Gravitational and non-gravitational energy: the need for background structures

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Abstract

The aim of this paper is to discuss some aspects of the nature gravitational energy within the general theory of relativity. Some aspects of the difficulties to ascribe the usual features of localization and conservation to gravitational energy are reviewed and considered in the light of the dual of role of the dynamical gravitational field, which encodes both inertio-gravitational effects and the chronogeometrical structures of spacetime. These considerations will lead us to discuss the fact that the very notion of energy - gravitational or not - is actually well-defined in the theory only with respect to some background structure.

1 Introduction

Since it encodes both the inertio-gravitational effects and the space-time structure, the gravitational field has a very peculiar nature within the general theory of relativty (GTR). An important aspect of the peculiarity of the gravitational field lies in the nature of its energy (and momentum). Intuitively, and from a non-relativistic point of view, it does not seem very surprising that the gravitational field possesses some energy - we experience the transfer of gravitational energy into kinetic energy all the time in everyday life, every time we drop something on the floor for instance. Within GTR, because of the dual role of the gravitational field, it seems that gravitational energy could equally be understood as the energy of the space-time structure itself. As will be discussed in this paper, such understanding of gravitational energy may shed some interesting light on the difficulties to ascribe the usual (related) features of localization and conservation to gravitational energy.

I will review some well-known aspects of these difficulties (sections 2-4) and will discuss them in relation to fundamental issues such as conservation laws, uniqueness (section 5), background independence (section 6) and the nature and status of energy in general within GTR (section 7). The upshot of the discussion is to highlight the fact that the fundamental property of background independence, which is an aspect of the dual nature of the gravitational field, might consitute the real crux of the difficulties with energy localization; moreover, strictly speaking, the introduction of background structures is actually mandatory for the very definition of energy within GTR, then understood as a global notion. We

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first start to look at the way standard, non-gravitational energy is defined according to the theory.

2 Energy-momentum tensor

Within the special and general theories of relativity, non-gravitational (mass-)energy and momentum densities (as well as stress) of any physical system (matter fields indeed) are described by the (stress-)energy-momentum tensor field. The vacuum Einstein field equations can be derived by means of a variational principle from the Einstein-Hilbert action S_{EH}. Now if the matter field equations can also be derived from a Lagrangian density $\mathcal{L}_M$ (like in many physically interesting cases such as electrodynamics), then the energy-momentum tensor can be defined as the variation of $\mathcal{L}_M$ with respect to the metric

$$T_{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_M}{\delta g^{\mu\nu}} \tag{1}$$

so that the variation of the total gravity-matter action $S = S_{EH} + S_M$ with respect to the metric leads to the full Einstein field equations

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} \tag{2}$$

This is a very general definition of the energy-momentum tensor associated with any given matter fields, whose dynamics is derived from a Lagrangian density $\mathcal{L}_M$.[1] An important aspect of this definition is that it entails, assuming the matter field equations (the Euler-Lagrange equations for $\mathcal{L}_M$) and the diffeomorphism invariance of $\mathcal{L}_M$, that the covariant divergence of the energy-momentum tensor vanishes:

$$\nabla_\mu T^{\mu\nu} = 0. \tag{3}$$

By analogy to special relativity, this identity is often and loosely called energy-momentum 'conservation law'. Indeed, within special relativity, the vanishing of the ordinary divergence of the energy-momentum tensor

$$\partial_\mu T^{\mu\nu} = 0 \tag{4}$$

can be straightforwardly interpreted as a genuine conservation law in the sense that for any space-time region the total energy-momentum flux is zero (in rough terms, 'all that enters the space-time region goes out').[2] But such interpretation of the vanishing of the covariant divergence (3) of the energy-momentum tensor is in general not available within GTR. There are several interrelated ways to see this. From an intuitive point of view, a mere look at the rewriting of the covariant divergence (3) in terms

[1] Within special relativistic Lagrangian field theories, there is another way to construct an energy-momentum tensor, the so-called canonical energy-momentum tensor, using Noether's (first) theorem with respect to space-time translation invariance (see section 3); however, this procedure does not lead to a gauge invariant object in the presence of gravitation and the definition (1) of the energy-momentum tensor is the natural one deriving from the Lagrangian formulation of GTR, see Wald (1984, 455-457).

[2] This can be easliy shown using suitable inegration and Gauss' theorem; (3) and (4) are sometimes referred to as differential conservation laws and what I call genuine or proper conservation law on any space-time region is called integral conservation law (see section 3).

of ordinary divergence shows that there are additional terms due to the gravitational field:

$$\nabla_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu} + \Gamma^\mu_{\mu\lambda} T^{\lambda\nu} + \Gamma^\nu_{\mu\lambda} T^{\mu\lambda} = 0 \quad (5)$$

These terms, containing the metric connection coefficients (or Christoffel symbols) $\Gamma^\alpha_{\beta\gamma}$, can be understood as resulting from the interaction between the gravitational field and matter fields; there is a strong analogy between the Christoffel symbols and the gauge potential in non-abelian Yang-Mills theories (both are coefficients of a connection on the respective relevant principal fibre bundle), where the Yang-Mills covariant derivative $D_\mu \psi = (\partial_\mu + A_\mu)\psi$, resulting from the interaction between matter fields and a gauge field, is defined in complete analogy with the GTR covariant derivative $\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda$.

So, the vanishing of the covariant divergence (3) does not constitute a genuine conservation law since it contains interaction terms with the gravitational field, but does not account for any gravitational energy-momentum. In the light of the dual nature of the gravitational field, it might therefore be helpful to discuss this failure of (non-gravitational) energy-momentum conservation from the point of view of the space-time structure.

3 Conservation and symmetries

Indeed, energy and momentum conservation are fundamentally linked to invariant properties of the space-time structure. This can be seen within the Lagrangian and Hamiltonian formalism as an instance of Noether's first theorem, which states (without entering into the details) that if the action is invariant under a continuous group of (global) transformations that depend on p constant parameters, then there are p associated vanishing divergences $\partial_\mu j^\mu_{(k)} = 0$ of the so-called Noether currents $j^\mu_{(k)}$, $k = 1, \ldots, p$ (assuming that all Euler-Lagrange equations are satisfied); these p Noether currents naturally lead to p conserved quantities ('charges') by suitable integration using Gauss' theorem (see below). For instance, within special relativity, for an action that is invariant under the Poincaré group, which has ten parameters, there are then ten conserved quantities, among which energy, linear and angular momentum. Energy and linear momentum conservation is specifically associated with the invariance under temporal and spatial translations.[3] So energy(-mometum) conservation does not only depend on properties of the matter fields but is also deeply related to symmetries of the space-time structure. Such symmetries (isometries indeed) can be characterized by their generators, that is, by their associated Killing vector fields.[4] The existence of a genuine (that is, integral) energy(-mometum) conservation law is then bound to the existence of a timelike Killing vector field, since this latter is the

[3]The canonical energy-momentum tensor (see footnote 1) can be understood as the Noether currents associated with the space-time translations; see for instance Wald (1984, 457).

[4]A vector field K^μ on $(M, g_{\mu\nu})$ is a Killing vector field if the associated Lie derivative of the metric vanishes, $\mathcal{L}_K g_{\mu\nu} = 0$, that is, if it is associated with a one-parameter group of isometries (the vanishing of the Lie derivative of the metric with respect to K^μ means that the metric 'does not change' along the integral curves of K^μ); one can write $\mathcal{L}_K g_{\mu\nu} = \nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$, where ∇_μ is the derivative operator associated with $g_{\mu\nu}$. One can show that Killing vector fields form a Lie algebra. The Killing vector fields of Minkwoski space-time generate the Lie algebra of the Poincaré group.

generator of an isometry that can be undertsood in terms of temporal translation invariance: the metric 'does not change' along the timelike integral curves of the Killing vector field (such space-times are called stationary). More precisely, if a space-time $(M, g_{\mu\nu})$ possesses a timelike Killing vector field K^μ, then one can build the quantity $T^{\mu\nu}K_\nu$, which represents the energy-momentum current density relative to K_ν and such that its divergence vanishes:

$$\nabla_\mu(T^{\mu\nu}K_\nu) = (\underbrace{\nabla_\mu T^{\mu\nu}}_{=0})K_\nu + T^{\mu\nu}(\nabla_\mu K_\nu) \underbrace{=}_{T^{\mu\nu}=T^{\nu\mu}} \frac{1}{2}T^{\mu\nu}(\underbrace{\nabla_\mu K_\nu + \nabla_\nu K_\mu}_{=0}) = 0.$$

And this allows us to write for the 3-dimensional boundary S of any 4-dimensional volume V, using Gauss' theorem,

$$\int_{S=\partial V} T^{\mu\nu}K_\nu N_\mu dS = \int_V \nabla_\mu(T^{\mu\nu}K_\nu)dV = 0 \tag{6}$$

where N_μ is the unit normal to S (with standard 'outward pointing' orientation) and dS and dV are 3- and 4-volume elements. Now the integral formulation (6) genuinely means (non-gravitational) energy-mometum conservation (with respect to the intergal curves of the timelike vector field K^μ): for any 4-dimensional space-time volume V, all energy-momentum that enters V is balanced by what goes out (change in time equals the flux), which precisely means that energy-mometum is conserved. Another way to consider this is to see that the quantity of non-gravitational energy in a spatial (3-dimensional) region R contained in a spacelike hypersurface Σ, defined by

$$E[R] = \int_{R\subset\Sigma} T^{\mu\nu}K_\nu N_\mu d\Sigma, \tag{7}$$

is conserved (it does not depend on the choice of Σ).[5]

Such conservation law seems to be fundamental for the very notions of mass, energy and momentum (and it plays indeed a fundamental role in pre-general relativistic physics): conservation is what makes these notions so fundamental, at least to matter (according to the Newtonian characterization of matter for instance), since it grounds a notion of persistence.[6]

Now (6) shows that we can have genuine (non-gravitational) energy-momentum conservation in the presence of a gravitational field, that is, in curved space-time, but only in the cases where this latter instantiates certain global symmetries, namely when the space-time structure remains unchanging (stationary) along the integral curves of the timelike Killing field. This is a very peculiar feature indeed, which can be very useful in relevant approximations of many physical situations (from everyday laboratory life to Kerr-Newman black holes), but which is in general (and globally) not instantiated in our evolving universe.

One can analyse this failure of integral non-gravitational energy-momentum conservation from two interrelated points of view. First, the timelike Killing vector field associated with a stationary space-time structure can be understood in a certain sense as a non-dynamical background structure

[5] See for instance the discussion in Wald (1984, 62-63 & 286) and Jaramillo and Gourgoulhon (2010, §1.1).

[6] Norton (2000, 19) clearly summarizes the point: "The distinctive property of matter is that it carries energy and momentum, quantities that are conserved over time. A unit of energy cannot just disappear; it transmutes from one form to another, merely changing its outward manifestation. This property of conservation is what licenses the view that energy and momentum are fundamental ontologically."

with respect to which integral non-gravitational energy-momentum conservation can be obtained; indeed, a timelike Killing vector field can be understood as defining a global inertial frame, which can represent a global family of inertial observers all at rest with each other. A fully dynamical metric field would prevent the existence of such global symmetries. So, in other terms, certain fundamental aspects of the space-time structure itself prevent non-gravitational energy-momentum conservation. Second, as already mentioned above, from the point of view of the universally interacting gravitational field, the failure of integral non-gravitational energy-momentum conservation is an obvious consequence of not taking into account gravitational energy: strictly speaking, there cannot be non-gravitational energy-momentum conservation since any material system interacts with the gravitational field and its energy can transform into gravitational energy and vice versa. Because of the fundamental dual nature of the gravitational-metric field, these two points of view are indeed equivalent. In this sense, the nature of gravitational energy seems to be linked to the failure of certain global symmetries and, most importantly, to the lack non-dynamical background structures, that is, to background independence.

4 Pseudotensors and real ambiguities

So, it seems natural to ask whether the gravitational field possesses some energy(-momentum) that can also be described by an energy-mometum tensor, written $t^{\mu\nu}$, such that the total (matter and gravitational) energy-mometum is conserved; in differential form, this means that the ordinary divergence of the total energy-momentum $\mathcal{T}^{\mu\nu} := T^{\mu\nu} + t^{\mu\nu}$ vanishes,

$$\partial_\nu \mathcal{T}^{\mu\nu} = \partial_\nu (T^{\mu\nu} + t^{\mu\nu}) = 0 \tag{8}$$

so that integral conservation can be obtained. $t^{\mu\nu}$ can be defined using this conservation requirement and the Einstein field equations. Indeed, the vanishing of the ordinary divergence of the total energy-momentum tensor $\mathcal{T}^{\mu\nu}$ (8) can be encoded in terms of an anti-symmetric (so-called) 'superpotential' $U^{\mu\lambda\nu} = U^{\mu[\lambda\nu]}$, so that one can write

$$16\pi \mathcal{T}^{\mu\nu} = 16\pi (T^{\mu\nu} + t^{\mu\nu}) := \partial_\lambda U^{\mu\lambda\nu} \tag{9}$$

where (8) is guaranteed by the antisymmetry of the superpotential. Using Einstein field equations, $t^{\mu\nu}$ can then be defined in the following way:[7]

$$16\pi t^{\mu\nu} := \partial_\lambda U^{\mu\lambda\nu} - 2G^{\mu\nu} \tag{10}$$

where $G^{\mu\nu}$ is Einstein's tensor (2). There is a freedom in the choice of the superpotential (the conservation law (8) does not define $U^{\mu\lambda\nu}$ in an unambiguous way), which leads to distinct non-equivalent expressions for $t^{\mu\nu}$, such as the ones of Einstein and Landau-Lifschitz among others. Another way to understand the formal defintion (10) is to consider $t^{\mu\nu}$ in terms of the nonlinear corrections to the linearization of Einstein's tensor: from the point of view of the linearized theory, gravitational energy-momentum is encoded in the gravitational (nonlinar) corrections.[8]

[7] The conservation law (8) and this definition can be understood in the light of Noether theorems, see Trautman (1962) and Goldberg (1980).

[8] The definition (10) can be written $16\pi t^{\mu\nu} := \partial_\lambda(\partial_\alpha H^{\mu\lambda\nu\alpha}) - 2G^{\mu\nu}$, where $H^{\mu\lambda\nu\alpha}$ is defined in terms of the field $h_{\mu\nu} := g_{\mu\nu} - \eta_{\mu\nu}$ such that $\partial_{\lambda\alpha} H^{\mu\lambda\nu\alpha}$ is a linearization of $G^{\mu\nu}$ (expansion to linear order in $h_{\mu\nu}$); see Misner et al. (1973, §20.3).

Now, the important point to highlight is that the definition (10) of the geometric object $t^{\mu\nu}$ depends on a local coordinate system (partial derivatives are defined with respect to a coordinate system): $t^{\mu\nu}$ does not represent an invariant entity under coordinate transformations, so that it is not a tensor (it is called a pseudotensor)[9]. This means that $t^{\mu\nu}$ does not describe the amount of local gravitational energy-momentum in an unambiguous and unique way: it depends on the choice of a coordinate system.[10] In particular, at any spacetime point or along any worldline, there is a coordinate system in which $t^{\mu\nu}$ vanishes. Since different coordinate representations are just different mathematical descriptions, relevant physical entities are usually taken to correspond to coordinate-independent entities (which can be represented in different coordinate systems, then providing different descriptions of the same physical entity - clearly assuming some uniqueness about the latter, see section 5). So, according to this understanding, the coordinate (or background or gauge) dependence of $t^{\mu\nu}$ shows that there is indeed no local (unique) gravitational energy-momentum, in the sense that such quantity cannot be in general unambiguously defined at any spacetime point.

It should be intuitively clear that this diffculty of localizing gravitational energy has actually an impact on all forms of energy, since one form of energy can transform into another. As a consequence of the above discussion, the amount of gravitational energy-momentum present in any given spacetime region (and not only at spacetime points) cannot be unambiguously defined in the general case. This ambiguity is valid for total energy-momentum as well, since the integral conservation law that can be derived from (8) for any spacetime region is obviously coordinate-dependent too. In particular, it is important to emphasize that non-gravitational energy (and mass) as well cannot be unambiguously defined in the general case: in the absence of the relevant symmetries, the definition of matter energy (7) in the spatial region R is not unique and not conserved (it now depends on the choice of Σ).

5 Non-genuine or non-unique?

If one considers that the satisfaction of a proper conservation principle is an essential aspect of the definition of energy (see footnote 6), then it is clear from the discussion above that gravitational energy is not a genuine form of energy. Hoefer (2000) seems to defend such a view. He explicitely claims that, due to the lack of a tensorial definition of gravitational energy as well as the lack of proper energy-momentum conservation in GTR, gravitational energy is not fundamental. As a consequence, he argues that the standard argument in favour of spacetime substantivalism based on the attribution of some form of energy to spacetime itself is not sound. In the light of the difficulties to define in any spacetime region gravitational and non-gravitational energy-momentum, this objection is actually rather weak: we are entitled to believe (or not) that the gravitational field or spacetime itself possesses some energy to the same extent that we believe

[9]Wald (1984, 292) defines that "a *pseudotensor field* is a tensor field which requires for its definition additional structure on spacetime, such as a preferred coordinate system or a preferred background metric". This should not be confused with tensor densities, which are also sometimes called pseudotensors.

[10]It is indeed possible to define $t^{\mu\nu}$ in a coordinate independent way, but only by rendering it dependent on other arbitrary background entities.

(or not) that fundamental material entities such as matter fields possess some energy (some mass). So if one wants to hold that material entites do possess some well-defined energy, one needs to account for gravitational energy and the lack of proper energy conservation law.

Pitts (2009) recently discusses such an account based on the possible non-uniqueness of gravitational energy-momentum. The idea is that the difficulties linked with the definition of (local) gravitational energy-momentum and the lack of proper energy conservation can be removed if one drops the assumption of uniqueness for gravitational energy-momentum: we simply have to accept that there are infinitely many (local) gravitational energies, which can be represented by a mathematically precisely defined infinite-component (geometric) object. As discussed above, this move explicitly recognizes that any definition of (local) gravitational energy needs additional background structures such as a (flat) background metric or a coordinate system for instance: gauge invariance is then restored by collecting all the background-dependent expressions for a given background structure into one (infinite-component) object. For instance, such object can be constituted by the expressions for Einstein's pseudotensor in all coordinate systems; contrary to the different sets of components of a tensor in different coordinate systems, the different sets of components of this object in different coordinate systems do not constitute different descriptions of the same entity, but distinct entities, namely distinct but conserved energies (since (8) holds in the corresponding coordinate system).

Since the conceptual importance within GTR of coordinate-free methods have been widely recognized, it might seem a bit awkward to rely so heavily on background-dependent (coordinate-dependent) entities. However, as such, there does not seem to be any contradiction in such move. Indeed, it has the virtue of emphasizing the importance of additional background structure to make sense of gravitational energy (as well as of the very notion of energy itself - see section 7). What remains however obscure is the physical improvement brought by and the physical meaning of such infinite-component object. In particular, the definition of (matter or gravitational) energy contained in a given spatial region R remains ambiguous in the sense of non-unique (trivially, since the very assumption of uniqueness has been dropped): one cannot say how much energy there is in a given spatial region R (in the general case), since each component may provide a distinct answer. It might be the case that looking for such energy quantity is fundamentally hopeless since, as a matter of principle, local gravitational energy cannot be unambiguously defined.

6 From equivalence principle to background independence

Indeed, the ambiguity about local gravitational energy-momentum is in general taken in the litterature (at least in most GTR textbooks) as a consequence of some 'infinitesimal equivalence principle'. According to this principle, in any infinitesimal spacetime region, the gravitational field, and therefore gravitational energy-momentum, can always be transformed away by a suitable choice of coordinate system. The idea is that in considering infinitesimal spacetime regions, any arbitrary gravitational field can be considered homogenous and transformed away in virtue of the equivalence of inertial reference frames and uniformly accelerated ones. Despite its heuristic force, one should be a bit wary here of the significance of this argument. The notion of infintesimal region and the sense in which the gravitational field and above all its energy-momentum can be transformed away are not clear:[11] obviously, there is in general no coordinate system in which the components of the curvature tensor and the second derivatives of the metric components can be made to vanish. In other words, tidal (or 'geodesic deviation') effects, which are often invoked in examples aiming to show that the gravitational field possesses some substantial energy, cannot be made to vanish. Simply ignoring second and higher derivatives of the metric components by invoking sufficiently small (infinitesimal) regions (that is, no proper neighbourhoods) is indeed problematic, since such restriction does not allow to distinguish between arbitrary smooth curves and geodesics any more.

I suggest to consider rather the fundamental GTR property (call it 'general equivalence principle' if you like) according to which the metric or gravitational field $g_{\mu\nu}$ accounts for both inertial and gravitational effects. More precisely, the fact that the metric-gravitational field cannot be decomposed uniquely into an inertial (non-dynamical) part plus a gravitational (dynamical) part is called 'background independence'.[12] This property is taken by many physicists and philosophers of physics to be the distinctive feature of GTR and one of the main difficulties towards a cohrent view between GTR and quantum field theory (QFT), which relies on a non-dynamical spacetime background. Background independence (and diffeomorphism invariance in particular) forces us to modify our common notion of localization with respect to a non-dynamical background, in particular background spacetime points (represented by manifold points) within field theories. In the framework of GTR, physical entities can only be meaningfully localized with respect to the dynamical gravitational field and more generally (Dirac) observables of the theory might only be meaningfully defined in terms of relations between dynamical variables. From this point of view, the trouble with gravitational energy-momentum localization might well be linked to the fundamental GTR feature of background independence: there is no (non-dynamical) background with respect to which gravitational (and non-gravitational indeed) energy-momentum can be defined and localized. For practical purposes, the introduction of some background structure seems mandatory for defining and localizing gravitational (and non-gravitational) energy-momentum. The case is actually similar to the possibility of defining meaningful time evolution and initial value formulation within GTR: some 'background gauge fixing' in the form of a $3 + 1$ decomposition (together with the choice of some coordinate system) is required in order to implement any useful calculation. But this does not mean that such decomposition or such background structure is fundamental.

[11] This has been explicitly discussed in the philosophy of physics litterature by Norton (1985).

[12] Background independence can also be characterized in terms the absence of 'absolute' objects or structures, which are defined in a precise sense, incoding the fundamental diffeomorphism invariance of the theory, see recently Giulini (2007).

7 Asymptotic symmetries and quasi-localization: the need for background structures

In the general case, one cannot define unambigously the amount of total (gravitational and non-gravitational) energy(-momentum) there is in any spacetime region; obviously, there is no question of conservation for the (ill-defined) total energy 'contained in' a given spacelike 3-volume. Now, if one takes this conclusion seriously, that is, if one takes GTR 'seriously', it may well have important metaphysical consequences: according to some standard conception, conservation is what makes the very notions of energy and mass (and momentum) so fundamental (recall the quotation in footnote 6). In the light of the difficulties with energy conservation as well as with the very notion of energy itself, this standard conception leads us to the conclusion that energy and mass (and momentum) may not be fundamental properties (within GTR).

To be more precise, strictly speaking, these notions make only sense once appropriate background structures are introduced: especially useful for practical purposes is for instance the notion of asymptotic flatness, which encodes the notion of isolated system within GTR and where one therefore assumes that spacetime becomes flat as one 'approaches' (spatial or null) infinity. The relevant asymptotic symmetries with the corresponding conserved quantities at (spatial or null) infinity can then be defined, namely the Arnowitt-Deser-Misner (ADM) and Bondi energies. The ADM energy associated with some (asymptotically flat) spacelike hypersurface Σ_{t_1} (with respect to some foliation) represents the total energy 'contained' in (whole) Σ_{t_1} and is conserved under the 'time' evolution $\Sigma_{t_1} \rightarrow \Sigma_{t_2}$. These notions of energy depend crucially on the background structures inserted in the form of asymptotic symmetries and are fundamentally global in the sense that they are associated with the whole (asymptotically flat) spacetime in definitive.

The 'quasi-local' research program hopes for several decades to remedy this last point with its various attempts to define energy-momentum for extended but finite spacetime regions, such as, typically, a compact spacelike (3-dimensional) region (or its closed 2-dimensional boundary). There are mainly two different ways to define quasi-local energy: first, using the variational method within the Lagrangian or Hamiltonian setting and second, trying to 'quasi-localize' global expressions such as the ADM energy. Unfortunately, these different methods lead to different expressions (Brown-York energy, Bartnik mass, ...), the links between which are not always clear; as acknowledged in the recent review of Szabados (2009) on the topic, there is for the time being no consensus on how to build a 'quasi-local' notion of energy, that is, how to define the energy or mass associated with some finite spacetime region. More importantly, all the quasi-local expressions depend on particular background structures (or gauge choices), such as the dependence to some particular embedding or to some particular boundary conditions for instance.[13]

As a conclusion, I would like to highlight two points whose metaphysical implications need to be further investigated: first, in the cases where total energy-momentum is well-defined (and conserved), it is a global notion. Second, it seems that within GTR all meaningful notions of (gravi-

[13]Indeed, there is a relationship between quasi-local quantities that are constructed within the Hamiltonian approach and pseudotensors, which are coordinate-dependent entites (see section 4); see Chang and Nester (1999).

tational and non-gravitational) energy-momentum (and mass) require the introduction of some background structures,[14] which correspond to some particular ways of describing the world (gauge choices), but which might not be intrinsic to it. That is, energy and mass might not be fundamental properties of the world, in the sense that they make only sense in some particular (but very useful) settings; this does not lessen the fact that the notions of (local, quasi-local or global) energy and mass constitute extremely powerful tools for many concrete and practical cases.

[14] I fully concur with the conclusion of Jaramillo and Gourgoulhon (2010).

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