Unification in Field Theories

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Abstract

What precisely does it mean to unify fields like the electric and magnetic field to only one field? Are there different kinds of unification? Is there only 'unified' and 'not unified', or could a unififcation of fields also be partially succesful?

I will argue that what is normally referred to as the project of a unified field theory is actually a bundle of three research programmes that are logically independent of each other. The sub-programmes are those of a unified field theory (in a narrow sense), a complete field theory, and a geometrised field theory.

Here I will focus on the programme of a unified field theory in a narrow sense. In particular, I will make use of an almost unknown debate between Einstein and Pauli about which criteria a unification of fields should fulfil, in order to distinguish between different degrees of unification in field theories.

Contents

1 Introduction

It is often claimed that the Einstein field equations of General Relativity Theory (GR)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa_E T_{\mu\nu} \tag{1}$$

describe how spacetime geometry (left hand-side of the equation) and matter content (right hand-side of the equation) interact with each other. However, the mass-energy-momentum tensor $T_{\mu\nu}$ should be seen as describing a key *property* of material systems, rather than representing the material system itself.[1] Indeed, knowing the components of a mass-energy-momentum tensor does not in general tell us what *kind* of material system is present, just as somebody telling us the colour of an animal (and nothing else) does not in general tell us the type of animal in question.[2] An energy-momentum tensor can give us a representation of the energetic properties of fields like the electromagnetic field, of perfect and viscous fluids, and even of single particles.

On the one hand that makes the energy tensor a powerful representational tool, on the other hand it does not give us much help in determining what the world is made up of at the fundamental level.

This weakness of the energy tensor is one reason why Einstein considered GR as merely a stepping stone. In Einstein [1949], p. 74, he writes:[3]

> The right side [the energy-momentum tensor] is a formal condensation of all things whose comprehension was still problematic in a field-theoretic manner. Of course, I did not doubt for a second that this was only a makeshift in order to give the general principle of relativity a preliminary closed expression. After all, it

[1] See Lehmkuhl [forthcoming] for an argument to this extent; the article also argues that $T_{\mu\nu}$ should be seen as an *esential* and *relational* property of material systems.

[2] See Tupper [1981].

[3] "Die rechte Seite [der Energie-Impuls Tensor] ist eine formale Zusammenfassung aller Dinge ..., deren Erfassung im Sinne einer Feldtheorie noch problematisch war. Natürlich war ich keinen Augenblick im Zweifel, dass diese Fassung nur ein Notbehelf war, um dem allgemeinen Relativitätsprinzip einen vorläufigen geschlossenen Ausdruck zu geben. Es war ja nicht wesentlich *mehr* als eine Theorie des Gravitationsfeldes, das einigermassen künstlich von einem Gesamtfelde noch unbekannter Struktur isoliert wurde". (My translation.)

> was essentially not *more* than a theory of the gravitational field, which was somewhat artificially isolated from a total field of as yet unknown structure.

I think that it is in this context that a much more famous quote of Einstein has to be read, namely the one from Einstein [1936], p.370:[4]

> [General Relativity] is sufficient — as far as we know — for the representation of the observed facts of celestial mechanics. But it is similar to a building, one wing of which is made of fine marble (left part of the equation), but the other wing of which is built of low-grade wood (right side of equation). The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter.

This quotation is often interpreted as expressing Einstein's dissatisfaction at the missing geometric significance of $T_{\mu\nu}$ in GR, and it was claimed that this was his motivation for striving for a unified field theory of gravitation and electromagnetism. One manifestation of this interpretation can be found in one of the core texts on the history of unified field theories, namely in Goenner [2004], p.7:

> In view of this geometrization, Einstein considered the role of the stress-energy tensor T^{ik} (the source-term of his field equations $G_{ik} = T_{ik}$) a weak spot of the theory because it is a field devoid of any geometrical significance.

But Einstein also wrote, in a letter to Lincoln Barnett from June 19, 1948 (Document 6-058.1 of the Einstein Archives):[5]

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[4] [Die allgemeine Relativitätstheorie] gleicht aber einem Gebäude, dessen einer Flügel aus vorzüglichem Marmor (linke Seite der Gleichung), dessen anderer Flügel aus minderwertigem Holz gebaut ist (rechte Seite der Gleichung). Die phänemologische Darstellung der Materie ist nämlich nur ein roher Ersatz für eine Darstellung, welche allen bekannten Eigenschaften der Materie gerecht würde.

[5] The English version is the one actually sent; it differs from the German draft in some points: "Ich kann nicht mit der weitverbreiteten Auffassung übereinstimmen, dass die allgemeine Relativitätstheorie die Physik "geometrisiere". Die Begriffe der Physik sind nämlich von jeher "geometrisch" gewesen, und ich kann nicht sehen, warum das g_{ik}-Feld

> I do not agree with the idea that the general theory of relativity is geometrizing Physics or the gravitational field. The concepts of Physics have always been geometrical concepts and I cannot see why the g_{ik} field should be called more geometrical than f.[or] i.[nstance] the electromagnetic field or the distance of bodies in Newtonian Mechanics. The notion comes probably from the fact that the mathematical origin of the g_{ik} field is the Gauss-Riemann theory of the metrical continuum which we are wont to look at as a part of geometry. I am convinced, however, that the distinction between geometrical and other kinds of fields is not logically founded.

What is happening here? What was the shortcoming of the representation of matter in GR according to Einstein? What could be expected from a unified field theory that would overcome this shortcoming? And is it related to geometrisation — or not?

2 Three kinds of Unified Field Theories

The articles published by physicists working on unified field theories, and by historians and philosophers publishing about unified field theories implicitly refer to three logically *independent* research programmes rather than one: the programmes aiming for a *unified field theory* (in the narrow sense), a *complete field theory* and a *geometrised field theory*. Once this classification is in place, and has been made precise (see below), one can see that a given unified field theory can do very well with respect to one of these programmes and rather badly with respect to another — and how Einstein could have strived for a unified field theory while denying the importance of geometrisation. In order to distinguish between a unified field theory in the wide sense and one in the narrow sense, I will just speak of the former by using the abbreviated form 'UFT'; whenever I speak of a 'unified field theory' I refer to the narrow sense

"geomtrischer" sein soll als das elektromagnetische Feld oder die Distanz von Körpern in Newtons Mechanik. Wahrscheinlich stammt die Ausdrucksweise aus dem Umstand, dass das g_{ik}-Feld seinen mathematischen Ursprung (Gauss, Riemann) Begriffen entstammt, die man als geometrisch zu betrachten gewoht ist. Genauere Überlegung zeigt aber, dass die Unterscheidung zwischen geometrischen und anderen Feldbegriffen sich nicht aufrecht erhalten lässt objektiv begründen lässt."

of the term.[6]

Let us assume that we start with a physical theory that has two or more fundamental fields in its ontological inventory, be it the electric and the magnetic fields or the electromagnetic and the gravitational fields. I will call a new theory a *unified field theory* (in the narrow sense) of these two fields if it shows that what was formerly thought of as two fields are rather two aspects of one and the same field (in a sense to be specified). I will call a theory a *unitary field theory* if the process of unification leaves only one fundamental field in the ontological inventory, for example a gravitational-inertial-electro-magnetic field; a unitary field theory is thus 'the best kind' of unified field theory.

But note that not every unified field theory is a unitary field theory, we may have weaker degrees of unification: relativistic electrodynamics is a unified field theory of the electric and magnetic fields, but it is not a unitary field theory. For even in a world in which electrodynamics was a complete account of the physical state of affairs we could still allow for more than one fundamental field: the electromagnetic field represented by the Faraday tensor $F_{\mu\nu}$ and the vector field j^{μ} giving an account of the sources of the field, not to mention the field(s) needed to account for spacetime structure.

We could also allow for particle-like sources of the field. A separate aim from that of a unified field theory is thus what I will call that of a *complete field theory*, a theory in which *only* fields are part of the fundamental ontological inventory of the world, and in which particle-like matter emerges from the ontologically primary fields.[7] A complete field theory does not have to be a unified or a unitary field theory: one could have ten fundamental fields so long as particles (and other kinds of entities) are derivative of the fields and the laws that govern them. Neither General Relativity nor (standard relativistic) electrodynamics are complete field theories, for particles are allowed to be ontologically independent of the fields, and act as their sources. It seems that what Einstein *really* wanted was a *complete and unified* field

[6]The landscape of classical UFTs alone is vast, let alone quantum UFTs. Standard works on the history and structure of Classical UFTs are Vizgin [1994], Goldstein [2003], Bergia [1993], Goenner [2004], Sauer [2009], Pauli [1958], Lichnerowicz [1955], Tonnelat [1966], Goenner and Wünsch [2003], van Dongen [2004] and Goenner [2009].

[7]This sense of 'complete field theory' is different from another kind of theory one could denote thus, namely a theory according to which (if it was describing the real world accurately) there are no particles in the first place; I will term such a theory a '*pure* field theory'.

theory, if possible a unitary one, and that, contrary to folklore, he did not care too much whether the respective theory would also be a geometrised field theory.[8]

Other workers in the field primarily strived for this third subprogramme, and, arguably falsely, regarded Einstein as their champion: Weyl and Kaluza, for example, strived for a geometrised field theory. I will call a theory a *geometrised field theory* if all the fields in question have geometrical significance in a sense to be specified, similar to the geometric significance the Riemann curvature tensor has in standard GR. Again, a geometrised field theory does not need to be unified or complete: if the Faraday tensor $F_{\mu\nu}$ had a geometric significance similar to the Riemann tensor $R_{\mu\nu\sigma}{}^{\omega}$, then the Einstein-Maxwell equations would constitute a non-unified and non-complete, but geometrised field theory.

We have found three orthogonal categories of field theories that are normally summarised under one heading: unified, complete and geometrised field theories. Within each of these categories, one could furthermore distinguish different 'degrees'; one theory may be more unified, more complete or more geometrised than another.

In the following section I will be concerned with the different degrees of unification of fields attainable.

3 Six Kinds Unification in Field Theories

In his article 'On the Unification of Physics', Maudlin [1996] names the (special) relativistic theory of electromagnetism and the general theory of general relativity two examples for 'perfect unification' of fields that had previously been thought of as separate. About special-relativistic electrodynamics he writes that (p. 132)

> the formerly distinct electric and magnetic fields are so commingled that a more complete integration is impossible to imagine'. [...] It is not that the electric field is reduced to the magnetic,

[8]Indeed, the above quote seems to suggest that Einstein may have seen the idea of 'geometrisation' as a rather trivial concept. However, these claims about Einstein are historical claims, while the current paper's main aim is to clarify a systematic question, namely: What should we take 'Unification' to mean in the context of field theory. The historical claims will be argued for in Lehmkuhl and Sauer [forthcoming].

> but that both are shown to be merely frame-dependent artifacts, inessential and misleading ways to describe the single objective reality, [the electromagnetic field]. [...] Electric and magnetic fields *are not objectively real*, they "arise" only when one chooses a certain reference frame relative to which the phenomena are to be described.

Maudlin also states the possibility that the unification of the electromagnetic field and the weak nuclear field within Glashow-Weinberg-Salam theory is a unification not as perfect as that achieved in special relativitic electromagnetism (unification of electric and magnetic field) and general relativity (unification of inertial and gravitational field): Maudlin hints at the possibility of distinguishing between different *degrees* of unification in field theory.[9]

In the following, my aim will be to eleborate on this idea. As a means to this end, I will report and expand on a hitherto almost unknown debate between Einstein and Pauli (with contributions by Weyl and Jordan). For even though it is true that Pauli was an adamant follower of quantum mechanics and (one form of) the Copenhagen interpretation, as well as a merciless critic of Einstein's quest for a classical unified and complete field theory, he may also have been the only founding father of quantum mechanics who also kept up to date with attempts towards a classical unified field theory. What is almost unknown is that Pauli was skeptical with regard to some aspects of Einstein's research programme, in particular to what I have called the subproject of a complete field theory, but he was sympathetic towards others, namely that of unification and even that of geometrisation (in the weak sense of focusing on differential geometry as a tool). In a letter to Fierz from July 1953, Pauli writes:[10]

> I think of [Einstein's] obsession with differential geometry insofar as justfied in that *all* "forces" (not only those of gravity but *also* those of electromagnetism *and* the nuclear ones) demand a geometrisation (wherefore then space + time have to extended).

[9] As Maudlin correctly points out, one can doubt whether GR actually constitutes a unification of 'gravitational and inertial field' or rather a reduction of gravity to (now) dynamical inertial structure.

[10] See Meyenn [1999], p.187 (my translation)

> "Meschugge" — like Pais says — is only Einstein's idea of reducing the quantum of action and the quantum of electricity to classical fields.

However, much earlier Pauli had started to criticise Einstein not only for striving for a *complete classical field theory* per se, but also for the *way* in which he strived for a unified field in the narrow sense. Pauli argued that Einstein (and before him Kaluza) did not do justice to their own aims: he thought that Einstein's late attempts towards a unified field theory of the electromagnetic and the gravitational field would constitute only an *apparent* rather than a *proper* unification, even if all of Einstein's hopes for the approach in question would be fullfiled. Interestingly, Einstein conceded at least to some extent: in Einstein [1945], p.578, he agrees that even a theory that managed to carry through his ansatz to its end would be "unified only in a limited sense". The difference between Einstein and Pauli in this regard was that Einstein regarded this 'unification in a limited sense' as something still worth striving for, while Pauli did not.

In the following two subsections, I will introduce Einstein's and Pauli's criteria for a 'good' unified field theory, respectively. I will argue that the criteria are richer than Einstein and Pauli stated: Einstein's criterion allows to distinguish between two different degrees of unification of fields, Pauli's criterion allows for three more degrees of unification, each of which more powerful than those introduced by Einstein's criterion. In the next section, I will then apply these criteria to special-relativistic electromagnetism (unification of the electric and the magnetic field), Glashow-Salam-Weinberg theory (weaker unification of the electromagnetic and the weak nuclear field), and Kaluza-Klein theory (no unification at all of the gravitational and the electromagnetic field).

3.1 Einstein's criterion for Unification: Unification Strength 2 and 3

What I will call Einstein's criterion for a 'good' unified field theory is already present in Einstein's first paper on a UFT approach, a paper co-written with Jakob Grommer (Einstein and Grommer [1923]). In the opening page, Einstein and Grommer point out what they see as the disadvantages of Weyl's theory. Apart from claiming that the theory is "not doing justice to the independence of rods and clocks, respectively atoms, of their prior history", they write:[11]

> Probably the most important current question of the general theory of relativity is the one of the essential unity of the gravitational and the electromagnetic field. [...] [Weyl's theory] does not get rid of this dualism, since his Hamiltonian function [i.e. his Lagrangian] is composed by addition of two terms, one electromagnetic and one gravitational, which are logically independent of each other.[12]

What made Einstein and Grommer want a Lagrangian that is not a sum of two terms? It seems fair to say that what must have driven them is the aim to get (from this Lagrangian) *one* set of field equations, one set for a unified field rather than two sets of field equations, one for the electromagnetic and one for the gravitational field. Let us see whether one form or other of Einstein's criterion gives us such a set of gravi-electric field equations.

Einstein and Grommer do not only demand that it must be possible for the Lagrangian of a properly unified field theory to be written in a non-decomposed form. That is trivially possible, just set $\mathscr{L}_G + \mathscr{L}_{EM} := \mathscr{L}_{\text{total}}$. What they seem to have in mind is to demand a theory with a non-decomposed Lagrangian which cannot be decomposed *in such a way* that it would consist of an 'electromagnetic term' and a 'gravitational term'. The latter would be the case if under certain assumptions corresponding to the other field being absent (or, in the case of the metric field, it being static) we get 'free' field equations for each of the two fields.

For example, the Lagrangian for the Einstein-Maxwell equations can be decomposed into a sum of two terms, one of which we regard as the gravitational part of the total Lagrangian and the other as the electromagnetic

[11] "Wohl die wichtigste gegenwärtige Frage der allgemeinen Relativitästheorie ist die nach der Wesenseinheit des Gravitationsfeldes und des elektromagnetischen Feldes. [...] [Die Weyl'sche Theorie] beseitigt ferner insofern jenen Dualismus nicht, als sich ihre Hamilton'sche Funktion additiv aus zwei Teilen, einem elektromagnetischen und einem gravitationellen, zusammensetzt, die logisch voneinander unabhängig sind."

[12] The authors go on to point out that both drawbacks are not present in Kaluza-Klein theory (the topic of the paper), but that it suffers from another serious drawback (the absence of particle solutions to the field equations), i.e. that Kaluza's theory fails as a *complete* field theory. For a historical investigation of Einstein's search for a complete field theory see Lehmkuhl and Sauer [forthcoming].

part *even though* the electromagnetic Lagrangian contains both the Faraday tensor and the metric field.

The reason for regarding this part of the total Lagrangian as the electromagnetic part is that variation with respect to the electromagnetic vector potential gives us the free Maxwell equations, no matter whether the total Lagrangian also contains a gravitational term or not. In the case of the gravitational term, we need the electromagnetic term to vanish in order to get the vacuum Einstein equations, the equations for a 'free' metric field, when varying with respect to the metric tensor.

I will speak of a theory of two fields fulfilling the *weak Einstein criterion* if the theory does *not* have a Lagrangian which can be decomposed in such a way that each of the two terms can be associated to the vacuum field equations of one and only only one of the two fields allegedly unified. In other words, for a theory fulfilling the weak Einstein criterion, there is no way to give two separate sets of field equations even under idealising assumptions. I will call a theory that overcomes the dualism between two fields like the electromagnetic and the gravitational field to this extent a theory of *unification strength* 2.[13]

If we had such a theory for gravitational and electromagnetic fields, the two fields would definitely be closer to each other than in Einstein-Maxwell theory. But it would still be possible to speak of *two* fields that are, even though intimately related, still not two aspects of one and the same field.

For the criterion allows for more than one tensor field in the non-decomposed (form of the) Lagrangian. We could still have a Lagrangian that looks similar to the first term of the Lagrangian of Jordan-Brans-Dicke theory in the Jordan frame[14]

$$\mathscr{L} = \sqrt{-g}\phi^2 R \tag{2}$$

where ϕ is a scalar field and R the Ricci scalar of the Levi-Civita connection defined in terms of the metric field $g_{\mu\nu}$. The Lagrangian would give us field

[13]Two fields are related with Unification Strength 0 if they do not interact at all (as gravitational and electric fields in Newton's theory of gravity), and of Unification Strength 1 if both fields have separate 'free' Lagrangians but if there is also an extra ineraction term in the total Lagrangian (as in Einstein-Maxwell theory, on which Einstein and Grommer try to improve here).

[14]The original sources of Jordan-Brans-Dicke theory, see Jordan [1955], Brans and Dicke [1961] and Dicke [1962]; for a refelction on its development see Brans [2005]. For discussions in the philosophical literature see e.g. Weinstein [1996] and Lehmkuhl [2008].

equations (if varied with respect to ϕ and $g_{\mu\nu}$ separately) that always contain both fields. Now, suppose we have good reason to think of $g_{\mu\nu}$ as representing the gravitational field and of ϕ as representing the electromagnetic field. Then varying with respect to either ϕ or $g_{\mu\nu}$ will give us field equations in which both dynamical fields feature under all circumstances, and so we cannot divide the total set of field equations into two sets, one of which describing the equations of motion of the 'free gravitational field' and the other describing the equations of motion of the 'free electromagnetic field' — the two fields are never free of each other. Thus, the weak Einstein criterion is fulfilled, and still it seems we do not have a good reason to speak of a unified gravi-electromagnetic field; we just have two fields which always occur together.

We could strengthen the weak Einstein criterion by demanding that the Lagrangian must contain a tensor field U which is either fundamental itself or defined in terms of only one fundamental field u. If we then vary with respect to u, we would get a single set of field equations for which it is not only impossible to split them into two sets of 'vacuum equations' for the two original fields, but where it is not even possible to distinguish between the two original fields in an objective manner. I will speak of a theory fulfilling the *strong Einstein criterion* if this is the case. I will not associate a *unification strength* to a theory fulfilling the strong Einstein criterion, for, as we will see, it needs Pauli's criterion (elaborated in the the next subsection) in order to be made precise; and any theory fulfilling Pauli's criterion that has a Lagrangian formulation will fulfill the strong Einstein criterion.

The criterion in question centers on the properties of the field(s) occurring in the Lagrangian and the field equations, rather than on the field equations governing the field(s).

3.2 Pauli's criterion: Unification Strength 3, 4 and 5

In a 1958 Addendum to his famous encyclopedia article (Pauli [1921]), Pauli published a criterion that he thought 'properly' unified field theories have to fulfill, and which he deemed had been blatantly neglected by many researchers in the field. Pauli had long argued for this criterion, and was supported by Weyl [1950], p. 83. Pauli writes that "only irreducible quantities should be used in field theories" (Pauli [1958], p.273), while Weyl puts it in the following way:[15]

> If there is one lesson that one can draw from mathematics about the formulation of physical theories, then it is the one that only quantities which are irreducible under their respective transformation law represent a unitary physical entity.

Pauli and Weyl were particularly attacking theories in which the gravitational and the electromagnetic field were supposed to be unified by the use of an asymmetric metric tensor $g_{\mu\nu} = s_{\mu\nu} + a_{\mu\nu}$. In the early approaches of this kind, the symmetric part $s_{\mu\nu}$ was supposed to represent the gravitational potential and the antisymmetric part $a_{\mu\nu}$ the electromagnetic field. Such accounts where favoured by Einstein for a long time; indeed Einstein worked on this approach on and off and focused on it for about the last 15 years of his life.[16]

According to Pauli, such a theory does not describe a unified field, but is a theory of two fields disguised as a theory of one field. The reason is the following: while in Maxwell's theory with its antisymmetric electromagnetic field tensor it is not in general possible to transform the 'electric part of the total field' without automatically transforming its 'magnetic part', this *is*

[15] "Wenn man der Mathematik eine für die Aufstellung physikalischer Theorien wichtige Lehre entnehmen kann, so ist es die, dass nur Grössen, die unter ihrem spezifischen Transformationsgesetz unzerlegbar sind, eine einheitliche physikalische Entität darstellen."

[16] The approach dates back until at least November 1917, when Rudolf Förster proposed it in a letter to Einstein. In his answer from January 1918 Einstein writes that he had thought along similar lines, but that he "lost hope of getting behind the secret of unity (gravitation, electromagnetism) in this way."(For the correspondence between Förster and Einstein on this issue see Documents 398, 400, 420 and 439 in Volume 8A and 8B of Einstein's Collected Papers.) Einstein started working on the approach properly in 1925, but dropped it again soon. After 20 years of working on yet other UFTs, Einstein came back to the asymmetric idea in the 1940s, and in his autobiographical notes even writes that after having tried all these different approaches, he came to think of the idea based on an asymmetric metric tensor (and, now, an independent asymmetric connection ${\Gamma^{\nu}}_{\mu\sigma}$) in 4-dimensional spacetime as "the one that is logically most satisfying" (Einstein [1949], pages 86/87). In the years until his death in 1955, Einstein focused his work on this approach, and it seems very likely that many of his calculations from the time are contained in a collection of still largely unidentified manuscripts that, according to anecdotal evidence reported in Sauer [2004], turned up behind a filing cabinet when Einstein's literary estate was prepared to be moved from Princeton to Jerusalem in 1982. For more information on the asymmetric approach see Bergia [1993], Goenner [2004], and in particular Goenner [2009].

possible for a field which is decomposable in the above way: I can transform the symmetric part of the metric tensor (gravity) without changing anything about its antisymmetric part (electromagnetism).[17] Hence, according to Pauli, there really are two fields in such a case, rather than one unified field. (Indeed, Pauli's criterion means that we should *never* regard asymmetric tensors as representing something fundamental, for the above argument applies to every asymmetric tensor.)

All this can be made precise in the language of representation theory and the reducible and irreducible representations of groups, which is surely the way Pauli and Weyl, jointly the champions of the use of group theory in physics, intended it. Indeed, Jordan [1955], p.156, referred to Pauli's criterion in exactly this way:[18]

> There is a principle voiced by *Pauli*, which seems to deserve the most serious consideration; *Weyl* whole-heartedly agreed with this principle: *Only irreducible n tensors must be used as the fundamental concepts of a theory.* [...] A tensor shall be called *irreducible* (corresponding to the concepts of representation theory), if it is *not* possible to additively compose it from tensors each of which posseses symmetry properties which are not exhibited by the sum. The *Pauli*an demand thus obliges us, if we accept it, to straightforwardly refuse the basic thought of the current *Einstein*ian theory..., which works with an antisymmetric second-rank tensor — which is the sum of a symmetric and an antisymmetric one.

What Pauli and Jordan refer to is the fact that the sets of symmetric and antisymmetric tensors correspond to two *irreducible representations* of the

[17] Of course, it will change the total value of the asymmetric $g_{\mu\nu}$ — but that is not at issue here.

[18] "[Es gibt einen] von *Pauli* ausgesprochenen Grundsatz, der mir ernsteste Beachtung scheint; *Weyl* hat diesem Grundsatz nchdrücklich zugestimmt: *Als Grundbegriffe einer überzeugenden Theorie dürfen nur irreduzible n Tensoren benutzt werden.* [...] "*Irreduzibel*" soll ein Tensor (entsprechend den Begriffen der Darstellungstheorie) heissen, wenn es *nicht* möglich ist, ihn additiv zusammenzusetzen aus solchen Tensoren, von denen jeder einzelne Symmetrie-Eigenschaften besitzt, die der Summe nicht mehr zukommen. Die *Pauli*schen [sic!] Forderung nötigt also, wenn wie sie anerkennen, unmittelbar zur Verwerfung des Grundgedankens der jetztigen *Einstein*schen Theorie ..., welche mit einem *unsymmetrischen* von zwei Indizes rechnet — der die Summe eines symmetrischen und eines antisymmetrischen ist."

group of coordinate transformations, whereas asymmetric tensors correspond to a *reducible representation*.[19]

Weyl [1950] quotes Pauli as having summarised the principle by saying: "Men shall not join what god has torn assunder".[20] Indeed, Einstein [1945] conceded that only Maxwell's theory had achieved this level of unification envisaged by Pauli, in contrast to the asymmetric theory he was about to present.

Pauli's criterion of course does not only apply to the group of coordinate transformations, and we will thus be able to make ample use of it in the subsequent sections.

It has to be noted that fulfilment of Pauli's criterion in itself does not tell us whether a given theory is even a candidate unified field theory. For we could well have a theory of electric and magnetic fields where both are *separately* irreducible in the above sense. So strictly speaking, in order to make Pauli's criterion into a criterion for *unified* field theories, we have to demand that not only physical entities that were always thought of as entities in their own right are described by irreducible quantities, but even attempts trying to unify two fields in a single field should use only irreducible quantities for representing the newly unified total field. I will from now on refer to this slightly modified version of the criterion (which I think is what Pauli had in mind) as Pauli's criterion.[21]

[19] Even though this may be intuitive given Jordan's comment about symmetry properties, the proof is not trivial. It can be delivered by using the Schur-Weyl theorem, which establishes a relation between the irreducible finite-dimensional representations of the general linear group $GL(n)$ on the one hand, and the symmetric group S_n on the other hand. $GL(n)$ is often represented by the set of all $n \times n$ matrices together with the operation of matrix multiplication (by which we can also represent tensors); S_n is standardly represented by a set of n arbitrary elements and the operation of permutation of the elements. For second-rank tensors (generalisable to tensors of arbitrary rank), we can use the theorem and the fact that S_2 has only two irreducible representations to show that there are only two irreducible representations of $GL(2)$, which correspond to symmetric and anti-symmetric tensors. (I do not know any reference of where this proof is carried out; I am indebted to Geordi Williamsson for showing me a sketch of the proof.)

[20] "Was Gott getrennt hat, soll der Mensch nicht zusammenfügen". Translation from Pais [2000], p.350.

[21] It seems sensible to see Pauli's criterion for classical unified field theories as a complement to Wigner's use of representation theory in classifying the particles of the standard model of (quantum) particle physics. In a nutshell, Wigner identifies the Poincare group as one of the relevant symmetries of the standard model, and associates a fundamental particle to every irreducible representation of the Poincare group. Pauli's strategy is the

It is straightforward that any theory fulfilling Pauli's criterion with respect to the mathematical field unifying two fields previously thought of as distinct will also satisfy the weak and strong Einstein criterion if it has a Lagrangian formulation: if there is no way to 'split up' the field itself (because it is irreducible in the above sense), then there is surely no Lagrangian decomposable into two 'field-parts' (the weak Einstein criterion), and the field equations are obtained by varying with respect to the unified field only (strong Einstein criterion).

Note that although symmetric and antisymmetric tensors fulfill Pauli's criterion with respect to the group of coordinate transformations while asymmetric tensors do not, in the general case it may demand very hard work before we can judge whether a given mathematical object occurring in the field equations of the theory corresponds to (an element of) an irreducible representation of the symmetry group of the equations. If we have an arbitrary field equation and find that it has a symmetry group X, then it is a non-trivial question whether a given mathematical object occurring in the field equation corresponds to an element of an irreducible representation of the symmetry group.

The example discussed above is one in which the symmetry group of the theory is a simple group G rather than a product group $H = H_1 \times H_2$. If the latter is the case, then we have two possibilities to fulfill Pauli's criterion: search for objects that correspond to irreducible representations of H_1 and H_2 *separately*, or search for irreducible representations of $H_1 \times H_2$.

Thus, Pauli's criterion can be applied to different kinds of groups. For the fundamental fields can cooerespond to the irreducible representation of a *simple group* or a *product group*. We have seen an example for the former case above (symmetric and anitsymmetric tensors as irreducible representarions of the group of 4-dimensional coordinate transformations), but we will meet an example for the latter case in the next section: the Glashow-Weinberg-Salam theory.

Thus, I will use Pauli's criterion to distinguish between 3 degrees of unification. I will speak of a field theory to be of *unification strength* 3 if the fundamental fields correspond to irreducible representations of a product

other way round (and of course in a different domain of physics): when you look for field equations of a unified gravitational-electromagnetic field, always look for field equations such that the *fundamental* fields are irreducible representations of the symmetry group of your theory. (It does not need to be the total symmetry group; in Wigner's classification of particles, the Poincare group is of course not the total symmetry group either.)

group. A theory will be said to be of *unification strength* 4 if the fundamental field supposed to unify field A and field B corresponds to an irreducible representation of a number of *simple groups*, and of *unification strength* 5 if the unified field corresponds to an irreducible representation of only *one* simple group.[22]

In all three cases we obtain the following picture: the two fields A and B previously thought of as two distinct entities/fields, even though possibly interacting, turn out to be artifacts of a particular conventional choice, and only the newly unified field has a completely objective (non-conventional) existence. The choice in question can be the choice of a particular coordinate system, a particular scale system, or the choice of any other kind of gauge.

Most successful unified field theories fulfill a further criterion: they make new empirical predictions, they are not empirically equivalent to their predecessor(s), the non-unified theorie(s). It is an interesting question whether new empirical content necessarily comes with a certain unification strength, while the theory may fail to have new empirical content (qua unification) if the theory stays below this level of unification.

Is a unified field theory of strength 2, 3, 4 or 5 an example for perfect unification? From which unification strength onwards should we speak of a *unitary* unified field theory? Are the above criteria for unification sensible, and are further degrees of unification needed in order to classify the landscape of theories? I will apply the above scheme to different field theories in the next section, and postpone the answer to these questions until later. For now, let me summarise the different degrees of field-unification we have found to be attainable so far.

A UFT is of $US1$ if there are two clearly distinct, but interacting fields, and if the total Lagrangian is decomposable into a 'field 1'-term, a 'field 2'-term, and an interaction term. The weak Einstein criterion says that the Lagrangian of a theory cannot be decomposed into a 'field 1'-term and a 'field 2'-term; its fulfillment (giving a theory $US2$) is a sufficient but not a necessary condition for a theory to be of $US3$. A theory fulfilling the strong Einstein criterion has only one fundamental field featuring in the non-decomposable Lagrangian. But this can only truly be so if this field fulfills a version of Pauli's criterion. A theory is of $US3$, $US4$ or $US5$ if

[22]Note that in either case the truly unified nature of the respective fields may be hidden from us because of spontaneous symmetry breaking: in this case they would only 'look' unified under certain conditions, mostly assumed to be that of high temperatures.

it fulfills Pauli's criterion, stating that the unified field must correspond to an irreducible representation of the symmetry group of the theory. In the case of $US4$ it corresponds to an irreducible representation of a product group, in the case of $US5$ to irreducible representations of the simple groups forming a product group and in the case of $US6$ if the total symmetry group is a simple group, and the unified field corresponds to one of its irreducible representations.

4 Pauli's criterion applied and questioned

In this section, we will see how Einstein's and Pauli's criterion fare in practice. I will look at three alleged cases of unification in turn and discuss *to what degree* they achieve a unification of the physical fields in question. In particular, I will discuss the alleged unification of electric and magnetic field in special-relativistic electromagnetism, that of electromagnetic and weak nuclear fields in Glashow-Salam-Weiberg theory and that of electromagnetic and gravitational fields in the original version of Kaluza-Klein theory. I will argue that the unification of electric and magnetic field is of strength 5, that of electromagnetic and weak fields of strength 3 and that of gravitational and electromagnetic fields of strength 2 — thus despite first appearances not going beyond the Einstein-Maxwell equations.

4.1 Special relativistic electrodynamics

The unification of electric and magnetic fields we find in electromagnetic theory surely fulfills the weak Einstein criterion: the Lagrangian of the theory cannot be decomposed such that it would give us one set of field equations for the electric and one for the magnetic field. But it also fulfills the strong Einstein criterion: there is no adequate way to split up the electromagnetic field tensor $F_{\mu\nu}$ in a way that would justify regarding it as 'really' being only a formal summary of two fields, the electric and magnetic field.

The reason is that $F_{\mu\nu}$ fulfills Pauli's criterion with respect to both the Poincare group and the general group of coordinate transformations; it corresponds to an irreducible representation and hence cannot be broken up into smaller parts in a manner invariant under group operations.

Indeed, the theory fulfils Pauli's criterion in two respects.

1. the electromagnetic field $F_{\mu\nu}$ is an element of an irreducible representation of the general group of coordinate transformations (and the Poincare group, a subgroup of the former), simply because it is a 4-dimensional antisymmetric tensor. (Indeed, the electromagnetic vector potential is also irreducible, because it is a one-form.) Thus, there is no way to break up $F_{\mu\nu}$ into electric field $\vec{E}$ and magnetic field $\vec{B}$ in a way that is invariant under coordinate transformations; thus, $F_{\mu\nu}$ itself is an irreducible element of reality and presents us with a non-trivial unification of electric and magnetic field.

2. the electromagnetic field $F_{\mu\nu}$ is invariant under gauge transformations of the vector potential; thus all such transformations map $F_{\mu\nu}$ into itself, and the latter hence trivially corresponds to an irreducible representation of the gauge group.

Consequently, electromagnetic theory fulfills Pauli's criterion with respect to both groups separately, and is hence a unified field theory of strength 5.

It should be noted, however, that an asymmetry between electric and magnetic components in a given coordinate system remains: once a reference frame is chosen, there is still a fact of the matter whether a given component of the Faraday tensor is an electric or a magnetic component. For since there are electric, but no magnetic monopoles that give rise to the electromagnetic field, an electric component $\vec{E}$ is a polar vector, whereas a magnetic component $\vec{B}$ is an axial vector. If there were magnetic monopoles, the Maxwell equations would be more symmetrical. Arguably it would then not be possible to distinguish objectively between electric and magnetic components even with respect to a chosen coordinate system, and our theory would be of even higher unification strength, which we might call unification strength 6.

4.2 Electroweak field Theory and the standard model

Let us now have a look at the goup-theoretic structure of the standard model of elementary particle physics. Here, there is no Pauli-unified electro-weak-strong force according, for the generators of the quantum fields of the full standard model do not correspond to irreducible representations of the (internal) symmetry group of the full standard model, $SU(3) \times SU(2) \times U(1)$. Instead, the generators fall into two classes, a set of generators of an irreducible representation of $SU(3)$ (corresponding to the 8 gauge bosons of the strong field) and a set of generators which are partly irreducible with respect to $SU(2)$ and party irreducible with respect to $SU(2) \times U(1)$ (corresponding to the 4 gauge bosons of electroweak field).[23]

The trouble is that even a unified field theory resting on an irreducible representation of a product group like $SU(2) \times U(1)$ does not give us a *perfect* unification. As Georgi and Glashow [1974], p. 438, point out:[24]

> [The theory of electroweak unification] is defective in one important respect: it does not truly unify weak and electromagnetic interactions. The $SU(2) \times U(1)$ gauge couplings describe two interactions with two independent coupling constants; a true unification would involve only one.

Georgi and Glashow then go on to propose $SU(5)$ as the gauge symmetry group of a new standard model. Their theory was ultimately unsuccessful because of predictions concerning proton decay that were not verified by experience, but to this day it remains a paradigm for a truly unified theory of elementary particle phyiscs. As it stands, however, Glashow-Salam-Weinberg theory (i.e. electroweak theory) is of unification strength 3, because one generator of the electromagnetic-weak field corresponds to an irreducible representation of the symmetry goup of the theory; the unification achieved is of a weaker kind than that obtained in special-relativitic electrodynamics.[25]

4.3 Kaluza-Klein Theory

We have now seen one example for a theory in which two fields are unified to a very high degree (special relativistic electromagnetism) and one in which the degree of unification is less high (GSW-theory). We are now turning to the original version of Kaluza-Klein theory as an example of a theory which

[23]Not all generators of the electroweak force are generators of the full product group $SU(2) \times U(1)$, only the generator of the Z boson is. Also, it is important to note that in the standard model of elementary particle physics the $SU(2) \times U(1)$ symmetry is spontaneously broken at low temparatures in order for the particles to aqurie mass via the Higgs mechanism. I am not yet sure whether spontaneous symmetry breaking changes anything about the argumentation in the main text.

[24]See also Maudlin [1996], section 4.

[25]One could argue that there is an intermediate leven of unification between unification strength 3 and 4, constituted by the case that *all* rather than *some* of the generators of the allegedly unified field correspond to irreducible representations of the product group.

seems to unify the gravitational and the electromagnetic field in a way very similar (and to an equally high degree) as electric and magentic field had been unified in special-relativistic electrodynamics. For in constrast to Einstein-Maxwell theory, where one field is associated with electromagnetism (A_μ or $F_{\mu\nu}$) and one with gravity ($g_{\mu\nu}$), in Kaluza-Klein theory both physical fields are supposed to be unitedly represented by the following 5-dimensional metric g_{AB}:[26]

$$\bar{g}_{AB} = \begin{pmatrix} \bar{g}_{\mu\nu} + \bar{\phi}\bar{A}_\mu\bar{A}_\nu & \bar{g}_{5\mu} = \bar{\phi}\bar{A}_\mu \\ \bar{g}_{\nu 5} = \bar{\phi}\bar{A}_\mu & \bar{g}_{55} = \bar{\phi} \end{pmatrix} \tag{3}$$

Indeed, Klein [1926] showed that g_{AB} *must* be of this form if one demands that the symmetry transformations of the theory are given by

$$x^\mu = f(x^{\mu\prime}) \quad ; \quad x^5 = x^{5\prime} - h(x^{\mu\prime}) \tag{4}$$

rather than by the full group of 5-dimensioal coordinate systems. He thereby fulfiled what has been termed 'the sharpened cylinder condition', in coordinate-dependent lannguage:

$$\partial_5 \bar{g}_{AB} = 0 \quad ; \quad \bar{g}_{55} = const. \tag{5}$$

In coordniate-indepent language, Klein's demand amounts to spacetime being endowed with a spacelike Killing vector (first part of the condition), which is normalized (second part of the condition). The cylinder condition(s) were motivated first by Kaluza and then by Klein by pointing out that we never observe physical qunatities change in a 5th dimension; this lead Klein directly to the symmetry group (4) and eventually to the result that the 5-dimensional must be given by (3).

However, g_{AB} does *not* correspond to an irreducible representation of the total symmetry group of the theory, the product group given by the two independent transformations (4), *nor* to an irreducible representation of its parts. Instead, just as in Einstein-Maxwell theory, this role is fulfilled by the 4-dimensional metric, the 4-dimensional vector potential, and, in the case when the Killing vector is not normalised, by the extra scalar field ϕ.

For the objects invariant under Klein transformations are $\gamma_{\mu\nu} := g_{\sigma\rho} - \frac{g_{\sigma 5}g_{\rho 5}}{g_{55}} = g_{\mu\nu} - \frac{1}{\phi}A_\mu A_\nu$, the 4-vector $g_{5'\nu'} := A_\nu$ and the scalar $g_{55} := \phi$.

[26]The bar on top of a geometric object states that the components of that object are referred to.

Thus in so far as we accept $\gamma_{\mu\nu}$ rather than $g_{\mu\nu}$ as the metric of 4-dimensional (sub)-spacetime, because of demanding that the fundamental quantities have to be irreducible with respect to the product group rather than to its parts, we have a theory that is quite different from Einstein-Maxwell theory: $\gamma_{\mu\nu}$ corresponds to an irreducible representation of the full group, rather than to the parts forming the group.

If the only fundamental field needed in the theory was $\gamma_{\mu\nu}$, we might hence well regard the theory as giving a unified treatment of gravitation and electromagnetism. Unfortunately, the theory also needs A_μ (and, unless the Killing vector is demanded to be normalised as in Klein's theory, ϕ) separately, and we hence would not have only one unified gravi-electromagnetic field. Thus it seems we would have a case which is quite similar to the electroweak unification: one of the fundamental fields, $\gamma_{\mu\nu}$, containing both of the previously separate fields, corresponds to an irreducible representation of the product symmetry group of the theory, whereas the other fundamental fields correspond to irreducible representations of the parts of the product group.

The cases would be completely similar if the covariant derivative used in the definition of the Ricci tensor of the theory was the covariant derivative of $\gamma_{\mu\nu}$ rather than of $g_{\mu\nu}$. But it seems clear that the covariant derivative we find in the components of the 5-dimensional Ricci tensor is that of $g_{\mu\nu}$. For R_{AB} can be decomposed in the following way:[27]

$$\tilde{R}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\phi F_{\mu\sigma} F^{\sigma}_{\mu} - \frac{1}{\sqrt{\phi}} \nabla_\mu \partial_\nu \sqrt{\phi} \tag{6}$$

$$R_{\mu 5} = \frac{1}{2\phi} \nabla^\nu (\phi^{\frac{3}{2}} F_{\mu\nu}) \tag{7}$$

$$R_{55} = -\frac{1}{4}\phi F_{\mu\nu} F^{\mu\nu} + \frac{1}{\sqrt{\phi}} \nabla_\mu \nabla^\mu \sqrt{\phi} \quad . \tag{8}$$

Thus we have a curious imbalance in the theory: even though $\gamma_{\mu\nu}$ is invariant under Klein transformations and $g_{\mu\nu}$ is not, it is $g_{\mu\nu}$ that determines the geodesic structure of the 4-dimensional spacetime in which $\gamma_{\mu\nu}$, $g_{\mu\nu}$, A_μ and ϕ seem to live. Thus, we seem unjustified in taking $\gamma_{\mu\nu}$ as representing the 4-dimensional sub-spacetime because it is irreducible with respect to the Klein group; instead we have to content ourselves with taking $g_{\mu\nu}$, A_μ and ϕ, which

[27]Cf Goenner and Wünsch [2003], p.24

are irreducible with respect to the two parts of the Klein group separately, as the fundamental fields of the theory.

All this shows that Kaluza-KLein theory does not fulfill Pauli's criterion for unified field theories in any way: it seems a theory of two (or three, given ϕ) fields disguised as a theory of one field.

Thus Kaluza-Klein theory does not accord with Pauli's criterion — how about Einstein's criterion? The latter focuses on Lagrangians and field equations rather than on the symmetry properties of the fields. Indeed, as we have seen above, one of Einstein's main arguments to favour tentatively Kaluza's theory over Weyl's theory was that its Lagrangian was not composed of a sum of a gravitational and an electromagnetic term, allegedly in contrast to Weyl.

Unfortunately, this is only apparently so. The Lagrangian employed by both Kaluza and Klein is

$$L = R \tag{9}$$

where R is the 5-dimensional Ricci scalar to the metric g_{AB}. And the 5-dimensional Ricci tensor corresponding to Klein's metric can be decomposed into a sum of terms determined by irreducible fields $g_{\mu\nu}$, A_μ and ϕ.[28] In the case where $\phi = const$ this Lagrangian corresponds precisely to that of the Einstein-Maxwell equations, and hence gives separate field equations for gravitational and electromagnetic fields. Hence, despite Einstein's judgement, Kaluza's theory does not fulfill Einstein's criterion.

Maybe the above reasoning was what Pauli [1958] had in mind when he wrote in the late 'Supplementary notes' to his famous encyclopaedia article from 1921 (p.230, his emphasis):

> Kaluza's theory is *not at all a 'unification' of the electromagnetic and the gravitational field.* On the contrary, every generally covariant and gauge invariant theory can be represented in *Kaluza's* way.

One way to argue for this statement is given above. Another is to see Pauli's comments as a generalised version of Kretschmann's argument — it would be interesting to see whether Pauli's criterion can be linked to notions of substantive and formal general covariance, but this is a separate topic and would lead us too far afield.

[28] Cf Goenner and Wünsch [2003], p.24

Either way, I have to side with Pauli on the issue: the 'true', fundamental and irreducible, fields of Kaluza-Klein theory are $g_{\mu\nu}$, A_μ and ϕ, nicely packaged in the 5-dimensional metric . The theory is thus of the same unificationary strength as Einstein-Maxwell theory, merely of unification strength 1 (meaning that the electromagnetic and the gravitational field couple to each other). In Klein's version of the theory (the space-like Killing vector is normalised, thus $g_{55} = 1$) this is not surprising given that the two theories are mathematically equivalent; in Kaluza's case (Killing vector not normalised, thus $g_{55} = \phi \neq const$) we get an extra scalar field that makes the theory genuinely different to Einstein-Maxwell theory — but the extra scalar field does not make the theory more unified.[29]

5 Conclusion

When describing the different degrees of unification achievable in a unified field theory, I have explicitly only spoken of 'free fields' and how they could be unified to a total field. Many authors leave it at that and do not discuss the role of the *sources* of the original fields and what their features tell us about whether the fields are unified or not. Georgi and Glashow [1974], as quoted in section 4.2, are an exception: they point out that the imperfect unification of GSW-theory leaves us with two coupling constants. This is equivalent to pointing out that the theory leaves us with two kinds of charges: the electric charge giving rise to the electromagnetic field and the flavour charge giving rise to the weak field. In a truly unified field theory, there would be only one kind of charge. The pradigm case is again electrodynamics: there is only one kind of charge giving rise to electric and magnetic fields.

The unification criteria developed in the last section can be applied not only to *'free' fields* (fields which *can* occur without sources) like the electric and the magnetic field E_i and B_i, the electromagnetic field $F_{\mu\nu}$ and the metric field $g_{\mu\nu}$, but also to *'source' fields* (fields which, if there, produce

[29] Tonnelat [1966], p.9, claims that Klein's theory predicts a Blackett-like effect, namely that in the case where no electric charge but a non-vanishing mass density is present a magnetic field could be created. This could be interpreted as a sign of unification on the level of the sources of gravitational and electromagnetic field (see the next section). Unfortunately, Tonnelat does not give any detail. But even if the effect exists, it need not be a consequence of unification *per se*, but could result equally well from the coupling of the new field ϕ to the 'old fields' $g_{\mu\nu}$ and $F_{\mu\nu}$.

other fields) like the mass density ρ_m, the electric charge density ρ_e or the 4-current density j^μ. Indeed, it seems fair to say that the distinction between 'free' fields and 'source' fields does not go very deep anyhow (at least in relativistic physics); the only difference seems to be that we allow the latter to come in the form of either particles or continua, whereas the former are always continuous.[30] But even so: one and the same entity can be both a 'free' field in one equation and a source field in another: compare the roles of the electromagnetic field $F_{\mu\nu}$ in the Einstein-Maxwell equations.

One may now demand that in a properly unified field theory, it is not enough for the free fields to be more intimately related to each other than in the predecessor theory describing two separate fields, but that the relation between the field sources, as compared to the predecessor theory, must be more intimately linked also. Indeed, the comparison between the electromagnetic and the electroweak unification above suggests that a particular strength of *field unification* will have a *charge unification* as its consequence. The conjecture is this: if the newly unified field corresponds to an irreducible representation of *one simple* symmetry group rather than a plentitude of simple groups or a product group, then there will be only one kind charge.

One might expect that a properly unified field theory of electromagnetism and gravity would create exactly this kind of relationship between electromagnetic sources (electric charge) and gravitational sources (mass-energy-momentum). If the degree of unification between the two source fields j_μ and $T_{\mu\nu}$ were high enough, we would expect the theory to predict the possibility to create, say, magnetic fields even in the absence of electric charges, by the mere presence (or motion) of a gravitationally charged body, i.e. one possessing mass-energy-momentum.

Einstein himself seems to have expected such a relation between the different kinds of charge as a result of a unified field theory. In a letter to Barret from 6 January 1931 he writes (Document 25-019 Einstein Archives):[31]

> I am convinced, since quite a while, that the magnetic field of

[30] Again, the programme of a complete field theory wants to describe particles as particular field configurations; if this programme is successful then there would be no fundamental distinction between sources and (non-source) fields anymore. But as pointed out in the introduction, the research programme of a complete field theory is logically independent from that of a unified field theory of specific fields.

[31] "[I]ch bin seit langem überzeugt, dass das magnetische Feld der Erde nicht durch magnetische Körper erzeugt wird, sondern durch jegliche ponderable Materie im Zusammenhang mit der Erdrotation"

> the earth is created not by magnetic bodies, but by any kind of ponderable matter in connection with earth's rotation.

Even independently of unified field theories of gravitation and electromagnetism, such effects were postulated. Blackett [1947] proposes an algebraic relation between the angular and the magnetic momentum of the earth (or the sun). He then claims that not only do the Maxwell equations not account for the strength of earth's magnetic field given any realistic charge distribution in or on earth, he also did not see how Maxwell's equations, as electromagnetic field equations, could be modified such that they would give an account of the magnetic field of earth. He thus envisages an effect by which the motion of an electrically uncharged but very massive body could produce a magnetic field.

Blackett quickly points out that no translational motion could produce such an effect, if the Lorentz transformations are not to be violated (p.664).[32] He goes on to postulate that the laws of nature might be such that an electrically neutral but very massive body could produce a magnetic field if *rotating*.

What is important for us is that a Blackett-like effect would point to a link between electromagnetic and gravitational charge, and hence to a unification not only on the level of the free fields but also on that of the sources.

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[32] Of course, one could argue that if one is willing to give up the Maxwell equations, there is no reason to be conservative about the Lorentz transformations.

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