

THE OBJECTIVE INDEFINITENESS INTERPRETATION OF QUANTUM MECHANICS: Partition logic, logical information theory, and quantum mechanics

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When Boolean logic is formulated at the right level of generality as the logic of subsets (rather than the logic of propositions), then it is seen that there is dual logic of quotient sets, equivalence relations, or partitions. Subsets and quotient sets are dual in the category-theoretic sense, and the duality between subobjects and quotient objects has long been evident in abstract algebra. The dual *logic of partitions* has only been recently developed and published [Ellerman, David 2010. The Logic of Partitions: Introduction to the Dual of the Logic of Subsets. *Review of Symbolic Logic*. 3 (2 June): 287-350].

When it was realized long ago that quantum mechanics was not compatible with Boolean logic, then the natural thing to do would have been to check if it was compatible with the dual logic of partitions (had that logic been known). To make a long story short, partition logic *is* the logic of the world described by quantum mechanics.

There is a lifting (linearization) program by which the set-theoretic machinery of partition logic can be "lifted" to complex vector spaces where it yields the mathematical machinery of QM (but not the specifically physical assumptions). Moreover, one can go the other way over this sets-to-vector-spaces bridge, and derive a non-metrical "quantum mechanics over 2" that captures the logical framework of QM without any of the metrical aspects (as one would expect logic to do). Since operations on sets can be formulated in vector spaces over 2, the QM-on-sets essentially distills out the role of the complex base field of Hilbert spaces to restate the logical core using vector spaces with base 2. This QM-on-sets is well-qualified to be the "logic of quantum mechanics."

By defining the normalized counting measure on subsets of a universe U , Boole developed finite "logical" probability theory out of the logic of subsets. The same mathematical moves can be applied in partition logic. Then the normalized counting measure on partitions defines new notion of information content or entropy, *logical entropy* [see: Ellerman, David 2009. Counting Distinctions: On the Conceptual Foundations of Shannon's Information Theory. *Synthese*. 168 (1 May): 119-149], that generalizes immediately to the notion of *quantum logical entropy*. This entropy notion is much better adapted to quantum mechanics than the usual von Neumann entropy. It allows the direct interpretation of the changes in the density matrix in a measurement. In a measurement that converts a pure state to a mixed state, the logical entropy is the sum of the absolute squares of the off-diagonal coherence terms that are zeroed by the measurement, whereas the vN entropy is only qualitatively related to measurement (i.e., increases in a projective measurement).

Putting these developments together allows a new interpretation of QM that can be viewed as the back-story to Shimony's emphasis on objective indefiniteness as well as to the standard view that a superposition description of a quantum state is a complete description, and which thus might be called the *objective indefiniteness interpretation* of quantum mechanics. The idea of objective indefiniteness (in one form or another) is old; it is the back story from partition logic and logical information theory that is new.