptotically flat body; this gravitational field he saw as represented by the connection components $\Gamma^{\nu}{}_{\mu\sigma}$ of a metric $g_{\mu\nu}$ which deviated only slightly from the flat Minkowski metric. He then inserted these gravitational field components $\Gamma^{\nu}{}_{\mu\sigma}$ into the the geodesic equation (2). He showed that this law contained Newton's first law and Newton's second law with a gravitational potential giving rise to a force as a limiting case, and showed how the resulting Keplerian laws for orbits differ in his theory as compared to its Newtonian limit. In the end, he obtained that according to the new theory the perihelion ϵ of *any* geodesic orbit around the Sun is given by

$$\epsilon = 24\pi^3 \frac{a^2}{T^2 c^2 (1 - e^2)} \tag{5}$$

exegetical argument would have started with evidence that, from early on, he saw GR as a theory of the pure gravitational field without any constraints on what kinds of matter give rise to the gravitational field. Furthermore, I would have argued that even in the Einstein-Grommer paper he clearly forbids singularities *outside* of material particles (where the theory is supposed to give an adequate and deterministic representation of gravitational fields) but has no problem with them appearing *inside of* material systems, where the theory can provide at best phenomenological placeholders for a future 'proper' theory of matter anyhow. Thus, for Einstein energy-momentum tensors as alleged representatives of material systems were on a par with singularities: both were only placeholders for a proper theory of matter.

[20]I thank Jim Weatherall for putting this question to me.

Here a denotes the length of the semimajor axis of the orbit in question, e its eccentricity, c the speed of light, and T the orbital period of the planet in question. Einstein then *takes the astronomically known values for Mercury*, plugs them into equation (5), and thereby predicts that Mercury's perihelion changes by 43" per century.

Note that there is *nothing* in the theoretical description that singles out any particular path as that of Mercury. There is no theoretical representation of Mercury, no model. All that is there is the assumption that Mercury will move on one of the geodesics of the affine connection determined by the spherically symmetric field of the Sun. A general equation that all possible geodesic orbits have to fulfil is derived. And then *external knowledge* is used to single out one of these orbits as that of Mercury. Einstein trusts the astronomers that they have measured the orbital period, the semimajor axis and the eccentricity of Mercury correctly. It is this external knowledge, plugged into his theoretical model, which does not in itself contain a representation of Mercury or its path, that produces the prediction.

In many ways, the whole vacuum approach to the problem of motion is about the question whether in this kind of scenario we really have to assume the geodesic equation as the equation of motion of matter over and above the gravitational field equations. Indeed, let us look at the Sun-Mercury system within the 1927 Einstein-Grommer approach. The problem of motion, then, is the question whether Einstein really *had to* introduce the gravitational field equations (to describe the exterior gravitational field of the Sun) *and* the geodesic equation (to describe the path of Mercury subject to this gravitational field) as separate assumptions.[21] Could he have only assumed the gravitational field equations and *derived* that Mercury moves on a geodesic of the exterior field of the Sun? My point is that, just like in Einstein's 1915 treatment, the 1927 Einstein-Grommer approach does not *need to* commit to a theoretical model that allows us to localise Mercury internally. It is fine to ask whether the exterior gravitational field around a given curve 'forces' that curve to be a geodesic. Just like in the 1915 treatment, Einstein and Grommer could then use *external knowledge* about whether that particular curve is actually the curve of a material object, or of Mercury in particular.

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[21]Interestingly, Einstein did not yet have the final gravitational field equations in the Mercury paper; he found them a week later, in his fourth paper of November 1915. However, the approximation of the Schwarzschild metric that he uses in the Mercury paper is an approximative solution of both the field equations from the Mercury paper, and of the final Einstein field equations.

No inner metric, no singularity to represent the material body, is actually needed.

5   Conclusion

I started out by saying that whether we are realists or antirealists, we should aim for a careful interpretation, rather than a literal interpretation, of the scientific theory we want to be realists or anti-realists about. As a case study, I argued that the vacuum approach to the problem of motion in GR, and the Einstein-Grommer approach in particular, is far more sensible and promising if we interpret the singularities *not as representing* material bodies but as *placeholders* for a representation of material bodies that is not included in the model. Indeed, I argued that the approach does not even need the introduction of singularities to represent material bodies; their introduction does not do any work in answering the question at hand.

Given that in their paper Einstein and Grommer seem to take the singularities as representing material bodies, one might wonder whether this allegedly more careful interpretation does not fall prey to the criticism that the careful interpreter presumes to understand the theory/formalism in question better than its originators. This might seem at odds with the realist tenet of taking scientists and science 'seriously'. I do indeed think that putting the Einstein-Grommer paper into its proper historical context by analysing Einstein's correspondence leading up to the paper and by relating it to his overarching research project at the time *would* convincingly show that he subscribed to something very much like the 'placeholder interpretation' I defended above. Showing this in detail will have to wait for a much longer paper, and I do not ask the reader to just take my word for it. So let us say, for the sake of the argument, that Einstein and Grommer did indeed intend the singularities as representatives of material objects in a rather straightforward way. I believe that we should not take *their* word for it either. And neither did Einstein. Just a few years after the Einstein-Grommer paper, in his famed 1933 Spencer lectures at the University of Oxford, Einstein told us in his opening words: "If you wish to learn from the theoretical physicist anything about the methods which he uses, I would give you the following advice: Don't listen to his words, examine his achievements."[22]

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[22]See Einstein [1934], and van Dongen [2010] for a detailed analysis of the text.

In philosophy of science, I believe there is no better way of examining a scientists achievements than by looking for the best possible interpretation of his theories. To do that, we have to not just listen to the words of the scientist who created or discovered it; we have to see what the theory *does* in practice, how it is *used*; which of its parts really do the work.

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