On the Persistence of Entanglement in Relativistic Quantum Field Theory

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ABSTRACT

A notion of *entanglement between algebras* is defined here, which naturally applies to both relativistic quantum field theory and ordinary quantum mechanics. This puts one in a position to compare the behaviour of entangled correlations in the two theories. We then show that entanglement is more robust in the relativistic context. In particular, we develop a result by Clifton and Halvorson (2001) claiming that entanglement in algebraic quantum field theory would persist, no matter what local operations one performs.

1 Introduction

When introducing the concept of entanglement in 1935, Erwin Schrödinger identified it as *the* characteristic trait of quantum mechanics, having no analogue in the classical world. He pointed out some of its puzzling consequences, which reflect the non-local nature of quantum states. That even led him to doubt that the theory could be extended to a relativistic version. Yet, quantum field theory, that is the synthesis of quantum mechanics and special relativity, has been experimentally confirmed. Entangled states do exist in such a framework. It is then an intriguing problem for philosophy of physics whether adding relativistic constraints to quantum theory makes entangled correlations more robust; and if so, in what sense.

With the recent development of quantum information theory, entanglement, rather than being a source of conceptual difficulties, has become a resource to exploit. An issue which is thus interesting in its own is what an experimenter can or cannot do with entanglement in a relativistic context. Clifton and Halvorson (2001) maintained that, contrary to non-relativistic quantum mechanics, it is not always possible for an experimenter to destroy entanglement between spacelike separated quantum field systems by performing local operations. Their argument is cast in the framework of Algebraic Quantum Field Theory (AQFT), which is an axiomatic formulation of quantum field theory.

> We offer this result as one way to make precise the sense in which AQFT requires a radical change in paradigm - a change that, regrettably, has passed virtually unnoticed by philosophers of quantum theory. [Clifton-Halvorson (2001), p.5]

After spelling out their result, they added that

> ... the advantage of the formalism of AQFT is that it allows us to see clearly just how much more deeply entrenched entanglement is in *relativistic* quantum theory. At the very least, this should serve as a strong note of caution to those who would quickly assert that quantum nonlocality cannot peacefully exist with relativity. [Clifton-Halvorson (2001), p.28]

The formalism of AQFT is that of the algebraic approach to physical theories, which we briefly review in section 2. Within such a general mathematical setting, we define the notion of *entanglement between algebras*. This provides one with a criterion for comparing entangled correlations in ordinary quantum mechanics and quantum field theory. In section 2.2 we discuss one sense in which entanglement is more robust in the relativistic case. We then address the argument by Clifton and Halvorson and demonstrate that it is not as general as it may seem (section 3.1). After giving a precise formulation of the impossibility *in principle* of local disentanglement, we conclude the last section by showing how their result can be extended.

2 Entangled correlations: algebraically

In the algebraic approach physical observables are represented by self-adjoint operators which are elements of an algebra. The algebras describing classical systems are commutative, whereas those describing quantum systems are noncommutative. States of a physical system are given by positive, linear, normalized functionals on the algebra. Throughout the paper we assume that states are normal (i.e. countably additive) states. The algebraic formalism provides a rigorous framework for various physical theories. So, investigating structural features of the relevant algebras can reveal conceptual differences between such theories.

In particular, the theory of von Neumann algebra factors[1] encompasses both non-relativistic and relativistic quantum theories. The distinction between the corresponding algebraic structures is emphasized by Rudolf Haag, one of the founders of AQFT, in the following passage of his seminal book *Local Quantum Physics*:

> in quantum *mechanics* ... we can associate to each system or subsystem an algebra of type I, i.e. an algebra isomorphic to the set of all bounded operators on Hilbert space [i.e. the algebra $\mathcal{B}(\mathcal{H})$]. The change from the materially defined systems in mechanics to "open subsystems" corresponding to sharply defined regions in space-time in a relativistic local theory forces the change in the nature of the algebras from type I to type III_1. [Haag (1996), p.267]

According to Haag, at the fundamental level a division of the physical world into "subsystems" can be achieved in terms of spacetime regions. The primitive concept of AQFT is in fact the map

$$\mathcal{O} \longrightarrow \mathcal{A}(\mathcal{O})$$

associating any finite region of Minkowski space, in which a quantum field system would be localized, with a local algebra. $\mathcal{A}(\mathcal{O})$ contains all local observables that can be measured in $\mathcal{O}$. A set of physically and mathematically motivated axioms determines the type III character of the local algebras. Typical geometric configurations for bounded spacetime regions are double cones, formed by the intersection of the forward light-cone of a point with the backward light-cone of another timelike separated point. Spacelike separated regions are tangent if their closures intersect at one point; otherwise, they are strictly spacelike separated.

The postulates of special relativity are built in the theory by two axioms, namely relativistic covariance and microcausality. While the former assures that the group of symmetry is the Poincaré group, the latter expresses Einstein's principle of locality: it requires that local algebras corresponding to spacelike separated regions mutually commute, so that measurements of observables in the two algebras cannot disturb each other. To put it formally, if $\mathcal{O}_A$ is contained in the causal complement[2] of $\mathcal{O}_B$, that is $\mathcal{O}_A \subset \mathcal{O}'_B$, then

[1]See Redei and Summers (2007) for an accessible overview of von Neumann algebra factors and their classification in types.

[2]The causal complement of a region comprises the set of points in Minkowski space which are spacelike separated from all points in the region.

$\mathcal{A}(\mathcal{O}_A)$ is contained in the commutant[3] of $\mathcal{A}(\mathcal{O}_B)$, that is $\mathcal{A}(\mathcal{O}_A) \subset \mathcal{A}(\mathcal{O}_B)'$. Importantly, such a local relativistic character of the algebras peacefully co-exists with the quantum non-locality associated with the existence of entangled states. Bell's inequality is indeed violated in AQFT. Also, peculiar non-local effects are predicted by the Reeh-Schlieder theorem, which is a result of quantum field theory with no non-relativistic analogue. We thus agree with Clifton and Halvorson that what would make (sub-)systems open for Haag is the fact that "quantum field systems are *unavoidably* and *intrinsically* open to entanglement" (p.4). This claim is corroborated by entanglement being more robust in relativistic quantum field theory than in ordinary quantum mechanics. The algebraic approach supplies the means to demonstrate it in a neat and rigorous manner: one can describe entangled correlations in terms of general algebras and then contrast their behaviour in type I and type III factors.

2.1 Entanglement between algebras

Let $\mathcal{A}$ and $\mathcal{B}$ be two von Neumann algebras (with common unit) describing distinct physical systems, say A and B, respectively. The state ϕ defined on the joint algebra $\mathcal{A} \vee \mathcal{B}$ represents the global state of the composite system $A + B$. Such a state is said to be entangled across $(\mathcal{A}, \mathcal{B})$ just in case it cannot be written as the (weak-*) limit of convex combinations of product states on the joint algebra; otherwise, ϕ is called separable or unentangled. As product states amount to classical probabilities, it means that entangled states cannot be reconstructed out of classical correlations. This formalizes Schrödinger's remark that entanglement is a non-classical feature of quantum states. No uniquely defined measure of the degrees of entanglement is actually available in the literature. Yet, one can extract information as to whether a state is entangled from the behaviour of the maximal Bell correlation[4] $\beta(\phi, \mathcal{A}, \mathcal{B})$. Indeed, entanglement is a necessary condition for the failure of Bell's inequality. So, if $\beta(\phi, \mathcal{A}, \mathcal{B})$ attains a value greater than 1, the global state ϕ must be entangled across $(\mathcal{A}, \mathcal{B})$. Furthermore, if Bell's inequality is maximally violated, i.e. $\beta(\phi, \mathcal{A}, \mathcal{B}) = \sqrt{2}$, one infers that ϕ would

[3]The commutant of an algebra is defined as the set of all self-adjoint operators that commute with any element of the algebra.

[4]The maximal Bell correlation was introduced by Summers and Werner (1985) to derive the algebraic form of Bell's inequality. It is defined by the relation

$$\beta(\phi, \mathcal{A}, \mathcal{B}) = \frac{1}{2} \sup \phi(A_1(B_1 + B_2) + A_2(B_1 - B_2)) \tag{1}$$

where the supremum is taken over the observables $A_{1,2} \in \mathcal{A}$ and $B_{1,2} \in \mathcal{B}$. Accordingly, Bell's inequality is expressed by $\beta(\phi, \mathcal{A}, \mathcal{B}) \leq 1$.

be maximally entangled.

The notion of entanglement between algebras hinges on the fact that there exists a whole set of states across any pair of algebras. They correspond to all possible global states that the relevant physical systems can share. We can then introduce the following definitions, which presuppose the notion of entanglement of a state.

The algebras $\mathcal{A}$ and $\mathcal{B}$ are said to be *entangled* if and only if there exists some entangled state across them

The algebras $\mathcal{A}$ and $\mathcal{B}$ are said to be *deeply entangled* if and only if there is a dense set of entangled states across them

The algebras $\mathcal{A}$ and $\mathcal{B}$ are said to be *intrinsically entangled* if and only if all states across them are entangled

For two algebras to be entangled it is sufficient that at least one state across them, no matter which one, is entangled. Hence, $\mathcal{A}$ and $\mathcal{B}$ are *not* entangled just in case any global state is separable. A theorem by Raggio and Bacciagaluppi (1993) gives insight into this concept. Suppose the algebras are mutually commuting, then it follows that all states across them are unentangled if and only if $\mathcal{A}$ or $\mathcal{B}$ is commutative. Accordingly, even if just one of two physical systems is classical, no state across the relevant algebras can be entangled, thus enforcing the idea that entanglement is a peculiar feature of quantum mechanics. As an immediate corollary, one obtains a necessary and sufficient condition for entanglement between two algebras: $\mathcal{A}$ and $\mathcal{B}$ are entangled if and only if they are both noncommutative. The concept of a pair of algebras being intrinsically entangled stands at the opposite side of the spectrum as it requires that entanglement of a state is endemic for $(\mathcal{A}, \mathcal{B})$. In such a case, no global state is separable. Between these two extremes there is a range of possible configurations for the set of all states on the joint algebra. The notion of deeply entangled algebras becomes particularly interesting in that, under the choice of a suitable metric, it would lead one to establishing that the overwhelming majority of states across $\mathcal{A}$ and $\mathcal{B}$ are entangled.

A more refined characterization of these notions sues for specifying the degrees of entanglement between algebras. The relevant measure should possibly compute the number of global states which are entangled. A yet finer-grained quantification would also take into account the degrees of entanglement of each state. Here we just introduce the following definition.

The algebras $\mathcal{A}$ and $\mathcal{B}$ are said to be *maximally (intrinsically) entangled* if and only if they are intrinsically entangled and all states across them are maximally entangled

Pairs of algebras such that the Bell's inequality is maximally violated for all global states would then be maximally intrinsically entangled.

Notice that no assumption has been made regarding the structure of the algebras $\mathcal{A}$ and $\mathcal{B}$. The above definitions are indeed general. They naturally apply to both type I and type III factors.

2.2 Intrinsically entangled local algebras

In AQFT the state of a global field is a state ϕ defined on the joint algebra $\mathcal{A}(\mathcal{O}_A) \vee \mathcal{A}(\mathcal{O}_B)$, with $\mathcal{O}_A$ and $\mathcal{O}_B$ being spacelike separated regions of Minkowski space, in which two quantum field systems would be localized. Such a global state is said to be entangled across $(\mathcal{O}_A, \mathcal{O}_B)$ just in case it does not lie in the (weak-*) convex hull of product states on the joint algebra. With slight abuse of language, we refer to entanglement between two local algebras as entanglement across the corresponding regions.

Since $\mathcal{A}(\mathcal{O}_A)$ and $\mathcal{A}(\mathcal{O}_B)$ commute with each other by microcausality and they are noncommutative type III factors, the Raggio-Bacciagaluppi theorem guarantees the presence of at least one entangled global state ϕ across them. Accordingly, $\mathcal{O}_A$ and $\mathcal{O}_B$ are entangled. However, this fact is equally true for any pair of (mutually commuting) type I factors as well. A quite remarkable difference between the two cases arises, instead, with respect to intrinsically entangled algebras. In ordinary quantum mechanics, the tensor product structure holding for the description in Hilbert space of composite systems entails that there always exists some product state, and hence some separable global state. Accordingly, no pair of algebras describing distinct non-relativistic quantum systems can be intrinsically entangled. To the contrary, as we now show, in AQFT there are plenty of spacelike separated regions such that all states across them are entangled. It means that the quantum field systems localized in such regions are entangled in any possible global state. This is our first sense in which entanglement is more robust in the relativistic context.

Let us see a first concrete example. The environment of a quantum field is defined as the causal complement $\mathcal{O}'$ of the region $\mathcal{O}$ of Minkowski spacetime where the system is localized. Many models of free quantum field theories verify the so-called duality relation. That is a strengthening of microcausality which asserts that the local algebra associated with $\mathcal{O}'$ is equivalent to the commutant of the local algebra associated with $\mathcal{O}$. Summers (1990) proved that for any state across a type III factor and its commutant the maximal Bell correlation attains its maximal value. This algebraic fact combined with the duality relation yields $\beta(\phi, \mathcal{A}(\mathcal{O}), \mathcal{A}(\mathcal{O})') = \sqrt{2}$ for any state ϕ. It follows, strikingly enough, that any arbitrary spacetime region and its environment would be maximally intrinsically entangled, even for non-interacting fields.

The same conclusion can be drawn for two quantum field systems localized in tangent spacelike separated regions $\mathcal{O}_A$ and $\mathcal{O}_B$. All states across such regions are maximally entangled. In fact, as Summers and Werner (1988) demonstrated, they maximally violate Bell's inequality. Therefore, any pair of tangent regions, irrespective of their geometrical configuration, are maximally intrinsically entangled. For strictly spacelike separated regions, though, the situation may be less dramatic. The maximal Bell correlation is known to decrease with the minimum Lorentz distance $d(\mathcal{O}_A, \mathcal{O}_B)$ between the regions. Of course, when $\beta(\phi, \mathcal{A}(\mathcal{O}_A), \mathcal{A}(\mathcal{O}_B))$ takes its value in the open interval $]1, \sqrt{2}[$ for all global state ϕ, the Bell's inequality is neither satisfied nor maximally violated[5]. The corresponding regions would thus be intrinsically entangled, although not maximally entangled.

However, if $\mathcal{O}_A$ and $\mathcal{O}_B$ are strictly spacelike separated double cones there may be unentangled states across them, at least if they are sufficiently far apart. Specifically, there exists some product state if and only if the split property holds (Buchholz (1974)). The local algebras $\mathcal{A}(\mathcal{O}_A)$ and $\mathcal{A}(\mathcal{O}_B)$ are split[6] if, for any double cone $\tilde{\mathcal{O}}_A$ containing the closure of $\mathcal{O}_A$, there is a type I factor $\mathcal{M}$ such that

$$\mathcal{A}(\mathcal{O}_A) \subset \mathcal{M} \subset \mathcal{A}(\tilde{\mathcal{O}}_A) \subset \mathcal{A}(\mathcal{O}_B)' \tag{2}$$

The split property is verified by many concrete and physically relevant models of quantum field theories, such as that of free neutral massive scalar fields. Crudely put, such an algebraic configuration is the closest one would get to the non-relativistic case in a relativistic context. This intuition is captured by the type I character of the factor $\mathcal{M}$ approximating the local algebra $\mathcal{A}(\mathcal{O}_A)$, in the sense that $\tilde{\mathcal{O}}_A$ can be arbitrarily larger than $\mathcal{O}_A$. A tensor product structure between two type III factors can indeed be defined whenever they are split. That is the only circumstance in which spacelike separated regions fail to be intrinsically entangled in AQFT.

[5] Incidentally, this is the case for the so-called wedge regions, no matter what their degree of spacelike separation is (see Summers and Werner (1995)).

[6] Here we take up the characterization of the split property given by Summers (2009). He argues, among other things, that it yields a notion of independence that makes it meaningful to speak of "subsystems" in relativistic quantum theory.

Yet, even in such a case, entanglement proves very robust. As a general consequence of the Reeh-Schlieder theorem, there exists a dense set of entangled states across $(\mathcal{O}_A, \mathcal{O}_B)$. Recall that a vector-state x in the Hilbert space $\mathcal{H}$ underlying the von Neumann algebra $\mathcal{A}$ is cyclic for the latter just in case the set $\{Ax|A \in \mathcal{A}\}$ is dense in $\mathcal{H}$. Accordingly, one can approximate any arbitrary state by acting on x. Dixmier and Maréchal (1971) demonstrated that, if there is a cyclic vector for $\mathcal{A}$, then there is a dense set of cyclic vectors for the same algebra. A result by Clifton and Halvorson (2000) connects cyclicity and entanglement: the global state ϕ_x generated by x being cyclic for $\mathcal{A}$ is entangled across $(\mathcal{A}, \mathcal{B})$. The Rees-Schlieder theorem then guarantees the existence of cyclic vector-states in AQFT. Specifically, it asserts that a state of bounded energy is cyclic for *any* local algebra. In particular, it would be cyclic for $\mathcal{A}(\mathcal{O}_A)$. It follows, therefore, not only that every state of bounded energy, such as the vacuum, is entangled across $(\mathcal{O}_A, \mathcal{O}_B)$, but also that there is a dense set of entangled global states. This fact is quite independent from the particular geometry of the spacetime regions and their degree of spacelike separation. In the last analysis, any pair of spacelike separated regions are deeply entangled in AQFT.

3 The impossibility *in principle* of local disentanglement in AQFT

The idea of acting locally on one system is captured in a precise sense by the notion of local operations, which represent physical operations that an experimenter could perform on one system. A *local operation* in $\mathcal{A}$ is a completely positive[7] map $T : \mathcal{A} \vee \mathcal{B} \longrightarrow \mathcal{A} \vee \mathcal{B}$ satisfying $T(I) \leq I$ which leaves $\mathcal{A}$ invariant, in the sense that $T(A)$ belongs to $\mathcal{A}$ for any element A of $\mathcal{A}$. In other words, $T(\mathcal{A})$ would be a subalgebra of $\mathcal{A}$. The operation is non-selective if it preserves the unit I; it is selective otherwise. Given the initial state ϕ, the operation-conditioned state resulting from the application of T is denoted by $T^*\phi$. From an algebraic point of view, as T acts as the identity on the other algebra $\mathcal{B}$ which is assumed to commute with $\mathcal{A}$, its effect is to select the pair $(T(\mathcal{A}), \mathcal{B})$.

It is a well known result of quantum information theory due to Popescu and Rohrlich (1997) that it is impossible to create entanglement by performing local operations. To put it technically, if the global state ϕ is unentangled,

[7]Recall that the linear map T from a von Neumann algebra $\mathcal{C}$ onto itself is completely positive if it can be extended to a linear map $T_n : M_n(\mathcal{C}) \longrightarrow M_n(\mathcal{C})$ which is positive for every number n (where $M_n(\mathcal{C})$ is the set of n by n matrices with elements from $\mathcal{C}$). That is, T_n maps positive operators to positive operators.

the operation-conditioned state $T^*\phi$ will be unentangled too, for any local operation T in $\mathcal{A}$. It means that an experimenter cannot entangle any state between two quantum systems by acting locally. As a consequence, entanglement between algebras cannot be created either. This is a general fact, which is thus common to both type I and type III factors.

But, one may ask, can entanglement be destroyed by local operations? And does this mark any difference between ordinary quantum mechanics and relativistic quantum field theory?

3.1 Generalizing Clifton-Halvorson's *no go* result

Clifton and Halvorson (2001) addressed the above questions. Nonetheless, they do not provide a precise statement of what it takes for entanglement to be destroyed by local operations. We fill this gap below by defining the algebraic notion of local disentanglement. Subsequently, we show that their argument in AQFT lacks generality.

The global state ϕ entangled across $(\mathcal{A}, \mathcal{B})$ is said to be disentangled by the local operation T in $\mathcal{A}$ just in case the operation-conditioned state $T^*\phi$ is separable. Equivalently, T is a disentangling local operation for ϕ. The following notion captures the idea of destroying entanglement between algebras.

Local Disentanglement between algebras
$\mathcal{A}$ and $\mathcal{B}$ can be *locally disentangled* if and only if there is a local operation T such that all states ϕ across them are disentangled by T.

That is, there exists a T such that for all global states ϕ the operation-conditioned state $T^*\phi$ is separable. T is thus a disentangling local operation for the pair of algebras.

The following is a stronger, and perhaps less interesting, notion.

Strong Local Disentanglement between algebras
$\mathcal{A}$ and $\mathcal{B}$ can be *strongly locally disentangled* if and only if for every local operation T all states ϕ across them are disentangled by T.

That is, all T are such that for any global state ϕ the operation-conditioned state $T^*\phi$ is separable.

The impossibility of destroying entanglement between algebras by means of local operations is then defined as the denial of the above statements on local disentanglement. It would be an impossibility *in principle* if it is a consequence of some structural feature of the relevant algebras. One obtains respectively:

Impossibility of Local Disentanglement between algebras
No local operation T can disentangle all states ϕ across $(\mathcal{A}, \mathcal{B})$

That is, for every T there is some global state ϕ such that the operation-conditioned state $T^*\phi$ is entangled.

Impossibility of Strong Local Disentanglement between algebras
Not all local operations T can disentangle all states ϕ across $(\mathcal{A}, \mathcal{B})$

That is, there is some T and there is some global state ϕ such that the operation-conditioned state $T^*\phi$ is entangled.

Let us now reconstruct Clifton-Halvorson's *no go* result. They considered selective local operations in $\mathcal{A}$ of the form $T_P(\cdot) = P(\cdot)P$, with P being a projection. Recall that a projection P of $\mathcal{A}$ is *abelian* for a non-commutative von Neumann algebra $\mathcal{A}$ just in case the algebra $P\mathcal{A}P$ is commutative[8]. Any type I factor contains an abelian projection, whereas type III factors do not contain any. The argument exploits such a structural difference between the two algebraic configurations. One can show, indeed, that T_P disentangles $\mathcal{A}$ and $\mathcal{B}$ if and only if P is an abelian projection for $\mathcal{A}$. The proof goes as follows. If P is abelian, $T_P(\mathcal{A})$ is a commutative algebra, and hence by the Raggio-Bacciagaluppi theorem all states across $(T_P(\mathcal{A}), \mathcal{B})$ would be unentangled. Therefore, for any state initial entangled state ϕ, the operation-conditioned state T_P^* must be separable. On the other hand, if P is not abelian, there is at least one entangled state state across $(T_P(\mathcal{A}), \mathcal{B})$, since both algebras would be non-commutative. Such a state is the image under T_P^* of an initial state ϕ on $\mathcal{A} \vee \mathcal{B}$. Since entanglement cannot be created by local operations, one can infer that ϕ must have been entangled in the first place too. Therefore, there exists *some* entangled state across $(\mathcal{A}, \mathcal{B})$ that cannot be disentangled by T_P^*. For type I factors one can chose P to be abelian, so that one can always construct a disentangling local operation for any pair of algebras isomorphic to $\mathcal{B}(\mathcal{H})$. The persistence of entanglement between local algebras under T_P^* results, instead, from the lack of any abelian projection in type III factors. It thus depends on a structural feature of the relevant algebras of observables and, as such, it would determine an impossibility *in principle* of destroying entanglement.

[8] Abelian projections correspond to the atoms in the projection lattice of a factor. Their absence implies the non-existence of pure state on the algebra of observables.

Clifton and Halvorson then strengthened the conclusion by proving that there is a dense set of global states across any pair of spacelike separated regions $(\mathcal{O}_A, \mathcal{O}_B)$ which remain entangled under the action of T_P. They also argued that this would be true for non-selective (pure) projective operations as well. Yet, as it stands, their result does not extend to *all* local operations[9]. Thus, although it is sufficient to assure the possibility of local disentanglement in non-relativistic quantum mechanics, it is *not* sufficient to establish the impossibility of local disentanglement in relativistic quantum field theory. It only proves the impossibility *in principle* of strong local disentanglement between the local algebras of AQFT. There could then be some other local operation, of a different form, disentangling all states across $(\mathcal{O}_A, \mathcal{O}_B)$. To rule out such a possibility one needs to construct a general *no go* result.

3.2 The persistence of entanglement under local operations

The first step to generalizing Clifton and Halvorson's result is to recognize that the Raggio-Bacciagaluppi theorem provides necessary and sufficient conditions for achieving local disentanglement between algebras. In fact, it guarantees that if, and only if, $T(\mathcal{A})$ or $\mathcal{B}$ is commutative then all states across them are separable. The recipe for local disentanglement thus reads: the noncommutative algebras $\mathcal{A}$ and $\mathcal{B}$ can be locally disentangled just in case the effect of the local operation T is to select a commutative subalgebra of $\mathcal{A}$. Next, we formulate a conjecture that would capture a structural feature of type III factors, of which the absence of abelian projections entails a special case.

Conjecture: It $\mathcal{A}$ is a type III factor, then there is no local operation in $\mathcal{A}$ such that $T(\mathcal{A})$ is commutative.

Applied to AQFT, this means that the subalgebra $T(\mathcal{A}(\mathcal{O}_A))$ of $\mathcal{A}(\mathcal{O}_A)$ can never be commutative for T being a local operation performed in the region $\mathcal{O}_A$. If Conjecture is true, then we can derive the following *no go* result, which demonstrates the impossibility *in principle* of local disentanglement between local algebras.

[9] As a partial remedy, they expressed some reservations on whether arbitrary, rather than just pure, mixing operations may constitute genuine disentangling maps. Length constraints prevent us from addressing this point here. Be it as it may, the fact remains that their result cannot cover the general case.

General Result: No local operation T in $\mathcal{O}_A$ can disentangle all states across $(\mathcal{O}_A, \mathcal{O}_B)$.

This completes the generalization of Clifton-Halvorson's result, thus establishing the persistence of entanglement under *any* local operation in the relativistic context.

Granted that there exists some state ϕ across $(\mathcal{O}_A, \mathcal{O}_B)$ which can never be locally disentangled, one can pose further interesting questions. First, what are the global states that cannot be disentangled by local operations? And, second, when can a local operation disentangle some global state?

Based on the above conjecture, we prove a proposition to the effect that cyclic states would remain entangled under any local operation, which provides ground for answering the first question.

Proposition: Let $\mathcal{A}$ and $\mathcal{B}$ be mutually commuting type III factors. If Conjecture holds, then the global state ϕ_x generated by a vector-state x cyclic for $\mathcal{B}$ cannot be disentangled by any local operation T in $\mathcal{A}$.

Proof. Let us assume, for the sake of reductio ad absurdum, that $T^*\phi_x$ is not entangled. By cyclicity one can approximate any state ϕ on $T(\mathcal{A}) \vee \mathcal{B}$ by applying to $T^*\phi_x$ local operations T_B given by the elements of $\mathcal{B}$ (the existence of such operations is assured by the Kraus representation theorem). Since entanglement cannot be created by local operations, all ϕ must be unentangled. Nevertheless, Conjecture assures that $T(\mathcal{A})$ is noncommutative. It thus follows from the Raggio-Bacciagaluppi theorem that there is at least one entangled state across these two algebras. Hence, a contradiction is obtained from the assumption. □

Such a proposition together with the result by Dixmier and Maréchal on the existence of a dense set of cyclic states imply that $T(\mathcal{A})$ and $\mathcal{B}$ are deeply entangled. The Rees-Schlieder theorem then gives a sufficient condition for the following fact to be true in AQFT.

Strong General Result: There is a dense set of states across $(\mathcal{O}_A, \mathcal{O}_B)$ that cannot be disentangled by any local operation T in $\mathcal{O}_A$.

This offers a qualitative characterization of (a large class of) those states across spacelike separated regions that can never be locally disentangled. Specifically, all states of bounded energy would remain entangled, no matter how one acts locally. Furthermore, by choosing a suitable metric, one may be able to extend such a claim to the overwhelming majority of global states.

Our Strong General Result has an appealing fall-out for the second question concerning local disentanglement of a state too. Indeed, it seems to pose severe practical difficulties on an experimenter to destroying the entanglement of a global state of the field. Even if she manages to disentangle ϕ by performing the local operation T, it would be very hard for her to distinguish the resulting state $T^*\phi$ from any of the dense set of global states which remain entangled. The quandary becomes even made more dramatic in the absence of any well-defined measure of the degrees of entanglement in AQFT. Be it as it may, one can give a necessary and sufficient condition for local disentanglement of some global state in terms of the split property. That is, a local operation T in $\mathcal{O}_A$ disentangles some state ϕ across $(\mathcal{O}_A, \mathcal{O}_B)$ if and only if the pair of algebras $T(\mathcal{A}(\mathcal{O}_A))$ and $\mathcal{A}(\mathcal{O}_B)$ are split. It is still an open problem to determining under what circumstances this is actually the case.

We conclude by discussing Clifton and Halvorson's final remark. Let $\tilde{\mathcal{O}}_A$ be an arbitrarily large region of Minkowski spacetime which properly contains $\mathcal{O}_A$. Any local operation $\tilde{T}$ in $\tilde{\mathcal{O}}_A$ is said to be approximatively local in $\mathcal{O}_A$. One can demonstrate that, if $\mathcal{A}(\mathcal{O}_A)$ and $\mathcal{A}(\mathcal{O}_B)$ satisfy the split property, then there exists an approximatively local operation in $\mathcal{O}_A$ disentangling all states across the corresponding regions. Such a map can be constructed by exploiting the type I character of the von Neumann algebra $\mathcal{M}$ splitting the local algebras associated with $\mathcal{O}_A$ and $\tilde{\mathcal{O}}_A$. In particular, $\tilde{T}$ would be the extension to $\mathcal{A}(\tilde{\mathcal{O}}_A)$ of a projective operation defined by an abelian projection in $\mathcal{M}$. In the last analysis, disentanglement of one region from another spacelike separated one would be possible by performing a local operation in an arbitrarily larger region containing it[10].

> So as soon as we allow Alice [an experimenter] to perform *approximatively* local operations on her field system, she *can* isolate it from entanglement with other strictly-separated field systems... God is subtle, but not malicious. [Clifton-Halvorson (2001), p.29]

Let us stress, though, that such a quasi-local procedure is subjected to some non-trivial restrictions. The argument goes through just in case $\mathcal{O}_B$ is contained in the causal complement of $\tilde{\mathcal{O}}_A$. Whether this condition is fulfilled depends on the size of the region $\mathcal{O}_A$ and its degree of spacelike separation from $\mathcal{O}_B$. Furthermore, from a practical point of view, in order to perform

[10]To be sure, the action of $\tilde{T}$ would not disentangle $\tilde{\mathcal{O}}_A$ from $\mathcal{O}_B$! Indeed, while $\tilde{T}(\mathcal{A}(\mathcal{O}_A))$ is commutative, $\tilde{T}(\mathcal{A}(\tilde{\mathcal{O}}_A))$ ought to be noncommutative by Conjecture.

$\tilde{T}$ the experimenter must be allowed to step out of her original region and, so to speak, operate in a larger laboratory. It implies, however, that she would invade the causal complement of $\mathcal{O}_A$. It is not quite obvious, both at a theoretical and an operative level, whether this may be permissible at all.

4 Conclusion

We showed that entanglement is more robust in relativistic quantum field theory than in ordinary quantum mechanics in two well-defined senses. First, there are many pairs of local algebras in AQFT that are intrinsically entangled. Second, it is impossible *in principle* to disentangle two spacelike separated regions by performing local operations. Under certain conditions disentanglement could be achieved by means of some approximately local operation. However, this is proven only in the special circumstances in which the split property holds. *God is indeed benevolent, but quite demanding!*

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