UnBorn: Probability in Bohmian Mechanics

Abstract Why are quantum probabilities encoded in measures corresponding to wave functions, rather than by a more general class of measures? Call this question WHY BORN?. Orthodox quantum mechanics has a compelling answer to WHY BORN?, I argue, but Bohmian mechanics might not. I trace Bohmian difficulties with WHY BORN? to its *antistructuralism*, its denial of physical significance to the algebraic structure of quantum observables, and propose other cases where Bohmian antistructuralism might have an explanatory cost.

1 Introduction

Probabilities run rampant in the quantum world. Here I raise a question, WHY BORN?, about quantum probabilities for which orthodox quantum mechanics (hereafter QM) has a compelling answer, but Bohmian mechanics might not. I blame Bohmian mechanics' difficulty with WHY BORN? on its reluctance to take the algebraic structure of quantum observables seriously, and sketch other cases where this reluctance might be an explanatory deficiency.

I don't take the considerations developed here to deal Bohmian mechanics a fatal blow. Rather, I take them to draw attention to features of the approach that might just be bugs. Adjudicating the status of these features should interest not only fans and foes of Bohmian approaches, but also students of the project of making physical sense of quantum theories.

Here's the plan: §2 offers a speedy review of classical, quantum, and Bohmian mechanics, one that foregrounds aspects of those theories that matter to what comes next. Prominent among these is what I term the *antistructuralism* of Bohmian mechanics. §3 explains and illustrates antistructuralism. §4 poses WHY BORN?, and reviews answers quantum and Bohmian. QM's answer, I suggest, constitutes the better explanation. To inspire suspicion that this is not an isolated incident, §5 describes other cases where Bohmian antistructuralism might inhibit its explanatory reach.

2 Mechanics, Three Ways

The three ways are: classical, quantum, and Bohmian. Briefly reviewing each, this section highlights a sense in which Bohmian mechanics is *antistructuralist*.

I focus throughout on the case of a non-relativistic mass m point particle moving in one linear dimension. The much-prosecuted question of the *ontology* of the wave function gets stickier for more complicated systems; relativistic systems trigger anxiety about how Bohmian mechanics meshes with relativity theory. Those issues I can bracket. WHY BORN? arises even in this simplest of cases.

2.1 Classical

Classical (Hamiltonian) mechanics (encapsulated in Hall 2013, §2.5) assigns our particle a state in a phase space of ordered pairs of position and momentum values. The position observable Q and the momentum observable P are the obvious functions from points of phase space to the real numbers $\mathbb{R}$; all other classical observables are functions $f(Q, P)$ of these canonical observables. Given an energy observable H, Hamilton's equations impose dynamical trajectories on phase space. Mapping observables f and g to $\{f, g\} = \frac{\partial f}{\partial Q}\frac{\partial g}{\partial P} - \frac{\partial f}{\partial P}\frac{\partial g}{\partial Q}$, the Poisson bracket equips the collection of classical observables with physically significant structure: the Poisson bracket encodes dynamics via $\frac{df}{dt} = \{H, f\}$ (with Hamilton's equations resulting when f is set to P and Q); Hamiltonian symmetries are transformations that preserve H and the Poisson bracket. The Poisson brackets between canonical observables assume a pleasingly spare form: $\{Q, Q\} = \{P, P\} = 0$; $\{Q, P\} = 1$. Position and momentum are canonically conjugate *because* their Poisson brackets assume this form.

2.2 Quantum

One route to Quantum mechanics (charted by Wald 1994, Ch 2) is canonical quantization, a tried and true recipe for basing a quantum theory of our particle on the classical theory just presented. The recipe exhorts us to find a Hilbert space $\mathcal{H}$ on which act symmetric position and momentum operators $\hat{q}$ and $\hat{p}$ satisfying the "lovely and ubiquitous" (Griffiths 2018, 41) canonical commutation relations:

$$[\hat{q}, \hat{q}] = [\hat{p}, \hat{p}] = 0, \quad [\hat{q}, \hat{p}] = i\hat{I} \quad \text{CCRs}$$

which mirror the classical Poisson brackets between P and Q. (Notation: $[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$; $\hat{I}$ the identity operator on $\mathcal{H}$; Planck's constant equals 1.) $\hat{q}$ and $\hat{p}$ represent, respectively, our particle's canonical position and momentum observables. To obtain other observables pertaining to our particle, start with $\hat{p}$ and $\hat{q}$ (or more precisely the operators in their spectral measures) and close under products, linear combinations, and limits (in $\mathcal{H}$'s weak topology). The von Neumann algebra $\mathfrak{B}(\mathcal{H})$ of bounded operators acting on $\mathcal{H}$ results.

$\mathfrak{B}(\mathcal{H})$'s self-adjoint elements correspond to quantum observables. Thus each observable can be understood as physical in virtue of standing in an articulate algebraic-cum-functional relationship to the (presumptively physical) canonical observables $\hat{p}$ and $\hat{q}$. If we need to make a case for the physical significance of the canonical observables, their role in canonical quantization and the success of that recipe give us plenty of material to work with.

Once a Hamiltonian (energy) observable $\hat{H}$ is specified, the Schrödinger equation determines a one parameter unitary family of dynamical automorphisms $U(t) = exp(-i\hat{H}t)$. This vests not just $\hat{q}$ and $\hat{p}$, but also their products and linear combinations, with physical import: for a free particle, $\hat{H}$ is proportional to $\hat{p}^2$; subjecting our particle to a spring-like restoring force adds a term proportional to $\hat{q}^2$ to its Hamiltonian; plugging this full Hamiltonian into $U(t) = exp(-i\hat{H}t)$ and calculating the right hand side via Taylor series expansion involves all sorts of products and linear combinations of $\hat{q}$ and $\hat{p}$.

Extending throughout the observable algebra, the commutator bracket CCRs teems with information about quantum dynamics and quantum symmetries: $\frac{d\hat{A}}{dt} = [\hat{A}, \hat{H}]$ expresses (Heisenberg picture) Schödinger dynamics; as Folland (2016) outlines, CCRs, the Lie algebra for the Heisenberg symmetry group, explains why position and momentum are Fourier-connected. Quantum observables form a collective with a physically potent structure of interrelationships, linking them to one another and the quantum theory to the classical one. Against the backdrop of this structure, quantum states are neatly identified as normed, positive, countably additive linear functionals $\omega : \mathfrak{B}(\mathcal{H}) \rightarrow \mathbb{C}$, where $\omega(\hat{A})$ gives the expectation value of the observable $\hat{A}$. The set of states is convex; its extremal elements are pure states.

The Schrödinger representation is the standard — (almost) unique (Hall 2013, Ch. 14) — way to realize all this. It's set in the Hilbert space $L^2(\mathbb{R})$ of square integrable complex-valued functions of $\mathbb{R}$, with canonical observables acting as follows on an arbitrary vector $|\phi(x)\rangle$:

$$\hat{q}|\phi(x)\rangle = x|\phi(x)\rangle \qquad \hat{p}|\phi(x)\rangle = -i\frac{d|\phi(x)\rangle}{dx}$$

If ω is a pure state on $\mathfrak{B}(L^2(\mathbb{R}))$, there's a unit vector $|\psi(x)\rangle$ such that $\omega(\hat{A}) = \langle\psi(x)|\hat{A}|\psi(x)\rangle$ for all observables $\hat{A}$. (Notation: $\langle\psi(x)|\phi(x)\rangle = \int \psi^*(x)\phi(x)dx$ gives the inner product on $L^2(\mathbb{R})$.) Via the Schrödinger equation, the dynamical automorphisms $U(t) = exp(-i\hat{H}t)$ implement time evolution for states: an initial state $|\psi(x,0)\rangle$ evolves over a time t to the state $U(t)|\psi(x,0)\rangle = |\psi(x,t)\rangle$.

Let Γ be a subinterval of the real line. The spectral measure of the position observable $\hat{q}$ maps Γ to a projection operator, call it $\hat{P}_\Gamma$, in $\mathfrak{B}(L^2(\mathbb{R}))$. A state $|\psi(x)\rangle$ assigns this projection operator an expectation value that coincides with the probability for obtaining an outcome in Γ upon subjecting a system in $|\psi(x)\rangle$ to a position measurement:

$$Pr(q \in \Gamma) = \int_\Gamma \psi^*(x)\psi(x)dx \qquad \text{BORN RULE}$$

Perhaps the most familiar form of the Born rule, this illustrates the truism that QM probabilities are expectation values of projection operators.

If all the ps and qs, hatted or otherwise, in the foregoing two paragraphs trade places, we obtain a representation of CCRs on $L^2(\mathbb{R})$, whose elements $\psi(p)$ we now understand as *momentum space* wave functions. BORN RULE, $\hat{p}$-style, gives a probability distribution over possible momenta of our particle. Considered as quantum theories, the configuration space and momentum space representations of CCRs are equivalent in the following sense (explicated by unitary equivalence; Hall 2013, Ch. 14, is a careful statement): their observable algebras are isomorphic, and there's a bijection between configuration space states $\psi(x)$ and momentum space states $\psi(p)$ that under the isomorphism preserves expectation value assignments. Fourier analysis being a familiar technique for translating between configuration space and momentum space states, the equivalence of position and momentum space representations is known as *Fourier duality*.

Most quantum states decline to predict the values of most quantum observables with certainty. And for most pairs of quantum observables, there's a tradeoff between how accurately a state can predict their values —a tradeoff the terms of which our trusty commutator sets. Classical mechanics is decidedly more forthcoming: for each classical observable, each classical state predicts its value with certainty. One might wonder, concerning quantum observables, whether they always have precise values, notwithstanding the incapacity of quantum states to say what those values are. Sadly, a variety of "No-Go results" indicate that wholesale programs for assigning determinate values to quantum observables, if they abide by reasonable-looking constraints (such as restricting the range of possessed values to the range of values revealed upon measurement), fail.

2.3 Bohmian

Bohmian mechanics (reviewed by Barret 2019, Ch. 11) is a selective program for entertaining determinate observable values not articulated by QM . The observable selected is *position*. Bohmian mechanics assigns our particle a (normed) configuration space wave function $\psi(x) \in L^2(\mathbb{R})$ and also a determinate position (aka *configuration*) q, *even if* $\psi(x)$ *is not a* $\hat{q}$ *eigenstate*. The wave function evolves via

$$i\frac{d\psi(x)}{dt} = (\frac{1}{2m}\frac{d^2}{dx^2} + V(x))\psi(x) \qquad \text{SCHRÖDINGER?}$$

where $V(x)$ is a potential energy function—say $\frac{1}{2}kx^2$ if our system is subject to a restoring force with spring constant k.

SCHRÖDINGER? bears an uncanny resemblance to the Schrödinger equation. Indeed, blithely recast using Schrödinger representation position and momentum operators, with $\frac{\hat{p}^2}{2m} + V(\hat{q})$ playing the role of $\hat{H}$, it *is* the Schrödinger equation. For reasons we'll dwell upon presently, some Bohmians resist such recasting. They depict SCHRÖDINGER? not as a unitary Hilbert space evolution generated (as the Schrödinger equation demands) by the Hamiltonian operator, but merely as "the simplest choice of covariant equation for the guiding field ψ" (Dürr, Goldstein, and Zanghí 1992b, 8).

A Bohmian particle's configuration evolves via a *guidance equation* defining a *velocity* that depends on its position and its wave function:

$$V(\psi, q) = \frac{1}{m} Im \left(\frac{\nabla \psi(x)}{\psi(x)} \right)_{|x=q} \qquad \text{GUIDANCE}$$

(Notation: $\nabla\psi(x) = \frac{d\psi}{dx}$; Im extracts the imaginary part of its argument.) Our particle follows a continuous and deterministic trajectory, an integral curve of the velocity field GUIDANCE defines: it "gets carried along with the flows of the ... wave function, just like a cork floating on a river" (Albert 1992, 139).

Bohm's 1952 debut of his theory cast it as a version of Hamiltonian mechanics, augmented by a distinctively quantum potential term. Most contemporary Bohmians prefer the guidance equation formulation just sketched, for reasons both philosophical and aesthetic. "Pure anachronism" (2011, 111) Maudlin calls the quantum potential formulation. (See Cushing, Fine, and Goldstein 1996 for other perspectives.) I'll stick with guidance equation formulation.

In that formulation, position and the wave function are the only dynamically salient variables. Observables other than position play a secondary role, not just mathematically but also (meta)physically. Consider a quantum observable $\hat{A}$. We can contrive situations where our particle gets together with a friend in such a way that their composite wave function correlates distinct $\hat{A}$ eigenstates of the friend with disjointly supported wave functions $\psi_L(x)$ and $\psi_R(x)$ of our particle— $\psi_L(x)$ is non-zero only on the left half the room, say, and $\psi_R(x)$ only on the right half. Letting the spin and position degrees of freedom of an electron be different components of a composite system, a Stern-Gerlach measurement of electron spin has this basic plot. *In situations like this*, tracking the friend's $\hat{A}$ eigenstates is a way to track our particle's position. But not a reliable way: which $\hat{A}$ eigenstates get correlated with which range of positions depends on how the measurement is set up — one expression of the "contextualism" through which Bohmian mechanics escapes the ravages of No-Go results. Highly dependent on details of the interaction is the prior question of *whether* $\hat{A}$ eigenstates get correlated with positions at all.

Bohmian systems always have, non-contextually, their positions. Other orthodox quantum magnitudes are unrobustly and intermittently vehicles of situationally convenient shorthands for talking about positions. Some Bohmians are adamant that

> *'Properties' that are merely contextual are not properties at all; they do not exist and their failure to do so is in the strongest sense possible!* (Dürr, Goldstein, and Zanghí 2004, 1045; underlining mine)

Now we can see the point of the question mark in SCHRÖDINGER?. Observables appearing in the quantum Hamiltonian threaten to sneak through a back door to the (otherwise very lonely) inner sanctum of truly physical properties: insofar as those observables drive the Schrödinger evolution of the guiding field, they lay claim to dynamical salience. The machinations following the introduction of SCHRÖDINGER? serve to slam this back door shut.

If position is the only genuine physical observable, there just aren't other quantities to which position might stand in robust and illuminating relationships codified by an algebra of enfranchised-as-physical quantities equipped by CCRs with a physically potent structure. Denying physical significance to observables other than position, Bohmian mechanics leaches physical significance from *collectives* of observables and their algebraic structure. This is the sense in which Bohmian mechanics is *antistructuralist*. The next section takes a closer look at Bohmian antistructuralism. §§4-5 assess its explanatory costs. (North 2021 and Wallace 2021 make other cases for the significance of structure.)

3 Antistructuralism

It might be tempting to think that Bohmian mechanics is QM and then some—that it's a sort of fan fiction that discloses more about certain central charismatic quantum characters than the official text, QM on its own, does. On this way of fitting the approaches together, Bohmian position is just QM position $\hat{q}$, but with an illuminating backstory. Underwriting that backstory is a theoretical apparatus that enables us to say more about $\hat{q}$ than QM says— to say, for instance, whether or not a system is located in Γ, even if its wave function fails to be a $\hat{P}_\Gamma$ eigenstate. It's crucial to fan fiction of this sort that the backstory is an elaborative commentary that refrains from doing wanton violence to the original. This section reviews a few (well-known) manifestations of Bohmian antistructuralism that upset this fan fiction model.

3.1 Velocity

Heuristically, eigenstates of the quantum momentum observable $\hat{p}$ are plane waves $exp(ikx)$, with k the associated eigenvalue. This is only heuristic because $exp(ikx)$ isn't square integrable.[1] Pleasingly, it falls directly out of Guidance that, no matter what its position, a Bohmian particle with wave function $exp(ikx)$ has a Bohmian velocity equal to the $\hat{p}$ eigenvalue k divided by m. Let *Bohmentum* be Bohmian velocity times mass. In this case, Bohmentum behaves like QM momentum $\hat{p}$, fostering a fan fiction picture. In this piece of fan fiction, Guidance defines Bohmentums for systems not in momentum eigenstates, and binds those Bohmentums to our particle's wave function and position in ways QM does not—thereby disclosing more about $\hat{p}$ than QM itself does.

Other cases upset the fan fiction picture. Placate fussbudgets by confining our particle to a circle, the subinterval $[-\pi, \pi] \subset \mathbb{R}$ with endpoints identified. Periodic boundary conditions mean that $\hat{p}$ has honest-to-goodness eigenstates and a discrete spectrum. And it's easy to find a wave function for our particle—one that goes as $exp(-ix^2)$ for $x \in [-1, 1]$, for instance—for which Guidance implies a *configuration-dependent* Bohmentum whose values aren't confined to $\hat{p}$'s spectrum.

[1] See Sen 2022 for an account, intersecting this essay's themes, of how Bohmian mechanics might handle non-normalizable states. I am obliged to Chip Sebens for bringing this work to my attention.

In QM, the events it's the duty of physics to assign probabilities have counterparts in the algebra of observables. Momentum values outside $\hat{p}$'s spectrum have no counterpart in the algebra. So Bohmian mechanics isn't merely saying more about quantum characters than QM itself does. It's introducing new characters, impossible quantum mechanically, and making them central to its narrative. That's doing violence to the original story. Bohmentum isn't quantum momentum $\hat{p}$. Bohmian antistructuralism, its failure to respect the structure of quantum observables, undermines the fan fiction model.

3.2 Position

Bohmentum isn't quantum momentum — but neither is it a fundamental observable of Bohmian mechanics. Position, however, is. And even for position, the fan fiction model fails. Bohmian position diverges from its eponymous quantum counterpart, associated with the operator $\hat{q}$. By assigning our particle an exact position, Bohmian mechanics does wanton violence, of a distinctly antistructuralist stripe, to QM's narrative.

We're conducting QM in the ambit of the Schrödinger representation, where $\hat{q}$'s spectral resolution maps $\Gamma \subset \mathbb{R}$ to a projection operator $\hat{P}_\Gamma$, and $\langle\psi(x)|\hat{P}_\Gamma|\psi(x)\rangle$ is the Born rule probability the wave function $\psi(x)$ assigns a position measurement outcome in Γ. Consider a precise position $q \in \mathbb{R}$. Its counterpart in the quantum observable algebra is $\hat{P}_{\Gamma=\{q\}}$, the element $\hat{q}$'s spectral resolution associates with the point set $\{q\}$. But $\hat{P}_{\Gamma=\{q\}}$ coincides with the zero operator (Halvorson 2001 explains why). The zero operator corresponds to the null event assigned probability 0 by every quantum state, the self-contradiction of quantum logic. Attributing our particle a precise position q, Bohmian mechanics countenances a condition without counterpart in the quantum observable algebra. It's not just saying something that QM doesn't about the QM position observable $\hat{q}$. It's saying something that QM can't, on pain of contradiction, say about that observable. The Bohmian account of position isn't fan fiction but an entirely different story.

This argument, that Bohmian position and QM position come apart, unfolds in the Schrödinger representation. Its conclusion, more carefully stated, is that Bohmian mechanics says something that Schrödinger representation QM can't about position. There are, however, other quantum resources (carefully mined by Halvorson 2001) for making sense of a position observable whose spectral resolution maps $\hat{P}_{\Gamma=\{q\}}$ to a non-zero projection operator. However, exploiting these resources requires abandoning the Schrödinger representation for a representation set in a non-separable Hilbert space. There are a good reasons for Bohmians to be leery of invoking this non-separable, non-Schrödinger, *position representation* to forge a connection between Bohmian position and QM position. One is, exactly because of its antistructuralism, Bohmian mechanics can't readily explain *why* the position representation is an appropriate setting for the theoretical treatment of our particle. By the lights of QM, the position representation passes muster because it affords a representation of an exponentiated version of CCRs known as the Weyl relations—and that's a basis for a quantum theory. However, it's a basis recognizable as such only through the lens of the algebraic relations realized in the representation—a lens Bohmian antistructuralism has shattered. Another reason for Bohmian leeriness of the position representation is that it's not clear how to formulate Bohmian dynamics there. Position representation wave functions, counterparts of configuration space wave functions in the Schrödinger representation, are square summable functions from countable subsets of the reals to the complex numbers—functions for which SCHRÖDINGER? makes no sense. So while there are quantum resources for making sense of precisely located point particles, Bohmian antistructuralism hinders access to those resources. This reinforces the conclusion that Bohmian antistructuralism upsets the fan fiction connection between Bohmian position and QM position.

3.3 Energy

GUIDANCE assigns velocity 0 to systems whose wave functions are real-valued. This bugged Einstein (see Fine 1996 or Myrvold 2003 for more). Those model organisms of physics, the harmonic oscillator and the particle in the box, have energy eigenstates that are real-valued wave functions: wave functions GUIDANCE assigns velocity 0. Confined by an infinite square well potential, the particle in a box has 0 potential energy. So all its energy is kinetic. But no matter how much kinetic energy it has, if its wave function is an energy eigenstate, its Bohmentum is 0.

Einstein fretted about obtaining the classical limit, a particle bouncing back and forth between the sides of the box, from a theory in which the particle velocity is resolutely 0. Conceding that "from a purely logical standpoint there is no principled objection to be made against Bohm's completion of the quantum theory," he continues

> Looked at from a physical standpoint, however, Bohm's way out does not seem to me acceptable. In all cases where the ψ function is not approximated in the neighborhood of each point by a *travelling* wave, one obtains values for the momenta that violate the requirement that the quantum theory should go over to classical mechanics *in the limit*. It is connected with this, that the Bohmian rule determines the momentum values not through a Fourier transform but rather through a *local* regularity in coordinate space. (1953 letter, translated by Fine 1996, 245)

The very same year, Pauli expresses the very same qualm (see Myrvold 2003 for more). Observing that the Bohm theory denies that both members of a pair of canonically conjugate variables are genuinely properties, Pauli objects that

> [this] strips physical meaning from the simple passage, by way of Fourier analysis, between one wave function and another expressed in terms of the conjugate variable ([a passage] which leaves us free to consider either function as "primary"). Or it introduces an asymmetry to the interpretation of canonically conjugate quantities, [an asymmetry] for which we find reason neither in the system of our experiments nor in the

mathematical formalism of wave mechanics. (Pauli 1953, 39; inexpertly translated)[2]

The connection forged by the Fourier transform is baked into CCRs. In both classical and quantum mechanics, velocity and kinetic energy are tightly interwoven; momentum and position are Fourier-connected. Another manifestation of Bohmian antistructuralism is to rupture this weave.

3.4 Rhetoric

Bohmians are wont to take examples like the foregoing to illustrate a virtue, not a vice, of their approach. Some argue that only those who have fallen prey to "the fallacy of naive realism about operators" (Daumer, Goldstein, and Zanghí 1997, 14) will be bothered by antistructuralism. Concerning energy and momentum, quantum observables just marshalled to dramatize antistructuralism, Bohmians aver that "in the transition from classical mechanics, they cease to remain properties at all" (Dürr, Goldstein, and Zanghí 1996, 27). So much for Fourier duality![3]

I didn't need to introduce "antistructuralism" as a term of art at all. I could have simply characterized orthodox QM as committing, and Bohmian mechanics as resisting, naive realism about operators. I've multiplied terminology beyond logical necessity. While I cheerfully acknowledge this, I hasten to observe that it's not unusual for different parties to a debate to deploy differently-valenced language to describe positions in the debate, as a way of leveling the dialectical playing field.[4] "Naive realism about operators" certainly sounds like a loser, as positions go. So I'll plead guilty to multiplying terminology beyond logical necessity, but cite rhetorical exigencies in my defense!

The next section attempts to lift the debate out of the register of pure rhetoric, by suggesting that Bohmian antistructuralism could have consequences unsettling to the foundationally-minded.

4 UnBorn

4.1 The Question

Here I characterize a foundational question about quantum probability that QM handles trippingly but Bohmian mechanics stumbles over. I blame the stumble on Bohmian antistructuralism.

Let $\hat{A}$ be a quantum observable other than position, and request a probability distribution over its values. Quantum mechanics will answer; but — absent a context rendering $\hat{A}$ an efficient way to talk about positions— Bohmian mechanics will not. We can hardly fault Bohmian mechanics for this

[2] Ceci dépouille de son sens physique le simple passage par analyse de Fourier d'une fonction d'onde à celle de la variable conjuguée (ce qui nous laisse le choix de consideérer soit l'une soit l'autre comme la fonction "primaire") ou introduit une asymétrie par rapport à l'interpretétation de grandeurs canoniquement conjuguées pour laquelle on ne trouve de raison ni dans le système de nos expériences ni dans le formalisme mathématique de la mécanique ondulatoire.

[3] Redacted Thanks to Francisco Calderon for this percussive characterization of antistructuralism.

[4] A circumstance explaining my title's evocation of rhetoric from US debates about abortion.

reticence. It's declining to answer because it's rejecting a presupposition of the question, that $\hat{A}$ corresponds to a genuine property physics should be in the business of treating directly.

So let's be fair. Let's find a question about probabilities that Bohmian mechanics doesn't have an obvious right to reject. A Bohmian particle has a determinate position $q \in \mathbb{R}$. A probability distribution over particle positions corresponds to a probability measure μ, a map from Borel subsets Γ of $\mathbb{R}$ to $[0, 1]$ that's normed (i.e. $\mu(\mathbb{R}) = 1$) and countably additive. Call this a *position measure*. Where $\psi(x)$ is a wave function, $\mu(\Gamma) = \int_\Gamma \psi^*(x)\psi(x)dx$ gives a position measure. Call position measures so generated *Born measures*. And note that not every position measure is a Born measure. For instance, for each $q \in \mathbb{R}$, there's the discrete (i.e.. supported on a countable subset of $\mathbb{R}$) and decisive measure that sends $\{q\}$ to 1. Call such a position measure a *q-measure*. A q-measure is not a Born measure: no element of $L^2(\mathbb{R})$ induces a q-measure through the Born rule.[5] (This reflects the absence, encountered in §3.2, of point-valued position eigenstates from $L^2(\mathbb{R})$.) Given that not all position measures are Born measures, and given that neither Bohmian nor quantum mechanics can reject questions about position measures on the grounds that position isn't a genuine observable, it's fair to ask them both

> Why Born? Why identify position measures with Born measures?

For Bohmians, an especially pointed version of Why Born? is: why associate an underinformed Born measure with a particle, when its actual position anchors a much better informed q-measure?

4.2 QM's lovely answer

QM's lovely response to Why Born? is mediated by Gleason's theorem (Hughes 1989 is an introduction). A *quantum probability measure* is a normed and countably additive map from projection operators on $\mathcal{H}$ to $[0, 1]$. Gleason's theorem alerts us that when $\mathcal{H} = L^2(\mathbb{R})$, quantum probability measures form a convex set, extremal elements of which correspond (via $\mu(\hat{P}) = \langle\psi(x)|\hat{P}|\psi(x)\rangle$) to normed wave functions. Restricting this map to projection operators in the spectral measure of the position operator $\hat{q}$ yields a position measure. From the standpoint of QM, wave functions and only wave functions code (pure) position measures because QM has structured quantum events in such a way that the only (pure) probability measures they admit are coded by wave functions.[6] All and only Born measures are (pure) position measures.

Its antistructuralism prevents Bohmian mechanics from coopting QM's lovely answer to Why Born?. QM's answer takes the algebraic structure of quantum observables seriously, and Bohmian

[5] There are also continuous measures that aren't Born measures, although the continuous measures in question aren't absolutely continuous with respect to the Lebesgue measure dx—that is, they give positive measure to sets dx declares measure 0. While continuous non-Born measures are in a topological sense generic, physics tends to privilege the Lebesgue measure, which plays well with the Euclidean distance metric.

[6] But what if we relieve quantum probability measures of the obligation to be countably additive? If I had space, I'd suggest that QM's account would still prevail. See Earman and Ruetsche (2020) for hints about how I'd use the space if I had it!

mechanics does not. Indeed, “non-commutative quantum probability theory”— the probability theory Gleason’s theorem characterizes—numbers among the approaches Daumer Goldstein, and Zanghí convict of “naive realism about operators” (1997, 14, 16)!

It is instructive to compare Bohmian mechanics to “hidden variable” approaches to quantum phenomena that aren’t antistructuralist. [7] I have in mind approaches, such as the modal interpretation and the family of strategies Bub (1997) studies, that *do* take observables to have determinate values on systems whose states aren’t eigenfunctions of those observables (these determinate values are the “hidden variables”) but *don’t* forsake the algebra of quantum observables. Bub honors that algebra through his pursuit of the question of how large a set of determinate observables can get before running afoul of No-Go results. Modal interpretations express allegiance through a state-dependent criterion for determinateness: with different observables determinate in different states, all the observables in the algebra share a potential for physical significance. While these approaches shine an ontological light, as it were, on elements of the algebra of quantum observables privileged as hidden variables, they don’t throw the remainder of that algebra away. This entitles them to follow QM in invoking the algebraic structure of quantum observables and Gleason’s theorem to answer Why Born?. Bohmian mechanics, by contrast, doesn’t merely shine an ontological light on position. It wields that light as a laser scalpel excising position from an observable algebra the rest of which it condemns to physical insignificance. Its antistructuralism, not its hidden variables or their dynamics, precludes Bohmian mechanics from giving QM’s lovely answer to Why Born?.

Before canvassing responses to Why Born? that are available to Bohmian mechanics, it’s worth considering why Bohmians might want to answer Why Born? at all. Bohmian mechanics has already dismissed a slew of questions about quantum probabilities on the grounds that they concern observables that lack objective existence. Couldn’t Bohmian mechanics dismiss Why Born? on the grounds that it concerns something else that, by its lights, lacks objective existence: probabilities?! Bohmian mechanics is a deterministic theory, and while we may distribute our subjective credences about our particle’s position however we wish, such distributions are a matter of psychology, not physics.

While these grounds for dismissing Why Born? are in principle available to Bohmians, it would be the height of imprudence to rush to stand on them. Bohmian mechanics seeks to characterize a world of which quantum mechanics is empirically adequate, a world where *where predictions based on the Born rule are upheld*. The most straightforward way to do so is to adopt Born measures. Fine sketches a “clever argument” (1996, 237) Bohmians can use to show that the empirical adequacy of conventional QM follows from the adoption. If Bohmian mechanics adopts Born measures, it confronts Why Born?.

Note that any approach to the quantum world that uses Born measures will for that reason enjoy empirical adequacy. Hence any approach that uses Born measures, if challenged to justify their use,

[7] Thanks to Doreen Fraser for suggesting the instructive comparison!

can vindicate that use by appeal to empirical adequacy. WHY BORN?, I want to insist, should be heard not as question of empirical vindication but as a question of theoretical validation. Observing that "Born measures work!" vindicates them empirically. To validate them theoretically is to explain—if you can!—what they're doing in your theory: to explain how and why your theoretical apparatus invites (or even better mandates!) Born measures. Heard as a question of empirical vindication, WHY BORN? is vital but not interesting. It's vital because sanctioning the empirical application of Born measures is a condition of survival for approaches to the quantum world. It's not interesting because surviving approaches — approaches on the table, approaches whose merits and frailties we're sorting through, approaches worth their salt—are tied with respect to the question of empirical vindication. All of them can reply, "Born measures work!" Heard as question of theoretical validation, by contrast, WHY BORN? promises to effect distinctions between approaches worth their salt. Some of them might give better—clearer, more cohesive, deeper, more robust—answers than others. I've been calling better answers to WHY BORN?, heard as a question of theoretical validation, better explanations. QM's lovely answer, just rehearsed, is a paradigm of theoretical validation. I'm about to argue that QM thereby sets an explanatory standard Bohmian mechanics doesn't meet.

4.3 Bohmian Answers

At least three strategies for dealing with WHY BORN? can be found in the literature.[8] Calling them *Stipulation*, *Relaxation*, and *Sublimation*, I'll briefly discuss each in turn, concluding—tentatively!, due to the brevity of my considerations—none answer WHY BORN? as well as QM does. Naturally, this doesn't settle a question that lies beyond the scope of this essay: how *all-things-considered* the merits of the approaches compare!

4.3.1 Stipulation

If Bohmian trajectories are deterministic, where do quantum probabilities come from? Bohm's original paper includes a suggestion.

> We do not predict or control the precise location of the particle, but have, in practice, a statistical ensemble with probability density $|\psi|^2$. The use of statistics is ... merely a consequence of our ignorance of the precise initial conditions of the particle. (1952, 171)

Bohm's idea is that quantum probabilities are ensemble averages, where the ensembles in question are swarms of particles sharing initial wave function ψ. *Stipulation* follows Bohm[9] by outfitting Bohmian mechanics with an additional axiom, *the distribution postulate*, governing how the positions of particles in such a swarm are distributed:

[8] For fourth and fifth ways, see Stoicu 2022 (a reconception of $\psi(x)$ that induces Born measures as a generalization of counting measures, which I'm obliged to Arthur Fine for knowing about) and Steeger 2022 (a Bohmian appropriation of the Deutsch-Wallace justification (Wallace 2003) of the Born Rule in the Everett interpretation).

[9] and others, including Albert 1987, 138; Bell 1982, "163"; Cushing 1996, 5; Bricmont 2016, 138.

DISTRIBUTION: for a particle with initial wave function ψ, the probability of its initial configuration being in the region Γ of configuration space is given by $\int_\Gamma \psi^*(x)\psi(x)dx$. (Compare Barrett 2019, 191)

Stipulation's answer to WHY BORN? is: Born measures are the only ones consistent with the distribution postulate!

QM answers WHY BORN? by appeal to the structure of quantum events, a structure it has a host of additional theoretical reasons to posit, along with an elegant theorem about admissible probability measures over events so structured. *Stipulation* answers WHY BORN? by ...stipulation. QM's answer shows how its central theoretical apparatus makes Born measures more or less inevitable. *Stipulation* takes the central Bohmian theoretical apparatus—SCHRÖDINGER? and GUIDANCE— and tacks Born measures on. QM's answer rests on reasons; *Stipulation*'s on fiat; widely-shared explanatory norms rate reasons more explanatory than decrees. I conclude that QM's answer to WHY BORN? constitutes a better explanation than *Stipulation*'s.

4.3.2 Interlude: Probability and equivariance

That's my main point about *Stipulation*. Auxiliary points concern how to interpret probabilities governed by DISTRIBUTION. Not knowing where my particle is entitles me to adopt subjective credences about its position. If I'm rational, those credences will constitute a position measure. But subjective credences are unattractive candidates for distribution postulate probabilities! Not only would this corrupt the Bohmian virtue (famously celebrated by Bell 1982) of foreswearing subjectivity, it would also give the Born rule undue influence over my credences (ordinarily constrained only by the probability calculus) and give my credences undue influence (regimented by GUIDANCE) over my particle! The version of *Stipulation* just sketched avoids these embarassments by interpreting distribution postulate probabilities swarm-wise, as ensemble averages that have nothing to do with us or our credences.

Such an interpretation has an attractive consequence. If each particle in a swarm of Bohmian particles initially distributed according to $|\psi(0)|^2$ evolves as Bohmian mechanics demands, at any later time t their positions will be distributed according to the Born measure $|\psi(t)|^2$, where $\psi(t)$ is the SCHRÖDINGER? evolute of $\psi(0)$. That is to say, Born measures are *equivariant* with respect to Bohmian dynamics. Equivariance assures that, if a Born measure ever accurately characterizes a swarm of Bohmian particles, its evolutes always do. (Note that equivariance is a virtue we can attribute probability measures even if we don't interpret those measures swarmwise. Recalling the circumstance that bugged Einstein, note also that assigning velocity 0 to systems associated with (stationary!) energy eigenstates is a pretty crafty thing to do if you care about equivariance.) Goldstein and Struyve (2007) argue that $|\psi|^2$ is the only equivariant ψ-dependent measure with nice locality features.

But consider the decisive (so decisive that it doesn't depend on ψ) and discrete q-measure concentrated on the actual initial position of our particle. Telling us how positions evolve, Bohmian mechanics tells us how this measure evolves: it follows the Bohmian trajectory $q(t)$ that passes through q at $t = 0$. Our particle follows that trajectory too! Thus the q-measure concentrated on our particle's actual position satisfies any equivariance demand it's fair to place on it: if that measure accurately describes the distribution of our particle at any time, its evolutes do so at all times. Equivariance is an attractive consequence of interpreting DISTRIBUTION's Born measures swarmwise—but Born measures aren't the only measures that are appealingly equivariant.

And interpreting distribution postulate probabilities as ensemble averages has some less attractive consequences. For one thing, it feels illicit if our particle is all alone in the world. There's no actual ensemble over which the Born measure defines a statistical average. How do swarm-appropriate probability notions apply to a lonely particle? Put another way, how exactly does DISTRIBUTION constrain worlds possible according to Bohmian mechanics? For finite n, *any* distribution of positions among n particles sharing initial wave function ψ (assumed, to simplify the example, to be everywhere supported) looks consistent with DISTRIBUTION, in the sense that no such configuration gets assigned probability 0 by the $|\psi|^2$ measure. DISTRIBUTION will have some bite when it comes to possible worlds teeming with continuously many particles sharing an initial wave function. About worlds that are meaningfully compliant with DISTRIBUTION, it's tempting to raise a version of the *Euthyphro* question: is the humungous swarm distributed according to $|\psi|^2$ because each of its members has wave function ψ, or does each of its members have wave function ψ because they belong to a swarm distributed according to $|\psi|^2$? The symmetry is imperfect: the distribution constrains only the wave function's amplitude, but not its phase, at each point. Even so, I think there are two candidate directions of fit here.

Expositions of DISTRIBUTION don't always yoke it to an ensemble interpretation of the Born measures it posits. Barrett states DISTRIBUTION in terms of "prior epistemic probabilties", which spins the postulate as a recommendation about how to tune our credences (see also Albert 1992, 140); Albert tells a "fairy tale" about how God sprinkles particles across the initial timeslice of the universe (1992, 138-139).

I am trying to suggest that some mysteries attend the interpretation of distribution postulate probabilities. But that is not the main thing I want to say about *Stipulation*. The main thing is: *Stipulation*'s status as a stipulation renders its response to WHY BORN? less explanatory than QM's.

4.3.3 *Relaxation* and *Sublimation*

Recall the variant of WHY BORN? particularly pointed for Bohmian mechanics: why associate the underinformed Born measure $|\psi|^2$ with our particle when its actual position q anchors a much better informed q-measure? There is a proud and—as Ismael (2009) urges—warranted tradition in physics of using "underinformed" measures to describe deterministically-evolving systems in underlying states not fixed by those measures. That tradition is statistical mechanics, which imposes an equilibrium probability measure over microstates consistent with a system's macrostate.[10] Even though a system occupies some underlying microstate, its equilibrium measure isn't concentrated there. *Relaxation* and *Sublimation* are Bohmian responses to WHY BORN? that pursue analogies with equilibrium. But they pursue different analogies. For *Relaxation*, it's paramount that equlibrium in classical statistical mechanics is dynamically-induced. For *Sublimation* it's paramount that equilibrium informs appraisals of *typicality*. Here I'll offer introductions to *Relaxation* and *Sublimation* that, although exceedingly rough, disclose enough about their characters to suggest that their answers to WHY BORN? are less explanatory than QM's.

Relaxation. On a way of thinking originating with Boltzmann, equilibrium comes about dynamically: a system initially in a non-equlibrium state will evolve into an equilibrium state (see Uffink 2007 for a less barbaric overview of the foundations of statistical mechanics). Lending aid and comfort to this "relaxation to equilibrium" picture is the observation that equilibrium states occupy a volume of the system's available phase space huge—Goldstein gives a ratio of $10^{10^{20}}$ for macroscopic systems (2012, 6) — compared to the volume occupied by non-equilibrium states. These volumes are gauged by a measure, the Liouville measure, whose claim to physical relevance rests on its invariance under the system's classical Hamiltonian dynamics. Such gargantuan volume disparities make it plausible that dynamical trajectories originating in non-equilibrium regions will wander into the equilibrium region and stay a while. And for systems whose dynamics are ergodic, there is a precise result: apart from a set of initial states with Liouville measure 0, trajectories split their time between equilibrium and non-equilibrium states in the exact proportion the accessible region of phase space splits its volume between equilibrium and non-equilibrium states. Alas, next to no dynamical systems of physical interest are known to be ergodic. So the foregoing considerations fall short of a perfectly rigorous argument that real life systems relax to equilibrium. How far the considerations can be tightened, and by what means, are lively topics in the philosophy of statistical physics. It's not our job to settle such questions here. Enough has been said to introduce *Relaxation*'s response to WHY BORN?.

Rather than *postulating* Born measures, *Relaxation* undertakes to exhibit them as a consequence of Bohmian dynamics: just as statistical mechanical systems relax to equilibrium states, swarms of Bohmian particles relax to Born measure distributions. If initial distributions of swarms of particles sharing (but not distributed according to the Born measure encoded by) wave function $\psi(0)$ evolve over time, via the Bohmian dynamics, to the Born distribution $|\psi(t)|^2$, Bohmian particles wind up distributed as DISTRIBUTION would predict— obviating the need to postulate that requirement! Arguing that random collisions can bring about Born distributions, Bohm (1953) initiated the *Relaxation* approach; it's been developed extensively—including through numerical simulations —by Valentini and collaborators (Valentini 1991a, Valentini and Westman 2004; Valentini 2020, §3, is

[10] For instance, the classical Gibbs equilbrium measure $\rho = exp(-H/kT)/Z$, where H is the system Hamiltonian, T its temperature, k Boltzmann's constant, and Z a normalization factor constructed from the foregoing and called the partition function.

a recent review).

I've encountered a boneheaded version of *Relaxation* in the context of pedagogy that contradicts equivariance. If equivariance holds and $|\psi(t)|^2$ describes the position distribution at time t, then (where $\psi(0)$ is the SCHROÖDINGER? devolute of $\psi(t)$) $|\psi(0)|^2$—and no other distribution — describes it at time 0. Equivariance leaves no way for a range of initial fine-grained distributions to evolve so that they converge to the Born distribution.

But this simple-minded and obviously flawed version of *Relaxation* is not what its proponents espouse. It's *coarse-grained* measures—measures over finite cells partitioning $\mathbb{R}$, say— that they contend are driven dynamically driven toward Born measures. And they don't claim that convergence occurs for arbitrary initial distributions. Rather they show that sufficiently "smooth" initial distributions — roughly speaking, those that don't vary *within* the cells coarsening the grain — tend under the influence of the Bohmian dynamics toward coarse-grained Born measures. A hitch here is that the actual initial distribution of a finite swarm is anything but smooth: rather it's an exquisitely fine-grained mixture of q-measures corresponding to the initial positions of swarm members, weighted by the number of members at each position. Enter numerical simulations: simulations of the Bohmian evolution of *finite* swarms whose initial positions are randomly distributed w.r.t. "smooth" measures exhibit relaxation toward Born distributions.

Relaxation won't unwind arbitrary initial distributions. Imagine a boxfull of particles sharing a real-valued wave function ψ_n, the n^{th} eigenstate of the box Hamiltonian. And suppose their initial positions are not distributed according to $|\psi_n|^2$ but smoothly concentrated on one side of the box. Their wave function is stationary under SCHRÖDINGER?; their configurations are stationary under GUIDANCE because ψ_n is real-valued (§3.3). Staying put, they'll persist in defying even the coarse-grained Born measure $|\psi_n|^2$.

One way to deal with recalcitrant initial conditions is to produce measure theoretic reasons to dismiss them as "measure 0" red herrings. *Relaxation*'s advocates doubt that there are natural measures, sufficiently analogous to the dynamically privileged Liouville measure of statistical mechanics, to impose on the space of initial conditions. Instead, *Relaxation* suggests that if Born measures prevail nowadays, it's thanks to the contingent empirical fact that the actual initial distribution of particles in the universe was among those driven by Bohmian dynamics toward a Born measure distribution. It follows that *Relaxation* countenances Bohmian universes that don't instantiate Born measures due to "unlucky" initial conditions, or don't instantiate Born measures in regimes where the relaxation mechanisms are suppressed. Valentini (2020, §4) lists possible empirical signatures, all of them cosmological, of such suppression.

WHY BORN?: Why identify position measures with Born measures? *Relaxation*'s response rests an analysis, conducted at a level of rigor characteristic of working physics, of coarse-grained probability measures brought about by Bohmian dynamics launched from suitable initial states, coupled with the empirical posit that the initial state of our universe was suitable. This supports the conclusion that prevailing coarse-grained position measures are Born measures.

Sublimation. *Sublimation*'s analogy with equilibrium might best be appreciated against the backdrop of its overall approach to WHY BORN?. So let's start with *Sublimation*'s big picture, drawn in exceedingly broad brush strokes, because space prohibits a detailed rendering (see Dürr, Goldstein, and Zanghí 1992a for more, including expositions of terms italicized in what follows). The universe in its entirety is a Bohmian system, with initial wave function Ψ_0 and initial configuration Q_0. $|\Psi_0|^2$ gives a measure (indeed, a Born measure!) over possible initial configurations of the universe. For each subsystem of the universe, the universal wave function and the configuration of its supersystem defines a *conditional wave function* for that subsystem. There are circumstances — roughly, decoherence processes correlating subsystem wave functions with *macroscopically distinct* supersystem wavefunctions—in which conditional wave functions are also *effective wave functions*, which we can use to predict measurement statistics. The *coup de grace* is a result showing that,

> For the overwhelming majority—in the sense of the measure $|\Psi_0|^2$—of initial configurations of a Bohmian universe, the empirical distribution for the positions of particles ... in suitable real world ensembles of systems having [effective] wave function ψ is (approximately) [the Born measure] $|\psi|^2$. (Goldstein 2012, §4)[11]

WHY BORN asks: why identify position measures with Born measures? *Sublimation*'s answer: in "most"—as gauged by the $|\Psi_0|^2$ measure over initial configurations— Bohmian universes, Born measures encoded by effective wave functions describe how particle positions are distributed. *Sublimation*'s preferred[12] formulation of this "mostness" claim is: Born measures are *typical*. (Dürr, Goldstein, and Zanghí 1992a, 1992b).

"Most" just got shudder-quoted because *Sublimation* hasn't given us a reason to privilege the $|\Psi_0|^2$ measure as a gauge of the "size" of sets of initial configurations. Where $\Psi_0^\perp$ is an initial wave function of the universe orthogonal to Ψ_0, $|\Psi_0^\perp|^2$ is *also* a measure over initial configurations, as formally qualified to gauge "most-ness" as $|\Psi_0|^2$. But using the $|\Psi_0^\perp|^2$ measure in lieu of the $|\Psi_0|^2$ measure wreaks havoc with *Sublimation*'s answer to WHY BORN?. *Sublimation* needs a reason to privilege the $|\Psi_0|^2$ measure as distinctively *physically* significant.

Enter *Sublimation*'s analogy with classical "statistical mechanics where the stationary [Liouville] measure plays an important role" (Dürr, Goldstein, and Zanghí 1992b, 8). In statistical mechanics, equilibrium measures are physically privileged because they are stationary: the Gibbs equilibrium measure $\rho = exp(-H/kT)/Z$ (cf fn 10) is invariant under the dynamics (governed by H) with respect to which it is an equilibrium measure. *Sublimation* proposes that in Bohmian mechanics, the $|\Psi_0|^2$ measure enjoys analogous privilege. The analogy is only "rough" (Dürr, Goldstein, and Zanghí 1992b, 8). Although classical equilibrium measures are stationary, in Bohmian mechanics, "ψ will in

[11] Some notation and terminology altered for the sake of continuity with my exposition. Close observers will note that Goldstein calls ψ a "conditional wave function." This is in the course of brief exposition making no mention of the notion of effective wave function. Dürr, Goldstein, and Zanghí notes that the result stated "would not in general be valid for conditional wave functions" (1992a, 867). I read "suitable real world ensembles" to be ensembles of systems whose conditional wave functions are also effective wave functions.

[12] for subtle reasons I'm suppressing. Goldstein 2012 elaborates.

general be time-dependent . . . and we cannot expect the evolution on configuration space to possess a stationary probability distribution" (ibid). We can, however, expect measures to be *equivariant* (ibid.)—an expectation the Born measure $|\psi|^2$ meets with respect to dynamics where ψ plays the role of guiding field. The equivariance of the $|\Psi_0|^2$ measure renders it "the only natural measure available" (Dürr, Goldstein, and Zanghí 1992b, 10), *Sublimation* contends, and permits us to erase the shudder quotes marring *Sublimation*'s answer to WHY BORN.

Evaluation. It may not escape the reader's notice that I haven't exactly made either *Relaxation*'s or *Sublimation*'s answer to WHY BORN? transparent. The approaches are just too intricate, both technically and conceptually, for me to do justice here. I'm not ashamed of this: even a leading architect of *Sublimation* admits that its details are "delicate" and its success "controversial" (Goldstein 2021). Happily, this expository inadequacy needn't impede the agenda of this essay. The current item on the agenda is to ask how well, qua explanations, *Relaxation*'s and *Sublimation*'s answers to WHY BORN? compare to QMs? We don't need to dive too deeply into the details of the approaches to make progress on this agenda item.

Familiar norms of explanation weigh several considerations in favor of QM's answer. First, QM answers WHY BORN? *tout court*: the structure of the algebra of quantum observables, via Gleason's theorem, entails that any quantum system whatsoever has its position measure given by a Born measure. Both *Relaxation* and *Sublimation* reconfigure the explanandum: *Relaxation* explains why coarse-grained position measures are Born measures; *Sublimation* explains why position measures encrypted by effective wave functions are Born measures. And *Relaxation* and *Sublimation* both admit exceptions to the patterns they're explaining: for *Relaxation*, "unlucky" initial conditions can disrupt the relaxation to coarse-grained Born measures; for *Sublimation*, in atypical universes ensembles of systems with effective wave function ψ won't have their configurations distributed, even approximately, according to $|\psi|^2$. If broader, more unifying, and exceptionless answers are more explanatory, QM wins.

Second, whereas QM's answer to WHY BORN? is a rigorous derivation from its central theoretical posit of a structured algebra of quantum observables, neither *Relaxation* nor *Sublimation* operate wholly withinin the register of mathematical demonstration. Both incorporate demonstrations, of course, but each must purchase their relevance with additional currency not backed by a gold mathematical standard. *Relaxation* appeals to numerical simulations to close a logical lacuna between the smooth initial distributions to which its central result applies and the spiky distributions that describe the initial positions of finite swarms. *Sublimation* offers a mathematical demonstration of the result Goldstein distills above—but a demonstration whose relevance is predicated on a variety of accretions to Bohmian mechanics' theoretical core. Collectively, these accretions don't supply the obviously firm traction that mathematical demonstration does.[13] Thus neither *Relaxation* nor

[13] Some slickness I experience: 1. *Sublimation* entertains a *narrower* class of measures than its resolution to focus on *equivariant* measures might lead us to expect. §4.3.2 suggested that the q-measure concentrated on the actual position of our particle satisfies any equivariance demand fair to impose on it. So too does the Q_0-measure concentrated on the actual

Sublimation binds Born measures as tightly or directly to their central theoretical posits as QM does. If systematic rigor is an explanatory virtue, QM wins.

I'm not sure I'd unconditionally endorse any of the antecedents of the consequent "QM wins" in the foregoing paragraphs. For instance, I'm sympathetic to the idea that, notwithstanding its special pleading for favorable initial conditions, the Past Hypothesis does yeoman explanatory work in statistical mechanics (Loewer 2020). Still, I think it's significant that *Relaxation* and *Sublimation* suffer when judged against a variety of formal explanatory desiderata people take seriously. QM's answer to Why Born? does not. If you reject it, it's not because it fails to satisfy whatever formal criteria for explanation you favor. You reject it because you don't accept the algebraic structure of quantum observables that's QM's central theoretical posit. You don't need to! But you do need to recognize the potential cost, that you thereby reject formally adequate explanations mediated by that structure.

I'm not done comparing *Relaxation*'s and *Sublimation*'s responses to Why Born?, considered as explanations, to QM's. Circumstantial evidence for the superiority of QM's answer comes from the sociological fact that neither *Relaxation*'s nor *Sublimation*'s answers command assent, even from occupants of camps drawn to strategies that answer Why Born? by plumbing analogies with equilibrium. (Indeed, criticisms of each approach are perhaps most forcefully expressed by advocates of the other (Dürr and Stuyve 2021, Valentini 2020; Norsen 2018 is an emollient review).) I suspect that their failure to command assent is a symptom of an underlying condition: neither *Relaxation* nor *Sublimation* satisfy an explanatory desideratum that happens to be much celebrated by fans of Bohmian mechanics. Imagine, for the sake of argument, that each approach can quell worries (eg those voiced in fn 13) about its baseline integrity and coherence. I submit that I myself wouldn't praise the ensuing packages, the approaches swathed in the full glory of their multi-tiered protective belts of elaboration and defense, as "low-brow" and "unsubtle" (1992, 169)—just a few notes in a wonderful ode Albert sings to Bohmian mechanics. If simplicity, directness, transparency, and straightfowardness are explanatory virtues—and this is exactly what many adherents of Bohmian mechanics contend in support of the theory—QM wins.

Perhaps I've just disclosed more about the height of my brow than I have about Bohmian mechanics. Still I think that, compared to QM's answer to Why Born?, *Sublimation*'s and *Relaxation*'s suffer with respect to received explanatory virtues frequently invoked to celebrate Bohmian mechanics over its rivals. As with *Stipulation* my preliminary verdict is that *Sublimation*'s and *Relaxation*'s responses to Why Born?, assessed as explanations, compare unfavorably with QM's.

This is far from a conclusive argument against Bohmian mechanics. It's rather an indication that there may be Bohmian stories to be told about quantum probabilities more resourceful and more

initial configuration of the universe. 2. Doesn't positing $|\Psi_0|^2$ (rather than, say, Q_0) as the measure over possible initial configurations just push Why Born? back? 3. *Sublimation* exploits a bevy of resources (a division of the universe into subsystem and remainder, macroscopic registration, decoherence, approximation) Bohmians criticize other approaches for using. Why is this OK?

satisfying—perhaps even more highbrow and more subtle— than those considered here.

5 Explanation

Bohmian antistructuralism impedes it from answering Why Born? as compellingly as QM does. Or so I've argued. Whether my argument succeeds or not, it draws attention to the possibility that, withholding physical significance from the algebraic structure of quantum observables, Bohmian mechanics deprives itself of explanations mediated by those structures. What are the costs of this austerity? I conclude with exceedingly brief accounts of two important quantum phenomena admitting algebraic-structural explanations Bohmians can't, without apology, use. My point isn't that Bohmian mechanics *can't* explain these phenomena. It is rather that, as with answers to Why Born?, Bohmian accounts might suffer, qua explanations, in comparison to structuralist accounts.

The first phenomenon is superselection, wherein certain quantum systems are never found in coherent superpositions across distinct eigenspaces of certain quantum observables, known as superselection observables (see Giulini 2009). Examples of superselection observables include mass in non-relativistic QM and electric, leptonic, and baryonic charge in QFT. Superpositions of states of different mass for our lowly particle are empirically indistinguishable from the corresponding mixture; mutatis mutandis for states of a Dirac field associated with different charges. Mathematically, the states are superpositions; physically, the terms superposed don't interfere. A lovely explanation of superselection begins with the algebra of quantum observables pertaining to a system subject to superselection. The explanation identifies superselection observables with non-trivial observables in the algebra that commute with everybody else. The non-interference that is the empirical signature of superselection follows as a direct mathematical consequence. Resting as it does on the algebraic structure of quantum observables, this explanation of superselection is one Bohmian antistructuralism prevents Bohmian mechanics from adopting.

My choice of the other phenomenon is not innocent of rhetoric. It's the Aharonov-Bohm effect, in which a charged particle traveling outside a solenoid experiences a phase shift that depends on the magnetic flux through the solenoid (see Aharonov and Bohm 1959 and Healey 2007 for more about the effect and the foundational consternation it occasions). The empirical signature of the effect is interference between particle trajectories passing the solenoid on different sides. A lovely explanation of the phenomenon, introduced by Reeh 1988 and developed by Acai 1995, hinges on niceties of the commutation relations between different components $\hat{p}_x$ and $\hat{p}_y$ of the particle's momentum. Configuration space wave functions for the particle live in a Hilbert space of functions on the xy plane from which a circle (the region pierced by the solenoid) has been removed. This mutilation has consequences for how the canonical commutation relations get represented on that Hilbert space. In particular, although $[\hat{p}_x, \hat{p}_y]|\psi\rangle = 0$ for all vectors $|\psi\rangle$ in the common domain of $\hat{p}_x$ and $\hat{p}_y$, $e^{-ia\hat{p}_x}$ and $e^{-ib\hat{p}_y}$—operators heuristically corresponding to position *translations*, for instance along paths around the solenoid—fail to commute. And that failure suffices to derive the interference pattern that's the empirical signature of the effect. Relying as it does on the algebraic structure of quantum observables to which —both the structure and the observables! — Bohmian mechanics denies physical significance, this lovely explanation of the Aharonov-Bohm effect is of no avail to Bohmian mechanics.

I don't believe in an absolute ranking of explanatory virtues, or an algorithm for recognizing when theories instantiate these virtues. Nor do I aspire to legislate the extent to which explanatory virtue, whatever it is, should matter to the all-things-considered evaluation of theories. I do, however, insist that when it comes to mechanics quantum and Bohmian, reasonable people will disagree about these matters—where this disagreement should inform and illuminate disagreement not only about what explanation requires, but also about just how much a theory has to deliver to count as successful physics. I care about WHY BORN? because I think it's an arena where such agreements can be brought down to (something like) earth.

Acknowledgements

For helpful feedback and encouragement, I am grateful to Dave Baker, Gordon Belot, Francisco Calderòn, Eddie Chen, John Earman, Arthur Fine, Doreen Fraser, Alex Meehan, Jeremy Steeger, Noel Swanson, Chip Sebens, David Wallace, and Jingyi Wu.

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