Handout on Quantum Challenges for the Equivalence Principle

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1 Formulations of the Equivalence Principle (EP)

1.1 "Principles of Relativity Physics" by Anderson [1]

EP: To the extent that we can neglect their action back on the sources of the gravitational field, measurements made on any physical system will serve to determine the same affinity in a given space-time region.

Minimal coupling of the gravitational field:

- Local equivalence of gravitational and accelerative effects.
- Only the affinity determined by the gravitational field can appear in the dynamical laws, to the extent that these laws are local laws.
- It amounts to requiring that in a sufficiently small region of spacetime, the laws of special relativity are valid.

. . . we can imagine a dimensionless particle that is characterized by a mass m and, in addition, a spin tensor $\sigma^{\mu\nu}$, for example, an electron. Its trajectory might then be determined by the geodesic equation, to which would be added a term proportional to $R_{\mu\nu\iota\kappa}\sigma^{\mu\nu}\sigma^{\iota\kappa}$. Such a dynamical law would violate the principle of minimal coupling as we have stated it. However, one could argue that a particle such as the one we have considered is in reality not a local structure and so does not violate the principle. But, of course, this objection is in reality a quibble; if such a particle did exist in nature, one could never find a region of spacetime that was sufficiently small to cause such spin-dependent term to vanish.

Comments:

- EP allows arbitrary couplings to the metric (and its derivatives).
- Is minimal coupling really violated by a spinning point particle? Is the geometrical character of gravity lost?

1.2 *Foundations for a Theory of Gravitation Theories* by Thorne Lee and Lightman [4]

Gravitational phenomena: Those which either are absolute or "go away" as the amount of mass-energy in the experimental laboratory decreases.

Local test experiment: Any experiment, performed anywhere in spacetime, in the following manner. A shield is set up around the experiment. When analyzed using the concepts and experiments of special relativity, this shield must have arbitrarily small mass-energy and must be impermeable to electromagnetic fields, to neutrino fields and to real (as opposed to virtual) particles. The experiment is performed, with freely falling apparatus, in the center of the shielded laboratory, in a region so small that inhomogeneities in all external fields are unimportant. One makes sure that external homogeneities are unimportant by performing a sequence of experiments of successively smaller size (with size of shield and external conditions unchanged), until the experimental result approaches a constant value asymptotically.

Local, nongravitational, test experiment: A local test experiment with these properties: (i) When analyzed in the center-of-mass Galilean frame, using Newtonian theory of gravity, and using all forms of special relativistic mass-energy as sources for the Newtonian potential Φ, the matter fields inside the shield must produce a Φ with

$$|\Phi(\text{at any point inside shield})| - |\Phi(\text{at any point on shield})| \ll 1\,.$$

(ii) When the experiment is repeated, with successively smaller mass-energy inside the shield (as deduced using special relativity theory) - but leaving unchanged the characteristic sizes, intrinsic angular momenta, velocities and charges (electric, baryonic, leptonic, etc.) of its various parts - the experimental result does not change.

Uncharged test body: An object (i) that is shielded, in the sense used above in defining "local test experiments"; (ii) that has negligible self-gravitational energy, when analyzed using Newtonian theory; (iii) that is small enough in size so its coupling (via spin and multipole moments) to inhomogeneities of external fields can be ignored.

Dicke's weak EP: If an uncharged test body is placed at an initial event in spacetime, and is given an initial velocity there, then its subsequent world line will be independent of its internal structure and composition.

Einstein EP: (i) Weak EP is valid, and (ii) the outcome of any local, nongravitational test experiment is independent of where and when in the universe it is performed, and independent of the velocity of the (freely falling) apparatus.

Dicke's strong EP: (i) Weak EP is valid, and (ii) the outcome of any local test experiment, gravitational or nongravitational, is independent of where and when in the universe it is performed, and independent of the velocity of the (freely falling) apparatus.

A theory of gravity obeys the Strong EP if and only if it obeys Einstein EP, and it possesses no preferred-frame or preferred-location effects.

Metric theory of gravity: (i) Spacetime is endowed with a metric; (ii) the world lines of test bodies are the geodesics of that metric; and (iii) Einstein EP is satisfied, with the nongravitational laws in any freely falling frame reducing to the laws of special relativity.

Schiff,s conjecture: Any complete and self-consistent gravitation theory that obeys the weak EP must also, unavoidably, obey the Einstein EP.

Comments:

- Definitions too dependent on existing theories.
- Can we always make experiments smaller, keeping everything else unchanged?

- Do *uncharged test bodies* exist for arbitrary experimental accuracies? (small enough so coupling to inhomogeneities of external fields can be ignored.)

1.3 "Gravitation" by Misner, Thorne and Wheeler [5]

Uniqueness of free fall: The world line of a freely falling test body is independent of its composition or structure.

Test body: electrically neutral, small enough that (1) its self-gravitational energy, as calculated using standard Newtonian theory, can be neglected compared to its rest mass, and (2) the coupling of its multiple moments to inhomogeneities of the gravitational field can be neglected. (In GR we might abandon (1)).

EP: In any and every local Lorentz frame, anywhere and anytime in the universe, all the (non-gravitational) laws of physics must take on their familiar special-relativistic forms.

Comma-goes-to-semicolon: The laws of physics, written in component form, change on passage from flat spacetime to curved spacetime by a mere replacement of all comas to semicolons (no change at all physically or geometrically: change due only to switch in reference frame from Lorentz to non-Lorentz!). This statement, like the nonchanging of abstract geometric laws, is nothing but a rephrased version of EP.

Comments:

- Should a general principle be stated in terms of one of our theories?

1.4 "Essential Relativity" by Rindler [6]

Weak EP:

- For all particles the inertial and gravitational masses are equal.
- All particles experience the same acceleration in a given gravitational field.

Strong or Einstein's EP: All local, freely falling, nonrotating laboratories are fully equivalent for the performance of all physical experiments.

Semi-Strong EP: All local, freely falling, nonrotating laboratories are fully equivalent for the performance of all physical experiments, but with the possibility of different numerical constants.

1.5 "General Relativity" by Wald [7]

EP: All bodies are influenced by gravity and, indeed, all bodies fall precisely the same way in a gravitational field.

Comments:

- Should we keep it that simple?

1.6 *The Strong Equivalence Principle* by Bertotti and Grishchuk [2]

Local dynamical system: Confining ourselves to slowly moving systems, we say that they are local if the measurement errors are greater than the corresponding effects of tidal forces.

Weak EP:

- All test bodies fall in a gravitational field with the same acceleration, independently of their mass and composition.
- Measurements within a local dynamical system are not affected by gravity.

Einstein's EP:

- The physical (non-gravitational) laws of a "small", freely falling system are universal.
- The physical (non-gravitational) behavior of a freely falling system can be made, to a given accuracy, universal (and in agreement with special relativity), by making its size sufficiently small.
- A "small" freely falling non-gravitational system is shielded from the external world, in the sense that the local observations within the system do not provide any information about its location in time and space.
- Non-gravitational measurements within a local dynamical system provide no information about the external world.

Local gravitational experiment: One in which the measurement errors make it impossible to detect the Newtonian tidal effects of the external curvature.

Strong EP:

- Fulfilled if when the size r of a system is sufficiently small, its dynamical behavior, to a given accuracy, is universal and not affected by the external world.
- Holds if a local gravitational experiment is in no way affected by the external world and is governed by universal laws.

Comments:

- Why define *local gravitational experiments* in terms of Newtonian forces?
- Do *local dynamical systems* exist for arbitrary experimental accuracies?

1.7 "Theory and Experiment in Gravitational Physics" by Will [8]

Weak EP:

- All bodies fall in a gravitational field with the same acceleration regardless of their mass or internal structure
- In Newtonian physics, $m_I = m_P$.
- If an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition.

Local nongravitational test experiment: (i) performed in a freely falling lab that is shielded and is sufficiently small that inhomogeneities in the external fields can be ignored throughout its volume, and (ii) in which self-gravitational effects are negligible.

Einstein's EP:

- In a freely falling elevator, all the laws of physics behave as if gravity were absent.
- (i) Weak EP is valid, (ii) the outcome of any local nongravitational test experiment is independent of the velocity of the (freely falling) apparatus, and (iii) the outcome of any local nongravitational test experiment is independent of where and when in the universe it is performed.

Gravitational weak EP: Weak EP is valid for self-gravitating bodies as well as for test bodies.

Strong EP: (i) Gravitational Weak EP is valid, (ii) the outcome of any local test experiment is independent of the velocity of the (freely falling) apparatus, and (iii) the outcome of any local test experiment is independent of where and when in the universe it is performed.

Analog of Schiff's conjecture: Any theory that embodies the Gravitational weak EP also embodies the Strong EP.

1.8 "Gravitation and Inertia" by Ciufolini and Wheeler [3]

Test particle: electrically neutral, with negligible gravitational binding energy compared to its rest mass, with negligible angular momentum, and small enough that inhomogeneities of the gravitational field within its volume have negligible effect on its motion.

Weak EP: The motion of any freely falling test particle is independent of its composition and structure.

Medium-Strong or Einstein's EP:

- For every pointlike event of spacetime, there exists a sufficiently small neighborhood such that in every local, freely falling frame in that neighborhood, all the nongravitational laws of physics obey the laws of special relativity.
- For every spacetime event, for any experimental apparatus, with some limiting accuracy, there exists a neighborhood, in space and time, of the event, and infinitely many local freely falling frames, such that for every nongravitational phenomenon the difference between the measurements performed and the theoretical results predicted by special relativity is less than the limiting accuracy and therefore undetectable in the neighborhood.

Very Strong EP: For every pointlike event of spacetime, there exists a sufficiently small neighborhood such that in every local, freely falling frame in that neighborhood, all the laws of physics obey the laws of special relativity.

1.9 Mathematical Formulation

In a neighborhood of *any point* p of a pseudo-Riemannian manifold, the metric can be written, to second order in the separation δx^α from p, as

$$\begin{aligned} g_{00} &= -1 - R_{0i0j}\delta x^i \delta x^j \\ g_{0k} &= -\frac{2}{3} R_{0ikj}\delta x^i \delta x^j \\ g_{kl} &= \delta_{kl} - \frac{1}{3} R_{kilj}\delta x^i \delta x^j \,, \end{aligned}$$

with $(\frac{\partial g_{\alpha\beta}}{\partial \delta x^\mu})_p = 0$. A coordinate system with this property, called a *Fermi Normal Frame*, corresponds physically to a freely falling, nonrotating, local inertial frame.

2 Quantum Challenges to the EP

2.1 Localizability

- Quantum mechanics is inherently nonlocal (e.g. entanglement or teleportation).
- Uncertainty principle puts limits on localizability.
- Quantum objects are extended and, as such, could be sensitive to departures of spacetime from exact flatness.

2.2 "Test particle" character

Quantum objects are not "test" objects because:

1. We may not be able to make the energy of a particle as small as we want in order to avoid back-reaction on a background spacetime.
2. We cannot make the momentum of a particle as small as desired and continue to demand localization.
3. Objects may be affected by observations.

2.3 Is the geometrical character of gravitation preserved?

- Alleged existence of quantum violations of the equivalence principle (e.g. mass dependence in COW experiments).
- Well defined criteria must be provided in order to determine whether gravity maintains its geometrical character at the quantum level.

References

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