UnBorn: Probability in Bohmian mechanics

Why are quantum probabilities encoded in measures corresponding to wave functions, rather than by a more general (or more specific) class of measures? Whereas orthodox quantum mechanics has a compelling answer to this question, Bohmian mechanics might not.

Word count: 4996! (not counting this line)

Probabilities run rampant in quantum phenomena. Here I raise a question about quantum probabilities for which orthodox quantum mechanics (hereafter QM) has a compelling answer, but Bohmian mechanics might not. I blame Bohmian mechanics' difficulty with the question on its reluctance to take the algebraic structure of quantum observables seriously, and sketch a way of framing Bohmian mechanics so that it's less troubled by the question — but more commital, in some surprising ways, about the structure of space.

1 Mechanics, three ways

The three ways are: classical, quantum, and Bohmian. Briefly reviewing each, this section emphasizes a sense in which Bohmian mechanics is *anti-structuralist*.

Focus on the simple case of a mass m point particle moving in one linear dimension. The much-prosecuted question of the *ontology* of the wave function gets stickier for more complicated systems. The questions addressed here arise even in this simple case.

Classical (Hamiltonian) mechanics (nicely encapsulated in Hall 2013, §2.5) assigns our particle a state in a phase space of ordered pairs of position and momentum values. The position observable Q and the momentum observable P are the obvious functions from points of phase space to the real numbers $\mathbb{R}$; all other classical observables are functions $f(Q, P)$ of these canonical observables. Given an energy observable H, Hamilton's equations impose dynamical trajectories on phase space. Mapping observables f and g to $\{f, g\} = \frac{\partial f}{\partial Q}\frac{\partial g}{\partial P} - \frac{\partial f}{\partial P}\frac{\partial g}{\partial Q}$, the Poisson bracket equips the collection of classical observables with physically significant structure: the Poisson bracket encodes dynamics via $\frac{df}{dt} = \{H, f\}$ (with Hamilton's equations resulting when f is set to P and Q); Hamiltonian symmetries are transformations that preserve the Poisson bracket. The Poisson brackets between canonical observables assume a pleasingly spare form: $\{Q, Q\} = \{P, P\} = 0$; $\{Q, P\} = 1$.

One route to **Quantum** mechanics (nicely encapsulated in Wald 1994, Ch 2) is canonical quantization, a tried and true recipe for basing a quantum theory of our particle on the classical theory just presented. The recipe exhorts us to find a Hilbert space $\mathcal{H}$ on which act symmetric position and momentum operators $\hat{q}$ and $\hat{p}$ satisfying the "lovely and ubiquitous" (Griffiths 2018, 41) canonical commutation relations:

$$[\hat{q}, \hat{q}] = [\hat{p}, \hat{p}] = 0, \quad [\hat{q}, \hat{p}] = i\hat{I} \qquad \text{CCRs}$$

which mirror the classical Poisson brackets between P and Q. (Notation: $[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$;

$\hat{I}$ the identity operator on $\mathcal{H}$; Planck's constant equals 1.) $\hat{q}$ and $\hat{p}$ represent, respectively, our particle's canonical position and momentum observables. To obtain other observables pertaining to our particle, start with $\hat{p}$ and $\hat{q}$ (or more precisely the operators in their spectral measures) and close under products, linear combinations, and limits (in $\mathcal{H}$'s weak topology). The von Neumann algebra $\mathfrak{B}(\mathcal{H})$ of bounded operators acting on $\mathcal{H}$ results. Quantum observables correspond to this algebra's self-adjoint elements. Thus each observable can be understood as physical in virtue of standing in an articulate functional relationship to the (presumptively physical) canonical observables $\hat{p}$ and $\hat{q}$. Once a Hamiltonian or energy observable $\hat{H}$ is specified, the Schrödinger equation determines a one parameter unitary family of dynamical automorphisms via $U(t) = exp(-i\hat{H}t)$. Were our particle subject to a restoring force, its Hamiltonian would have terms proportional to both $\hat{p}^2$ and $\hat{q}^2$; thinking of the Taylor series expansion defining $exp(-i\hat{H}t)$ for this Hamiltonian provides further motivation to vest products and linear combinations of the canonical observables with physical significance.

Extending throughout the observable algebra, the commutator bracket CCRs teems with information about quantum dynamics and quantum symmetries: $\frac{d\hat{A}}{dt} = [\hat{A}, \hat{H}]$ expresses Schödinger dynamics, for instance, and CCRs explains why position and momentum are Fourier-connected (see Folland 2016, Ch 1). Quantum observables form a collective with a physically potent structure of interrelationships, linking them to one another and the quantum theory to the classical one. Against the backdrop of this structure, quantum states are readily identified as normed, positive, countably additive linear functionals $\omega : \mathfrak{B}(\mathcal{H}) \to \mathbb{C}$, where $\omega(\hat{A})$ gives the expectation value of the observable $\hat{A}$. The set of states is convex; its extremal elements are pure states.

The Schrödinger representation is the standard — (almost) unique (Hall 2013, Ch.

14) — way to realize all this. It's set in the Hilbert space $L^2(\mathbb{R})$ of square integrable complex-valued functions of $\mathbb{R}$, where canonical observables act as follows on an arbitrary vector $|\phi(x)\rangle$ in this space:

$$\hat{q}|\phi(x)\rangle = x|\phi(x)\rangle \qquad \hat{p}|\phi(x)\rangle = -i\frac{d|\phi(x)\rangle}{dx}$$

If ω is a pure state on $\mathfrak{B}(L^2(\mathbb{R}))$, there's a unit vector $|\psi(x)\rangle \in L^2(\mathbb{R})$ such that $\omega(\hat{A}) = \langle\psi(x)|\hat{A}|\psi(x)\rangle$ for all observables $\hat{A}$. According to the Schrödinger equation, the dynamical automorphisms above implement time evolution for states: an initial state $|\psi(x,0)\rangle$ evolves over a time t to the state $U(t)|\psi(x,0)\rangle = |\psi(x,t)\rangle$. Let Γ be a subinterval of the real line. The spectral measure of $\hat{q}$ maps Γ to a projection operator, call it $\hat{P}_\Gamma$, in $L^2(\mathbb{R})$. $|\psi(x)\rangle$ assigns this projection operator an expectation value that coincides with the Born rule probability for obtaining an outcome in Γ if one performs a position measurement on a system in $|\psi(x)\rangle$:

$$Pr(q \in \Gamma) = \int_\Gamma \psi^*(x)\psi(x)dx \qquad \text{Born Rule}$$

This illustrates the truism that quantum probabilities are expectation values of projection operators.

Most quantum states decline to predict the values of most quantum observables with certainty. And for most pairs of quantum observables, there's a tradeoff between how accurately a state can predict their values —a tradeoff the terms of which our trusty commutator sets. Classical mechanics is decidedly more forthcoming: for each classical observable, each classical state predicts its value with certainty. One might wonder, concerning quantum observables, whether they always have precise values, notwithstanding the incapacity of quantum states to say what those values are. Sadly, a variety of "No-Go results" indicate that wholesale programs for assigning determinate values to quantum observables, if they abide by reasonable-looking constraints (such as restricting the range of possessed values to the range of values revealed upon measurement), are bound to contradict QM's empirical predictions. Nature's predilection for upholding those predictions condemn such programs to failure.

Bohmian mechanics is a selective program for entertaining determinate observable values not articulated by QM. The observable selected is *position*. Bohmian mechanics assigns our particle a (normed) wave function $\psi(x) \in L^2(\mathbb{R})$ and also a determinate position q, *even if* $\psi(x)$ *is not a* $\hat{q}$ *eigenstate*. The wave function undergoes Schrödinger evolution; the *guidance equation* assigns our particle a *velocity* that depends on its position and its wave function:

$$V(\psi, q) = \frac{1}{m} Im \left(\frac{\nabla \psi(x)}{\psi(x)} \right)_{|x=q} \qquad \textsc{Guidance}$$

(Notation: $\nabla \psi(x) = \frac{d\psi}{dx}$; Im extracts the imaginary part of its argument.) Our particle follows a continuous and deterministic trajectory, an integral curve of the velocity field Guidance defines: it "gets carried along with the flows of the ... wave function, just like a cork floating on a river" (Albert 1992, 139).

Bohm's 1952 debut of his theory cast it as a version of Hamiltonian mechanics, augmented by a distinctively quantum potential term. Most contemporary Bohmians prefer the guidance equation formulation just sketched, for reasons both philosophical and aesthetic. "Pure anachronism" (2011, 111) Maudlin calls the quantum potential formulation (see Fine 1996 for other perspectives).

Position and the wave function are the only variables dynamically salient to the Bohm theory. Observables other than position play a secondary role, not just mathematically but also (meta)physically. Consider a bivalent quantum observable $\hat{A}$. We can contrive situations where our particle gets together with a friend in such a way that their composite wave function correlates distinct $\hat{A}$ eigenstates of the friend with wave functions $\psi_L(x)$ and $\psi_R(x)$ of our particle with disjoint spatial support ($\psi_L(x)$ is non-zero only on the left half the room, say, and $\psi_R(x)$ only on the right half). Thinking of the spin and position degrees of freedom of an electron as different components of a composite system, a Stern-Gerlach measurement of electron spin has this basic plot. *In situations like this*, keeping track of the friend's $\hat{A}$ eigenstates is a way to track our particle's position. But not a reliable way: which $\hat{A}$ eigenstates get correlated with which range of positions depends on how the measurement is set up — one expression of the famous "contextualism" through which Bohmian mechanics escapes the ravages of No-Go results. Highly dependent on details of the interaction is the prior question of *whether* $\hat{A}$ eigenstates get correlated with positions at all. Bohmian systems always have, non-contextually, their positions. Other orthodox quantum magnitudes are unrobustly and intermittently vehicles of situationally convenient shorthands for talking about positions.

If position is the only genuine physical observable, there just aren't other quantities to which position might stand in robust and physically illuminating structural relationships codified by an algebra of enfranchised-as-physical quantities equipped by CCRs with a commutator bracket structure. Denying fundamental physical significance to observables other than position, Bohmian mechanics leaches physical significance from *collectives* of observables and their algebraic structure. This is the sense in which Bohmian mechanics is *anti-structuralist*.

2 Antistructuralism

It might tempting to think that Bohmian mechanics is QM and then some—that it's a sort of fan fiction that discloses more about certain central charismatic quantum characters than the official text, QM on its own, does. On this way of fitting the approaches together, Bohmian position is just QM position $\hat{q}$, but with an illuminating backstory. Underwriting that backstory is a theoretical apparatus that enables us to say more about $\hat{q}$ than QM says— to say, for instance, whether a system is located in Γ, even if its wave function fails to be a $\hat{P}_\Gamma$ eigenstate. It's crucial to fan fiction of this sort that the backstory is an elaborative commentary that refrains from doing wanton violence to the original. This section presents a few (wellknown) manifestations of Bohmian antistructuralism that upset this fan-fiction model.

Velocity. Heuristically, eigenstates of the quantum momentum observable $\hat{p}$ are plane waves $exp(ikx)$, with k the associated eigenvalue. This is only heuristic because $exp(ikx)$ isn't square integrable. Pleasingly, it falls directly out of GUIDANCE that, no matter what its position, a Bohmian particle with wave function $exp(ikx)$ has a Bohmian velocity equal to the $\hat{p}$ eigenvalue k divided by m. Let *Bohmentum* be Bohmian velocity times mass. In this case, Bohmentum behaves like quantum momentum, fostering the fan-fiction picture that GUIDANCE, by defining Bohmentums for systems not in momentum eigenstates, is telling us more about $\hat{p}$ than QM itself does.

But other cases upset the fan-fiction picture. Placate fussbudgets by confining our particle to a circle, the subinterval $[-\pi, \pi] \subset \mathbb{R}$ with endpoints identified. Periodic boundary conditions mean that $\hat{p}$ has honest-to-goodness eigenstates and a discrete spectrum. And it's easy to find a wave function for our particle—one that goes as $exp(-ix^2)$ for $x \in [-1, 1]$, for instance—for which GUIDANCE implies a *configuration-dependent* Bohmentum whose values aren't confined to $\hat{p}$'s spectrum. In QM, the events it's the duty of physics to assign probabilities have counterparts in the algebra of observables. Momentum values outside $\hat{p}$'s spectrum have no counterpart in the algebra. So Bohmian mechanics isn't merely saying more about quantum characters than QM itself does. It's introducing new characters, impossible quantum mechanically, and making them central to its narrative. That's doing violence to the original story. Positing Bohmentum values outside $\hat{p}$'s spectrum is a manifestation of Bohmian antistructuralism, its failure to respect the structure of quantum observables.

Energy. GUIDANCE assigns velocity 0 to systems whose wave functions are real-valued. This bugged Einstein (Fine 1996 elaborates). Those model organisms of physics, the harmonic oscillator and the particle in the box, have energy eigenstates that are real-valued wave functions: wave functions GUIDANCE assigns Bohmian velocity 0. Confined by an infinite square well potential, the particle in a box has 0 potential energy. So all its energy is kinetic. But no matter how much kinetic energy it has, its Bohmian velocity is 0. So where is its kinetic energy coming from?

Identifying the puzzle as arising from "cases where the ψ function is not approximated in the neighborhood of each point by a *travelling* wave," Einstein commented, "It is connected with this, that the Bohmian rule determines the momentum values not through a Fourier transform but rather through a *local* regularity in coordinate space" (as quoted in Fine 1996, 245). The connection forged by the Fourier transform is baked into CCRs. In both classical and quantum mechanics, velocity and kinetic energy are tightly interwoven; momentum and position are Fourier-connected. Another manifestation of Bohmian antistructuralism is to rupture this weave.

Bohmians tend to take examples like the foregoing to illustrate features, not bugs, of their approach. Some argue that only those who have fallen prey to "the fallacy of naive realism about operators" (Daumer et al 1997, 14) will be bothered by what I've labelled antistructuralism. The next section suggests that antistructuralism could have further consequences the foundationally-minded might find unsettling.

3 UnBorn

Here I characterize a foundational question about quantum probability that QM handles trippingly but Bohmian mechanics stumbles over. I blame the stumble on Bohmian antistructuralism.

Let $\hat{A}$ be a quantum observable other than position, and request a probability distribution over its values. Quantum mechanics will answer; but — absent a context rendering $\hat{A}$ an efficient way to talk about positions— Bohmian mechanics will not. We can hardly fault Bohmian mechanics for this reticence. It's declining to answer because it's rejecting a presupposition of the question, that $\hat{A}$ corresponds to a genuine property physics should be in the business of treating directly.

So let's be fair. Let's look for a question about quantum probabilities that Bohmian mechanics doesn't have an obvious right to reject. A Bohmian particle has a determinate position $q \in \mathbb{R}$. A probability distribution over particle positions corresponds to a measure μ, a map from Borel subsets Γ of $\mathbb{R}$ to $[0, 1]$ that's normed (i.e. $\mu(\mathbb{R}) = 1$) and countably additive. Call this a *position measure*. Where $\psi(x)$ is a wave function, $\mu(\Gamma) = \int_\Gamma \psi^*(x)\psi(x)dx$ gives a position measure. Call position measures so generated *Born measures*. And note that not every position measure is a Born measure: for each $q \in \mathbb{R}$, there's the discrete (i.e.. supported on a countable subset of $\mathbb{R}$) and decisive measure that sends $\{q\}$ to 1. Call such a position measure a q-measure. A q-measure is not a Born measure: no element of $L^2(\mathbb{R})$ induces such a measure through the Born rule.[1] (This reflects the absence from $L^2(\mathbb{R})$ of point-valued position eigenstates (see Halvorson 2001 for more).) Given that not all position measures are Born measures, and given that neither Bohmian nor quantum mechanics can reject questions about position measures on the grounds that position isn't a genuine observable, it's fair to ask them both

WHY BORN? Why identify position measures with Born measures?

Quantum mechanics has a lovely answer, mediated by Gleason's theorem. A *quantum probability measure* is a normed and countably additive map from projection operators on $\mathcal{H}$ to $[0, 1]$. Gleason's theorem alerts us that when $\mathcal{H} = L^2(\mathbb{R})$, quantum probability measures form a convex set, extremal elements of which correspond (via $\mu(\hat{P}) = \langle\psi(x)|\hat{P}|\psi(x)\rangle$) to normed wave functions. Restricting this map to projection operators in the spectral measure of the position operator $\hat{q}$ yields a position measure. From the standpoint of QM, wave functions and only wave functions code (pure)

[1]There are also continuous measures that aren't Born measures, although the continuous measures in question aren't absolutely continuous with respect to the Lebesgue measure dx—that is, they give positive measure to sets dx declares measure 0. While continuous non-Born measures are in a topological sense generic, physics tends to privilege the Lebesgue measure, which plays well with the Euclidean distance metric.

position measures because QM has structured quantum events in such a way that the only (pure) probability measures they admit are coded by wave functions.

Its antistructuralism prevents Bohmian mechanics from coopting this answer to WHY BORN?. Before canvassing some answers it might give, it's worth considering why Bohmians might want to answer WHY BORN? at all. Bohmian mechanics has already dismissed a slew of questions about quantum probabilities on the grounds that they concern observables that lack objective existence. Couldn't Bohmian mechanics dismiss WHY BORN? on the grounds that it concerns something else that, by its lights, lacks objective existence: probabilities?! Bohmian mechanics is a deterministic theory, and while we may distribute our subjective credences about the positions of our particle however we wish, such distributions are a matter of psychology, not physics.

While these grounds for dismissing WHY BORN? are in principle available to Bohmians, it would be the height of imprudence to rush to stand on them. Bohmian mechanics seeks to characterize a world of which quantum mechanics is empirically adequate, a world where *where predictions based on the Born rule are upheld*. The most straightforward way to do so is to adopt Born measures. (Fine sketches a "clever argument" (1996, 237) Bohmians can use to show that the empirical adequacy of conventional QM follows from the adoption.) If Bohmian mechanics adopts Born measures, it confronts WHY BORN?.

At least three strategies for dealing with WHY BORN? can be found in the literature. Calling them *stipulation*, *derivation*, and *sublimation*, I'll briefly discuss each in turn, concluding—tentatively!, due to the brevity of my considerations—none answer WHY BORN? as well as QM does. (Naturally, this doesn't settle a question that lies beyond the scope of this essay: how *all-things-considered* the merits of the approaches compare!)

Stipulation joins Bohm 1952 in outfitting Bohmian mechanics with an additional axiom, sometimes called *the distribution postulate*:

> for a particle with wave function $\psi(x)$ the prior epistemic probability of the configuration being in the region Γ of configuration space is given by $\int_\Gamma \psi^*(x)\psi(x)dx$. (cf. Barrett 2019, 191)

Stipulation's answer to WHY BORN? is: the Born measures are the only ones consistent with the distribution postulate!

QM answers WHY BORN? by appeal to the quantum event structure and theorems about it. *Stipulation* answers by, well, stipulation. This makes QM's answer WHY BORN? has a lot more explanatory *oomph* than *Stipulation*'s.

That's my main point. Subsidiary points concern how to interpret probabilities governed by the distribution postulate. Not knowing where my particle is entitles me to adopt subjective credences about its position. If I'm rational, those credences will constitute a position measure. But subjective credences are unattractive candidates for distribution postulate probabilities! Not only would this corrupt the Bohmian virtue (famously celebrated by Bell 1982) of foreswearing subjectivity, it would also give the Born rule undue influence over my credences (ordinarily constrained only by the probability calculus) and give my credences undue influence (regimented by the guidance equation) over my particle!

Bohm himself suggests a more promising interpretation:

> We do not predict or control the precise location of the particle, but have, in practice, a statistical ensemble with probability density $|\psi|^2$. The use of statistics is . . . merely a consequence of our ignorance of the precise initial

conditions of the particle. (1952, 171)

The probabilities are ensemble averages; the ensembles in question are swarms of particles sharing initial wave function $\psi(0)$; their initial positions are distributed according to the Born measure $|\psi(0)|^2$.

If each particle in a swarm of Bohmian particles initially distributed according to $|\psi(0)|^2$ evolves as Bohmian mechanics demands, at any later time t their positions will be distributed according to the Born measure $|\psi(t)|^2$, where $\psi(t)$ is the Schrödinger evolute of $\psi(0)$. That is to say, Born measures are *equivariant* with respect to Bohmian dynamics. Equivariance assures that, if a Born measure ever accurately characterizes a swarm of Bohmian particles, its evolutes always do. Note that equivariance is a virtue we can attribute probability measures even if we don't interpret those measures swarmwise. (Recalling the feature of Bohmian mechanics that bugged Einstein, note as well that assigning velocity 0 to systems associated with (stationary!) energy eigenstates is a pretty crafty thing to do if you care about equivariance.) Goldstein and Struyve (2007) show that $|\psi|^2$ is the only equivariant ψ-dependent measure with nice locality features.

But consider the decisive (so decisive that it doesn't depend on ψ) and discrete q-measure concentrated on the actual initial position of our particle. Telling us how positions evolve, Bohmian mechanics tells us how this measure evolves: it follows the Bohmian trajectory $q(t)$ that passes through q at $t = 0$. Our particle follows that trajectory too! Thus the q-measure concentrated on our particle's actual position satisfies any equivariance demand it's fair to place on it: if that measure accurately describes the distribution of our particle at any time, its evolutes do so at all times.

Interpreting distribution postulate probabilities as ensemble averages leaves us with a mystery if our particle is all alone in the world. There's no actual ensemble over which the Born measure defines a statistical average. How do swarm-appropriate probability notions apply to a lonely particle? Differently-distributed swarms ground different ensemble statistics; given that there's no actual swarm our particle belongs to, which ensemble's statistics code the measure it's appropriate to apply to our particle? One way to handle these questions attributes the wave function a nomological role that's not limited to telling actual particles where to *go*, but extends to telling non-actual particles where to *be*, were they to become actual. Then the swarm whose ensemble statistics give the probability measure for our lonely particle is a modal swarm, the swarm our lonely particle *would* belong to if gazillions of its possible companions became actual (Belot 2011 discusses similar moves in the context of spacetime geometry).

These comments about the interpretation of probability in Bohmian mechanics are subsidiary. The primary complaint about *Stipulation* is that its response to WHY BORN? rests on fiat rather than reasons.

Derivation tackles this complaint headon, by presenting the distribution postulate not as an axiom but as a theorem of Bohmian mechanics—specifically, an "H-theorem" to the effect that (almost all) initially random distributions of swarms of particles with wave function $\psi(0)$ evolve over time, via the Bohmian dynamics, to the Born distribution $|\psi(t)|^2$ (Valentini 1991). Thus no matter what their initial distribution, particles obedient to Bohmian mechanics wind up distributed in accordance with the distribution postulate — obviating the need to postulate that requirement!

As the evocation of the H-theorem might have primed us to expect, *Derivation*'s answer to WHY BORN? sit uneasily with equivariance. If equivariance holds and $|\psi(t)|^2$ describes the position distribution at time t, then $|\psi(0)|^2$—and not some other distribution and certainly not most random distributions — describes it at time 0. The reasons *Derivation* gives in response to WHY BORN? aren't reasons most Bohmians can live with.

Sublimation. For Bohmians, a particularly pointed variation on WHY BORN? is: why associate the underinformed Born measure $|\psi|^2$ with our particle when its actual position q anchors a much better informed q-measure? There is, of course, a proud tradition in physics of using "underinformed" measures to describe deterministically-evolving systems in underlying states not fixed by those measures. That tradition is called statistical mechanics, which imposes an equilibrium probability measure over microstates consistent with a system's macrostate. Although the system occupies some underlying microstate, its equilibrium probability measure isn't concentrated there.

Sublimation answers WHY BORN? by developing an analogy between equilibrium measures in statistical mechanics and Born measures in Bohmian mechanics

> We shall call the probability distribution on configuration space given by $\rho = |\psi|^2$ the *quantum equilibrium distribution*. ...we say that a system is in quantum equilibrium when the quantum equilibrium distribution is appropriate for its description. It is a major goal of this paper to explain what exactly this might mean and to show that, indeed, when understood properly, it is *typically* the case that systems are in quantum equilibrium. (Dürr et al 1992b, 856)

Sublimation seeks to justify the use of the Born measure $|\psi|^2$ through "a qualitative statistical analysis, roughly analogous to that of statistical mechanics, where the stationary (Liouville) measure on phase space plays an important role" (1992a, 9)—e.g.

equilibrium measures are stationary with respect to the Liouville measure. The analogy is only rough because the guidance equation determines a time-dependent velocity field; hence "we cannot expect the evolution on configuration space to possess a stationary distribution" (1992a). It is, however, perfectly fair to expect measures to be equivariant—and, given an equivariant measure ρ^{ψ}, to "ask for a distribution stationary relative to ρ^{ψ}" (1992b, 8). Arguing that Born rule measures fit this bill, *Sublimation* answers WHY BORN? by motivating the application of "equilibrium-y" measures to Bohmian systems, and by arguing that Born measures play the equilibrium-y measure role.

Even leading architects of this approach admit that its details are "delicate" and its success "controversial" (Goldstein 2021). The full account is far too intricate to do justice here. I'll settle for a few adamantly preliminary big picture observations.

First, there's something mighty *selective* about equilibrium measures in statistical mechanics: a small slate of candidate measures are singled out by a few physical desiderata; there's a tractable sense in which these measures are "close to" one another in the space of measures. By contrast, apart from precluding the q-measures it might otherwise be natural to use for Bohmian systems, Born measures barely seem selective at all: they include all measures absolutely continuous w.r.t. the beloved Lebesgue measure.[2] Criteria applied in statistical mechanics to privilege equilibrium measures appeal to observables and structures disenfranchised by Bohmian antistructuralism. Bohmians spurn Hamilton's equations; it makes no never mind to them that the phase

[2]See the previous footnote for why dx is beloved. The inclusion follows from the Radon-Nikodym theorem, by letting $\psi(x) = \sqrt{f(x)}$, where f is the Nikodym derivative integrated against dx to obtain a measure absolutely continous w.r.t. dx.

space of a Hamiltonian system comes equipped with a symplectic form encoding its geometry. Statistical mechanics privileges the Liouville measure for being built from the symplectic form and preserved under Hamiltonian flows. Bohmians can't use exactly these structures to explain what's special about Born measures. They can— and do!— use their own dynamical resources. My observation is that their antistructuralism forces them to, and that these resources appear less selective than their structurally-supported statistical mechanics counterparts.

Second, *Sublimation* presents equivariant Born measures as analogues of equilibrium measures. But doesn't QM harbor an even better analogue, in the form of ...equilibrium measures?! A quantum system whose dynamics is generated by a Hamiltonian $\hat{H}$ in thermal equilibrium at a temperature T can be assigned a Gibbs equilibrium state. For our particle, once $\hat{H}$ and T are specified, its Gibbs equilibrium state is uniquely determined. This is another sense in which *Sublimation* entertains a wider class of measures (Born measures) than the class of measures (Gibbs measures) the equilibrium analogy might lead us to expect.

But *third*, *Sublimation* entertains a *narrower* class of measures than its resolution to focus on *equivariant* measures might lead us to expect. I suggested above that the q-measure concentrated on the actual position of our particle satisfies any equivariance demand fair to impose on it. So a version of WHY BORN? specific to *Sublimation* is: why cast Born measures and only Born measures in the role of equivariant measures, when (by your lights perfectly clear and sensible) discrete q-measures are equivariant as well?

Nothing if not resourceful, *Sublimation* has sustained responses to these questions and others. I'll submit that I myself wouldn't praise the ensuing package as "low-brow" and "unsubtle" (1992, 169)—just a few notes in a wonderful ode Albert sings to the Bohm theory. Perhaps that says more about the height of my brow than it does about Bohmian mechanics. Still I'll hazard that, compared to QM's answer to WHY BORN?, *Sublimation*'s package will suffer with respect to received explanatory virtues, such as clarity, directness, straightforwardness. As with *Stipulation* and *Derivation*, my preliminary verdict is that *Sublimation*'s response to WHY BORN? compares unfavorably with QM's.

4 A friendly amendment

Bohmian antistructuralism impedes it from answering WHY BORN? as compellingly as QM does. This is far from a conclusive argument against Bohmian mechanics. It's rather an indication that there may be Bohmian stories to be told about quantum probabilities more resourceful, and more interesting—maybe even more highbrow and more subtle—than those considered here.

Although the following may not be among them, it's a story I like. It's a story according to which the universe begins with a huge swarm of Bohmian particles, all of whom share a wave function $\psi(x, 0)$ *determined* as follows: the distribution of initial positions of particles in the swarm gives[3] $\psi(x, 0)$'s magnitude at each point x—note that this addresses Albert's (2015, Ch. 7) worry that Bohmian configurations are "epiphenomenal"—, and a mildly eerie field on the $t = 0$ slice of space specifies $\psi(x, 0)$'s

[3]OK, constrains — but some curve-fitting notion could evoke to close the gap between the spatial distribution of an immense swarm, of necessity supported on a countable subset of $\mathbb{R}$, and a magnitude defined at each point of $\mathbb{R}$.

phase at each x. Granted, this initial condition equips space with some unanticipated extra structure. The payoff is that positing this structure renders $\psi(x, 0)$ comprehensible as both physical and objective, insofar as $\psi(x, 0)$ is determined by the bonus spatial structure and the actual initial distribution of particles. When through the offices of the guidance equation, $\psi(x, 0)$ and its evolutes push the Bohmian particles around, they're not responding to our ignorance or our to nearly-freely-set credences, but to that initial condition through the intervention of Bohmian theory. And although the initial composite wave function is separable, nothing about the story precludes interesting dynamics eventuating in entanglement down the road. Finally, while WHY BORN? is still answered by something like stipulation, it's a stipulation that answers a number of other pressing questions as well!

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